Price Discovery in the U.S. Equity Market at Tick-Time Precision – Supplementary Material

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1 Additional simulation results

Table S.1: Simulation results for $\lambda = 0.01$ and $\delta_2 = -1$, High-resolution VECM estimated at 100-ms intervals

δ_1	IS_1^{tick}	IS_2^{tick}	$\widehat{HIS}_1$	$\widehat{HIS}_2$	$\hat{\rho}$
		$\sigma = 0.0025$			
-1	0.391	0.609	0.500	0.500	0.999
-0.5	0.563	0.426	0.499	0.501	0.994
0	0.606	0.382	0.389	0.612	0.199
0.5	0.606	0.381	0.384	0.616	0.062
1	0.606	0.381	0.384	0.616	0.049
		$\sigma = 0.025$			
-1	0.391	0.609	0.500	0.500	0.999
-0.5	0.569	0.431	0.500	0.500	0.999
0	0.613	0.386	0.481	0.519	0.915
0.5	0.614	0.386	0.388	0.612	0.199
1	0.614	0.386	0.384	0.616	0.069

Note: IS_1^{tick} and IS_2^{tick} denote the structural information share evaluated at the mean duration for market one and two, respectively. $\widehat{HIS}_1$ and $\widehat{HIS}_2$ are the corresponding midpoints between the upper and lower bound of the Hasbrouck information shares estimated from the high-resolution VECM. $\hat{\rho}$ denotes the estimated residual correlation, the main driver for the spread between upper and lower bound.

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Table S.2: Simulation results for $\lambda = 0.1$ and $\delta_2 = -1$, High-resolution VECM estimated at 100-ms intervals

δ_1	IS_1^{tick}	IS_2^{tick}	$\widehat{HIS}_1$	$\widehat{HIS}_2$	$\hat{\rho}$
		$\sigma = 0.0025$			
-1	0.372	0.580	0.500	0.500	0.999
-0.5	0.325	0.400	0.473	0.527	0.878
0	0.278	0.210	0.384	0.616	0.063
0.25	0.270	0.182	0.383	0.617	0.026
0.5	0.268	0.172	0.383	0.617	0.015
1	0.268	0.169	0.383	0.617	0.009
		$\sigma = 0.025$			
-1	0.391	0.609	0.500	0.500	0.999
-0.5	0.447	0.550	0.500	0.500	0.999
0	0.563	0.426	0.465	0.535	0.835
0.25	0.591	0.397	0.424	0.576	0.592
0.5	0.601	0.386	0.397	0.603	0.348
1	0.606	0.382	0.384	0.616	0.108

Note: IS_1^{tick} and IS_2^{tick} denote the structural information share evaluated at the mean duration for market one and two, respectively. $\widehat{HIS}_1$ and $\widehat{HIS}_2$ are the corresponding midpoints between the upper and lower bound of the Hasbrouck information shares estimated from the high-resolution VECM. $\hat{\rho}$ denotes the estimated residual correlation, the main driver for the spread between upper and lower bound.

Table S.3: Simulation results for $\lambda = 1$ and $\delta_2 = -1$, High-resolution VECM estimated at 100-ms intervals

δ_1	IS_1^{tick}	IS_2^{tick}	$\widehat{HIS}_1$	$\widehat{HIS}_2$	$\hat{\rho}$
		$\sigma = 0.0025$			
-1	0.065	0.100	0.500	0.500	0.999
-0.5	0.065	0.100	0.400	0.600	0.353
0	0.065	0.100	0.384	0.616	0.046
0.25	0.065	0.100	0.384	0.616	0.041
0.5	0.065	0.100	0.384	0.616	0.046
1	0.065	0.100	0.385	0.615	0.087
		$\sigma = 0.025$			
-1	0.372	0.580	0.500	0.500	0.999
-0.5	0.372	0.580	0.496	0.504	0.980
0	0.372	0.580	0.462	0.538	0.822
0.25	0.372	0.580	0.458	0.542	0.803
0.5	0.372	0.580	0.462	0.538	0.821
1	0.372	0.580	0.478	0.522	0.902

Note: IS_1^{tick} and IS_2^{tick} denote the structural information share evaluated at the mean duration for market one and two, respectively. $\widehat{HIS}_1$ and $\widehat{HIS}_2$ are the corresponding midpoints between the upper and lower bound of the Hasbrouck information shares estimated from the high-resolution VECM. $\hat{\rho}$ denotes the estimated residual correlation, the main driver for the spread between upper and lower bound.

2 Calendar time measures

After obtaining the tick time measures, it is possible to aggregate them to fixed calendar time intervals. To do so, we define the closest observation preceding time t with $j(t)$. In a fixed interval $[t-1,t]$, we then have $j(t)-j(t-1)$ quote updates. The dealer/market liquidity measure is given by

$$\beta(t) = \frac{\tilde{\sigma}^2(t)}{\sigma^2(t)}\iota + \frac{\sigma^2_{j(t)}}{\sigma^2(t)}\beta_{j(t)}, \tag{1}$$

where $\sigma^2(t) = \sum_{i=j(t-1)-1}^{j(t)} \sigma_i^2$ and $\tilde{\sigma}^2(t) = \sigma^2(t) - \sigma^2_{j(t)}$. The quote impact on the efficient price change measured in calendar time is

$$\gamma(t) = \Sigma^{-1}(t)\beta(t)\sigma^2(t), \tag{2}$$

where $\Sigma(t) = \tilde{\sigma}^2(t)\iota\iota' + \sigma^2_{j(t)}\beta_{j(t)}\beta'_{j(t)} + \Omega$ and finally, the information shares in calendar time are defined as

$$IS(t) = \beta(t) \circ \gamma(t). \tag{3}$$

For a given number of ticks observed over a trading day and a chosen fixed interval, we obtain a sample of values for each measure. We can either analyze the distributions graphically or compute sample characteristics like the mean or median.

3 Quote dynamics model: Kalman filter algorithm

The state space model can be represented by the measurement equation and the transition equation of a linear Gaussian model. Let d be the dimension of the observations, m be the dimension of the state variable, and T the number of observations. The measurement equation and the transition equation are given by

$$\begin{aligned} y_t &= c_t + Z_t\alpha_t + G_t\epsilon_t \\ \alpha_{t+1} &= d_t + T_t\alpha_t + H_t\eta_t, \end{aligned}$$

where ϵ_t and η_t are iid $N(0, I_d)$ and iid $N(0, I_m)$, respectively, and α_t denotes the state variable. The parameters admit the following dimensions:

$$\begin{aligned} &y_t, c_t, \epsilon_t \in \mathbb{R}^d, \qquad Z_t \in \mathbb{R}^{d\times m}, \qquad G_t \in \mathbb{R}^{d\times d}, \\ &\alpha_t, d_t, \eta_t \in \mathbb{R}^m, \qquad T_t, H_t \in \mathbb{R}^{m\times m}. \end{aligned}$$

We first define the expected value,

$$\begin{aligned} a_t &= E[\alpha_t | y_1, \ldots, y_{t-1}] \\ a_{t|t} &= E[\alpha_t | y_1, \ldots, y_t], \end{aligned}$$

and the corresponding variances,

$$\begin{aligned} P_t &= Var[\alpha_t | y_1, \ldots, y_{t-1}] \\ P_{t|t} &= Var[\alpha_t | y_1, \ldots, y_t], \end{aligned}$$

of the state α_t. In $t = 1$, we set $a_t = a_0$ and $P_t = P_0$. The updating equations are given by

$$\begin{aligned} v_t &= y_t - c_t - Z_t a_t \\ F_t &= Z_t P_t Z_t' + G_t G_t' \\ K_t &= P_t Z_t' F_t^{-1} \\ a_{t|t} &= a_t + K_t v_t \\ P_{t|t} &= P_t - P_t Z_t' K_t' \end{aligned}$$

and the prediction equations by

$$\begin{aligned} a_{t+1} &= d_t + T_t a_{t|t} \\ P_{t+1} &= T_t P_{t|t} T_t' + H_t H_t'. \end{aligned}$$

For the quote dynamics model, we obtain the following definitions

$$\begin{aligned} c_t = c, \qquad Z_t = (\iota, \alpha \tau_j^{\delta_2} \sigma), \qquad G_t G_t' = \Omega \\ d_t = 0, \qquad T_t = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad H_t = \begin{pmatrix} 0 & \sigma \tau_j^{\delta_1} \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

4 Additional empirical results

Symbol	**Company**	**Primary**	**Industry**	**Weight**
AAPL	Apple	NASDAQ	Information Technology	3.33%
AMGN	Amgen	NASDAQ	Biopharmaceutical	3.76%
AMZN	Amazon	NASDAQ	Retailing	3.02%
AXP	American Express	NYSE	Financial Services	4.12%
BA	Boeing	NYSE	Aerospace & Defense	2.15%

Symbol	Company	Primary	Industry	Weight
CAT	Caterpillar	NYSE	Construction & Mining	5.41%
CRM	Salesforce	NYSE	Information Technology	4.95%
CSCO	Cisco	NASDAQ	Information Technology	0.82%
CVX	Chevron	NYSE	Petroleum Industry	2.18%
DIS	Disney	NYSE	Entertainment	1.60%
GS	Goldman Sachs	NYSE	Financial Services	8.18%
HD	Home Depot	NYSE	Home Improvement	5.84%
HON	Honeywell	NASDAQ	Conglomerate	3.10%
IBM	IBM	NYSE	Information Technology	3.22 %
JNJ	Johnson & Johnson	NYSE	Pharmaceutical Industry	2.05%
JPM	JPMorgan Chase	NYSE	Financial Services	3.36%
KO	Coca-Cola	NYSE	Drink industry	0.86%
MCD	McDonald's	NYSE	Restaurant	4.10%
MMM	3M	NYSE	Conglomerate	1.83%
MRK	Merck	NYSE	Pharmaceutical Industry	1.42%
MSFT	Microsoft	NASDAQ	Information Technology	6.06%
NKE	Nike, Inc.	NYSE	Clothing industry	1.08%
NVDA	Nvidia	NASDAQ	Information technology	1.99%
PG	Procter & Gamble	NYSE	Consumer Goods	2.41%
SHW	Sherwin-Williams	NYSE	Speciality chemicals	5.28%
TRV	Travelers	NYSE	Insurance	3.61%
UNH	UnitedHealth Group	NYSE	Healthcare	7.93%
V	Visa	NYSE	Financial Services	4.23%
VZ	Verizon	NYSE	Telecommunications industry	0.58%
WMT	Walmart	NYSE	Retailing	1.31%

Figure S.1: Daily information shares of NYSE for IBM stock at 10-ms precision

This figure depicts the daily information shares of the primary listing exchange for IBM (NYSE) at the average duration. The information shares are estimated for trading days spanning from March 1, 2024, to May 31, 2024. The vertical bars indicate the 95% confidence interval.

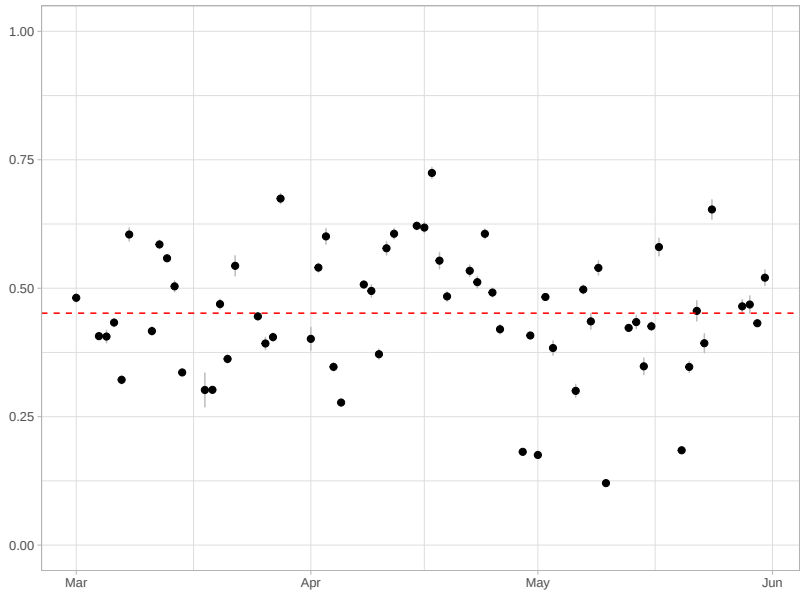

Table S.5: DJIA stocks, average number of ticks per trading day

	1-ns	10-ms	100-ms
AAPL	480776	228914	111629
AMGN	51266	35308	24664
AMZN	572014	264556	122361
AXP	112429	70857	45511
BA	92764	58077	38815
CAT	69538	46165	31768
CRM	110102	69879	45369
CSCO	34812	21538	16926
CVX	200268	115360	68681
DIS	130933	69846	44872
GS	50775	36018	26551
HD	103695	66187	44212
HON	72309	46760	33075
IBM	83230	51747	35454
JNJ	99152	62091	42389
JPM	175692	108352	66062
KO	31857	19342	15381
MCD	64384	41942	29535
MMM	69079	44090	30609
MRK	92852	58303	39349
MSFT	408613	222352	113859
NKE	114978	62352	40133
NVDA	430356	274405	134627
PG	113155	70722	47265
SHW	29353	21942	16979
TRV	33226	25038	19102
UNH	69162	53499	37316
V	148222	92655	57347
VZ	23154	12582	10262
WMT	42560	25667	19626

This table reports the average number of ticks per trading day at 1-nanosecond, 10-milliseconds, and 100-milliseconds precision.

Table S.6: Relative frequency of simultaneous quote changes in the DJIA panel

	100-ms					10-ms				
	single	2	3	4	all	single	2	3	4	all
AAPL	0.328	0.205	0.156	0.130	0.181	0.532	0.215	0.120	0.075	0.059
AMGN	0.693	0.176	0.071	0.040	0.020	0.787	0.134	0.048	0.023	0.008
AMZN	0.306	0.196	0.156	0.144	0.197	0.522	0.212	0.123	0.082	0.062
AXP	0.595	0.208	0.098	0.056	0.044	0.723	0.165	0.064	0.031	0.017
BA	0.599	0.204	0.093	0.060	0.044	0.710	0.172	0.066	0.034	0.018
CAT	0.637	0.210	0.083	0.049	0.020	0.746	0.164	0.055	0.027	0.008
CRM	0.609	0.206	0.086	0.052	0.047	0.734	0.159	0.056	0.031	0.020
CSCO	0.621	0.164	0.072	0.044	0.098	0.699	0.142	0.057	0.041	0.062
CVX	0.494	0.221	0.123	0.081	0.082	0.653	0.190	0.084	0.045	0.028
DIS	0.519	0.190	0.108	0.082	0.101	0.631	0.177	0.091	0.058	0.044
GS	0.650	0.220	0.081	0.036	0.013	0.758	0.174	0.048	0.016	0.004
HD	0.577	0.233	0.100	0.053	0.037	0.707	0.185	0.064	0.029	0.015
HON	0.649	0.191	0.082	0.049	0.028	0.751	0.152	0.057	0.028	0.012
IBM	0.600	0.204	0.093	0.062	0.040	0.709	0.171	0.067	0.036	0.017
JNJ	0.583	0.202	0.095	0.061	0.059	0.704	0.167	0.068	0.038	0.024
JPM	0.544	0.218	0.106	0.065	0.067	0.701	0.171	0.067	0.036	0.024
KO	0.620	0.152	0.073	0.048	0.109	0.695	0.134	0.064	0.046	0.062
MCD	0.635	0.195	0.079	0.048	0.043	0.734	0.159	0.058	0.031	0.018
MMM	0.628	0.180	0.082	0.058	0.051	0.730	0.151	0.062	0.036	0.021
MRK	0.597	0.197	0.088	0.056	0.062	0.713	0.162	0.063	0.036	0.026
MSFT	0.443	0.241	0.140	0.090	0.086	0.655	0.190	0.081	0.045	0.029
NKE	0.490	0.198	0.121	0.085	0.106	0.612	0.190	0.096	0.058	0.044
NVDA	0.523	0.282	0.110	0.052	0.032	0.743	0.177	0.050	0.021	0.009
PG	0.575	0.202	0.100	0.063	0.061	0.699	0.170	0.071	0.038	0.022
SHW	0.701	0.198	0.062	0.029	0.010	0.795	0.150	0.038	0.014	0.004
TRV	0.687	0.198	0.068	0.032	0.015	0.771	0.159	0.046	0.018	0.006
UNH	0.742	0.166	0.056	0.026	0.011	0.842	0.112	0.030	0.012	0.004
V	0.547	0.229	0.113	0.064	0.046	0.696	0.184	0.070	0.033	0.017
VZ	0.586	0.145	0.074	0.053	0.142	0.644	0.136	0.071	0.055	0.093
WMT	0.618	0.162	0.073	0.051	0.096	0.700	0.142	0.064	0.046	0.048

This table reports the average frequency of markets simultaneously changing their quotes at one 100-ms tick.

Table S.7: DJIA stocks: average duration and estimated quote dynamics parameters at 10-ms precision

	Avg. duration					Est. parameters		
	N	T	K	P	Z	σ	δ_1	δ_2
AAPL	0.057	0.032	0.039	0.051	0.035	0.077	0.586	2.412
AMGN	2.519	0.867	0.791	1.429	0.630	0.002	0.046	−0.538
AMZN	0.077	0.042	0.066	0.068	0.054	0.069	0.606	1.971
AXP	0.319	0.695	0.462	0.911	0.336	0.059	0.744	−0.374
BA	0.284	0.642	0.471	0.927	0.291	0.005	0.088	−0.548
CAT	0.489	1.122	0.813	0.756	0.470	0.011	0.111	−0.513
CRM	0.363	0.644	0.398	0.592	0.268	0.025	0.195	−0.606
CSCO	0.249	0.160	0.342	0.155	0.292	0.011	0.326	0.775
CVX	0.125	0.297	0.195	0.327	0.161	0.019	0.408	−0.286
DIS	0.274	0.292	0.288	0.320	0.205	0.011	0.240	0.029
GS	0.647	1.562	0.839	2.011	0.501	0.018	0.172	−0.292
HD	0.384	1.060	0.394	1.007	0.263	0.029	0.312	−0.548
HON	1.594	0.367	0.610	1.409	0.442	0.023	0.241	−0.367
IBM	0.328	0.704	0.584	1.073	0.348	0.008	0.232	−0.433
JNJ	0.240	0.422	0.447	0.621	0.259	0.009	0.255	−0.273
JPM	0.153	0.308	0.244	0.467	0.146	0.015	0.307	−0.440
KO	0.165	0.231	0.341	0.181	0.281	0.012	0.349	0.681
MCD	0.452	0.981	0.667	1.267	0.462	0.009	0.114	−0.522
MMM	0.400	0.697	0.744	1.059	0.407	0.002	0.045	−0.607
MRK	0.219	0.547	0.485	0.574	0.309	0.007	0.218	−0.410
MSFT	0.186	0.071	0.117	0.202	0.103	0.077	0.677	−0.179
NKE	0.256	0.301	0.354	0.382	0.243	0.011	0.256	0.338
NVDA	0.289	0.166	0.055	0.369	0.046	0.179	0.458	−0.062
PG	0.284	0.452	0.380	0.573	0.251	0.005	0.168	−0.287
SHW	1.172	2.835	1.903	2.543	0.973	0.005	0.089	−0.328
TRV	0.858	2.697	1.804	3.195	0.802	0.042	0.277	−0.390
UNH	0.372	1.411	0.775	1.952	0.409	0.065	0.407	−0.283
V	0.199	0.498	0.383	0.618	0.244	0.016	0.208	−0.450
VZ	0.212	0.281	0.416	0.206	0.336	0.022	0.402	0.638
WMT	0.222	0.262	0.385	0.259	0.299	0.012	0.323	0.731

The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.8: DJIA stocks: average duration and estimated quote dynamics parameters at 1-ns precision

	Avg. duration					Est. parameters		
	N	T	K	P	Z	σ	δ_1	δ_2
AAPL	0.057	0.032	0.039	0.051	0.035	0.111	0.494	2.721
AMGN	2.519	0.867	0.791	1.429	0.63	0.031	0.17	−0.207
AMZN	0.077	0.042	0.066	0.068	0.054	0.077	0.473	2.397
AXP	0.319	0.695	0.462	0.911	0.336	0.095	0.908	−0.13
BA	0.284	0.642	0.471	0.927	0.291	0.032	0.337	−0.126
CAT	0.489	1.122	0.813	0.756	0.47	0.041	0.215	−0.235
CRM	0.363	0.644	0.398	0.592	0.268	0.03	0.156	−0.209
CSCO	0.249	0.16	0.342	0.155	0.292	0.022	0.308	1.132
CVX	0.125	0.297	0.195	0.327	0.161	0.02	0.242	1.158
DIS	0.274	0.292	0.288	0.32	0.205	0.02	0.224	0.721
GS	0.647	1.562	0.839	2.011	0.501	0.092	0.341	−0.204
HD	0.384	1.06	0.394	1.007	0.263	0.045	0.324	−0.206
HON	1.594	0.367	0.61	1.409	0.442	0.008	0.291	−0.116
IBM	0.328	0.704	0.584	1.073	0.348	0.03	0.387	−0.058
JNJ	0.24	0.422	0.447	0.621	0.259	0.011	0.075	0.906
JPM	0.153	0.308	0.244	0.467	0.146	0.031	0.236	1.109
KO	0.165	0.231	0.341	0.181	0.281	0.034	0.406	0.826
MCD	0.452	0.981	0.667	1.267	0.462	0.009	0.131	−0.214
MMM	0.4	0.697	0.744	1.059	0.407	0.002	0.071	0.025
MRK	0.219	0.547	0.485	0.574	0.309	0.006	0.047	0.483
MSFT	0.186	0.071	0.117	0.202	0.103	0.096	0.556	0.689
NKE	0.256	0.301	0.354	0.382	0.243	0.011	0.194	0.755
NVDA	0.289	0.166	0.055	0.369	0.046	0.387	0.587	−0.211
PG	0.284	0.452	0.38	0.573	0.251	0.011	0.101	0.758
SHW	1.172	2.835	1.903	2.543	0.973	0.037	0.152	−0.195
TRV	0.858	2.697	1.804	3.195	0.802	0.012	0.122	−0.272
UNH	0.372	1.411	0.775	1.952	0.409	0.169	0.689	−0.225
V	0.199	0.498	0.383	0.618	0.244	0.059	0.605	−0.184
VZ	0.212	0.281	0.416	0.206	0.336	0.031	0.329	0.811
WMT	0.222	0.262	0.385	0.259	0.299	0.013	0.316	0.955

The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.9: DJIA stocks: estimated information shares and R^2 (1-ns precision)

	N	T	K	P	Z	R^2
AAPL	0.03	0.076	0.071	0.086	0.062	0.325
AMGN	0.035	0.146	0.155	0.113	0.037	0.485
AMZN	0.052	0.078	0.074	0.061	0.059	0.324
AXP	0.187	0.052	0.134	0.084	0.136	0.592
BA	0.297	0.069	0.068	0.132	0.053	0.619
CAT	0.278	0.05	0.092	0.08	0.019	0.519
CRM	0.254	0.077	0.072	0.072	0.042	0.517
CSCO	0.077	0.09	0.094	0.086	0.086	0.435
CVX	0.166	0.05	0.041	0.044	0.034	0.335
DIS	0.179	0.092	0.062	0.074	0.038	0.446
GS	0.14	0.135	0.044	0.147	0.056	0.523
HD	0.164	0.051	0.086	0.125	0.048	0.475
HON	0.064	0.258	0.104	0.096	0.076	0.598
IBM	0.291	0.059	0.097	0.069	0.065	0.581
JNJ	0.176	0.1	0.064	0.063	0.031	0.433
JPM	0.156	0.049	0.045	0.066	0.031	0.348
KO	0.1	0.076	0.091	0.086	0.078	0.43
MCD	0.27	0.04	0.079	0.08	0.028	0.498
MMM	0.356	0.051	0.063	0.054	0.071	0.596
MRK	0.272	0.042	0.062	0.054	0.048	0.479
MSFT	0.143	0.136	0.062	0.108	0.099	0.548
NKE	0.141	0.103	0.076	0.107	0.064	0.491
NVDA	0.037	0.084	0.123	0.339	0.195	0.779
PG	0.206	0.051	0.045	0.054	0.047	0.403
SHW	0.284	0.096	0.047	0.056	0.05	0.533
TRV	0.221	0.054	0.018	0.111	0.027	0.432
UNH	0.345	0.049	0.051	0.105	0.066	0.617
V	0.24	0.045	0.144	0.076	0.079	0.585
VZ	0.101	0.086	0.094	0.085	0.085	0.452
WMT	0.117	0.091	0.082	0.084	0.062	0.435

This table reports the average daily information shares at the median duration along with the total fraction of the variance of the efficient price change that can be attributed to quote innovations (sum over all five trading venues and denoted as R^2). The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.10: DJIA stocks: estimated Hasbrouck information shares and R^2 (100-ms intervals)

	N	T	K	P	Z	R^2
AAPL	0.159	0.217	0.207	0.21	0.206	1
AMGN	0.125	0.266	0.201	0.212	0.196	1
AMZN	0.175	0.219	0.205	0.201	0.2	1
AXP	0.213	0.213	0.188	0.199	0.187	1
BA	0.277	0.199	0.181	0.172	0.171	1
CAT	0.245	0.209	0.191	0.185	0.17	1
CRM	0.25	0.205	0.182	0.186	0.178	1
CSCO	0.192	0.187	0.199	0.199	0.223	1
CVX	0.25	0.197	0.186	0.185	0.183	1
DIS	0.219	0.204	0.196	0.192	0.188	1
GS	0.216	0.241	0.169	0.202	0.173	1
HD	0.25	0.185	0.175	0.208	0.182	1
HON	0.118	0.29	0.191	0.203	0.198	1
IBM	0.259	0.206	0.185	0.178	0.172	1
JNJ	0.244	0.201	0.188	0.185	0.182	1
JPM	0.236	0.194	0.186	0.198	0.185	1
KO	0.191	0.186	0.197	0.196	0.23	1
MCD	0.271	0.195	0.176	0.183	0.174	1
MMM	0.249	0.194	0.193	0.185	0.179	1
MRK	0.262	0.185	0.187	0.185	0.181	1
MSFT	0.133	0.264	0.204	0.2	0.198	1
NKE	0.22	0.201	0.193	0.195	0.192	1
NVDA	0.13	0.246	0.186	0.215	0.223	1
PG	0.259	0.192	0.184	0.184	0.181	1
SHW	0.256	0.24	0.169	0.173	0.163	1
TRV	0.246	0.211	0.163	0.208	0.171	1
UNH	0.26	0.226	0.167	0.181	0.166	1
V	0.289	0.191	0.174	0.176	0.17	1
VZ	0.199	0.185	0.2	0.196	0.219	1
WMT	0.21	0.196	0.201	0.195	0.198	1

This table reports the average daily Hasbrouck information shares midpoints along with the total fraction of the variance of the efficient price change that can be attributed to price innovations. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.11: DJIA stocks: estimated Hasbrouck information shares and R^2 (10-ms intervals)

	N	T	K	P	Z	R^2
AAPL	0.161	0.219	0.202	0.218	0.2	1
AMGN	0.137	0.265	0.201	0.209	0.189	1
AMZN	0.167	0.228	0.207	0.199	0.198	1
AXP	0.211	0.215	0.19	0.199	0.186	1
BA	0.277	0.205	0.18	0.17	0.168	1
CAT	0.243	0.215	0.193	0.183	0.166	1
CRM	0.248	0.206	0.181	0.189	0.176	1
CSCO	0.188	0.181	0.21	0.197	0.223	1
CVX	0.244	0.198	0.187	0.187	0.184	1
DIS	0.223	0.205	0.197	0.191	0.185	1
GS	0.212	0.243	0.17	0.203	0.171	1
HD	0.245	0.191	0.176	0.211	0.178	1
HON	0.131	0.285	0.188	0.203	0.193	1
IBM	0.255	0.209	0.188	0.179	0.169	1
JNJ	0.247	0.204	0.191	0.181	0.177	1
JPM	0.235	0.194	0.19	0.199	0.183	1
KO	0.176	0.192	0.195	0.203	0.233	1
MCD	0.258	0.203	0.178	0.187	0.174	1
MMM	0.239	0.197	0.199	0.19	0.176	1
MRK	0.243	0.191	0.194	0.188	0.184	1
MSFT	0.141	0.265	0.202	0.201	0.191	1
NKE	0.218	0.196	0.194	0.203	0.189	1
NVDA	0.143	0.245	0.181	0.209	0.222	1
PG	0.258	0.195	0.186	0.183	0.178	1
SHW	0.255	0.239	0.17	0.173	0.163	1
TRV	0.244	0.213	0.163	0.21	0.17	1
UNH	0.258	0.226	0.168	0.182	0.166	1
V	0.28	0.197	0.176	0.178	0.168	1
VZ	0.206	0.18	0.206	0.191	0.216	1
WMT	0.213	0.192	0.209	0.192	0.194	1

This table reports the average daily Hasbrouck information shares midpoints along with the total fraction of the variance of the efficient price change that can be attributed to price innovations. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.12: DJIA stocks: estimated asymmetry parameters (100-ms)

	N		T		K		P		Z	
AAPL	0.008	-0.001	0.01	0.004	0.008	0.002	0.008	0.002	0.011	0
AMGN	-0.805	0.727	-0.298	0.257	-0.527	0.469	-0.343	0.342	-0.561	0.353
AMZN	0	0.005	0.006	0.005	0.002	0.005	0.003	0.007	0	0.006
AXP	-0.229	0.197	-0.249	0.213	-0.389	0.353	-0.248	0.216	-0.33	0.321
BA	-0.314	0.342	-0.478	0.496	-0.533	0.581	-0.607	0.59	-0.608	0.567
CAT	-0.237	0.221	-0.288	0.269	-0.312	0.322	-0.341	0.272	-0.542	0.421
CRM	-0.19	0.251	-0.253	0.287	-0.341	0.337	-0.318	0.3	-0.416	0.39
CSCO	0.019	0.004	0.015	0.014	0.015	0.006	0.015	0.004	0.013	0.005
CVX	-0.059	0.098	-0.095	0.154	-0.133	0.175	-0.119	0.177	-0.136	0.2
DIS	-0.234	0.27	-0.254	0.304	-0.217	0.335	-0.299	0.329	-0.453	0.529
GS	-0.368	0.294	-0.559	0.222	-1.176	0.69	-0.469	0.306	-1.76	1.319
HD	-0.219	0.255	-0.289	0.335	-0.405	0.424	-0.225	0.257	-0.234	0.304
HON	-1.541	1.562	-0.57	0.452	-1.085	0.799	-0.787	0.6	-1.059	0.808
IBM	-0.297	0.353	-0.327	0.416	-0.535	0.629	-0.421	0.505	-0.771	0.848
JNJ	-0.155	0.19	-0.165	0.194	-0.258	0.3	-0.228	0.274	-0.262	0.313
JPM	-0.043	0.067	-0.055	0.08	-0.081	0.103	-0.064	0.075	-0.081	0.1
KO	0.018	0.004	0.016	0.004	0.02	0.003	0.014	0.002	0.027	0.004
MCD	-0.509	0.49	-0.615	0.686	-0.819	0.989	-0.728	0.665	-0.889	1.008
MMM	-0.431	0.49	-0.479	0.591	-0.51	0.717	-0.555	0.635	-0.97	1.115
MRK	-0.206	0.26	-0.31	0.387	-0.346	0.412	-0.281	0.363	-0.34	0.523
MSFT	-0.127	0.132	-0.044	0.044	-0.083	0.088	-0.081	0.085	-0.083	0.085
NKE	0.042	-0.032	0.052	-0.041	0.059	-0.059	0.061	-0.06	0.052	-0.045
NVDA	-0.172	0.169	-0.046	0.042	-0.075	0.091	-0.055	0.057	-0.056	0.064
PG	-0.143	0.188	-0.252	0.339	-0.296	0.425	-0.267	0.35	-0.4	0.569
SHW	-0.343	0.405	-0.474	0.478	-1.005	0.959	-0.719	0.677	-1.111	1.529
TRV	-0.857	0.752	-0.958	0.964	-2.364	2.125	-0.946	0.853	-1.607	1.507
UNH	-0.144	0.151	-0.241	0.246	-0.571	0.554	-0.262	0.3	-1.01	0.92
V	-0.191	0.282	-0.372	0.473	-0.51	0.582	-0.447	0.498	-0.468	0.657
VZ	0.017	0.003	0.013	0.004	0.017	0.002	0.014	0.003	0.018	0
WMT	0.023	0.004	0.018	0.005	0.031	0.003	0.02	-0.003	0.027	-0.005

This table reports the average daily asymmetry parameters. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.13: DJIA stocks: estimated asymmetry parameters (10-ms)

	N		T		K		P		Z	
AAPL	0.003	0.004	0.009	0.016	0.017	0.007	0.016	0.004	0.012	0.017
AMGN	-0.745	0.767	-0.253	0.232	-0.502	0.563	-0.3	0.366	-0.368	0.475
AMZN	-0.007	0.023	0.005	0.019	0	0.02	-0.006	0.024	-0.003	0.033
AXP	-0.181	0.155	-0.208	0.19	-0.304	0.335	-0.215	0.21	-0.327	0.33
BA	-0.221	0.259	-0.409	0.381	-0.367	0.39	-0.348	0.481	-0.481	0.445
CAT	-0.256	0.297	-0.344	0.309	-0.519	0.438	-0.321	0.315	-0.556	0.48
CRM	-0.141	0.167	-0.145	0.193	-0.23	0.243	-0.184	0.215	-0.22	0.259
CSCO	0.027	0.004	0.02	0.016	0.02	0.014	0.018	0.016	0.017	0.01
CVX	-0.09	0.103	-0.122	0.171	-0.189	0.219	-0.187	0.227	-0.209	0.294
DIS	-0.278	0.297	-0.269	0.276	-0.263	0.32	-0.336	0.358	-0.495	0.519
GS	-0.324	0.092	-0.211	0.166	-0.61	0.649	-0.257	0.196	-0.171	1.558
HD	-0.154	0.162	-0.188	0.203	-0.3	0.291	-0.142	0.17	-0.214	0.23
HON	-1.466	1.57	-0.407	0.372	-0.983	0.978	-0.676	0.57	-0.906	0.702
IBM	-0.202	0.222	-0.219	0.371	-0.41	0.545	-0.304	0.379	-0.66	0.647
JNJ	-0.136	0.167	-0.154	0.175	-0.243	0.272	-0.191	0.226	-0.246	0.292
JPM	-0.12	0.115	-0.107	0.115	-0.189	0.163	-0.129	0.138	-0.219	0.215
KO	0.001	0.019	0.001	0.02	0.001	0.022	0.003	0.019	0.001	0.026
MCD	-0.205	0.227	-0.289	0.35	-0.393	0.439	-0.307	0.301	-0.427	0.422
MMM	-0.245	0.295	-0.286	0.361	-0.328	0.451	-0.346	0.437	-0.583	0.663
MRK	-0.137	0.155	-0.236	0.268	-0.305	0.315	-0.24	0.292	-0.309	0.377
MSFT	-0.16	0.26	-0.035	0.038	-0.163	0.098	-0.178	0.167	-0.12	0.153
NKE	-0.03	0.07	-0.058	0.07	-0.086	0.128	-0.058	0.088	-0.107	0.131
NVDA	-0.285	0.227	-0.058	0.076	-0.096	0.154	-0.123	0.117	-0.246	0.043
PG	-0.084	0.124	-0.141	0.184	-0.2	0.274	-0.172	0.235	-0.245	0.371
SHW	-0.212	0.372	-0.507	0.512	-0.976	1.123	-0.598	0.807	-0.738	2.138
TRV	-0.164	0.24	-0.322	0.339	-0.605	0.975	-0.419	0.503	-0.79	0.458
UNH	-0.064	0.073	-0.122	0.281	-0.267	0.423	-0.115	0.268	-0.416	0.297
V	-0.291	0.422	-0.517	0.736	-0.827	0.949	-0.717	0.851	-0.74	1.089
VZ	0.02	0.01	0.019	0.022	0.018	0.013	0.02	0.014	0.019	0.017
WMT	0.027	0.013	0.017	0.019	0.019	0.013	0.02	0.019	0.018	0.019

This table reports the average daily asymmetry parameters. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table S.14: DJIA stocks, Estimated information shares and R^2 (100-ms precision, robustness check, tick time event if both bid and ask are updated)

	N	T	K	P	Z	R^2
AAPL	0.059	0.243	0.19	0.278	0.172	0.942
AMGN	0.057	0.379	0.115	0.163	0.099	0.813
AMZN	0.113	0.24	0.211	0.18	0.181	0.924
AXP	0.231	0.173	0.139	0.18	0.113	0.836
BA	0.423	0.143	0.126	0.071	0.092	0.856
CAT	0.353	0.187	0.13	0.104	0.034	0.808
CRM	0.273	0.159	0.153	0.167	0.104	0.855
CSCO	0.158	0.193	0.304	0.193	0.119	0.967
CVX	0.299	0.167	0.152	0.147	0.142	0.907
DIS	0.293	0.207	0.183	0.152	0.086	0.921
GS	0.181	0.197	0.069	0.181	0.084	0.712
HD	0.252	0.102	0.122	0.257	0.096	0.829
HON	0.061	0.452	0.095	0.145	0.118	0.871
IBM	0.404	0.178	0.147	0.095	0.045	0.869
JNJ	0.305	0.224	0.155	0.136	0.096	0.917
JPM	0.267	0.137	0.142	0.196	0.141	0.883
KO	0.353	0.137	0.259	0.139	0.082	0.97
MCD	0.4	0.121	0.118	0.154	0.071	0.864
MMM	0.401	0.165	0.172	0.098	0.068	0.904
MRK	0.382	0.12	0.136	0.171	0.09	0.9
MSFT	0.089	0.309	0.167	0.17	0.141	0.875
NKE	0.257	0.208	0.136	0.217	0.131	0.949
NVDA	0.036	0.111	0.061	0.166	0.473	0.847
PG	0.379	0.148	0.131	0.145	0.095	0.898
SHW	0.302	0.191	0.067	0.078	0.13	0.768
TRV	0.347	0.141	0.072	0.191	0.073	0.824
UNH	0.381	0.136	0.075	0.117	0.074	0.782
V	0.376	0.126	0.132	0.131	0.072	0.836
VZ	0.339	0.139	0.243	0.155	0.089	0.964
WMT	0.34	0.221	0.193	0.126	0.078	0.958

This table reports the average daily information shares at the median duration along with the total fraction of the variance of the efficient price change that can be attributed to quote innovations (sum over all five trading venues and denoted as R^2). The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Figure S.2: Dealer/market liquidity for durations ranging from 10 milliseconds to 5 seconds
This figure depicts $\beta(\tau)$ for IBM: NYSE (green), Nasdaq (gray), Arca (orange), Cboe EDGX (lightblue), and Cboe BZX (yellow) estimated for the full sampling period (March 1, 2024, to May 31, 2024) using data at 10-milliseconds, 100-milliseconds, and 1-second precision.

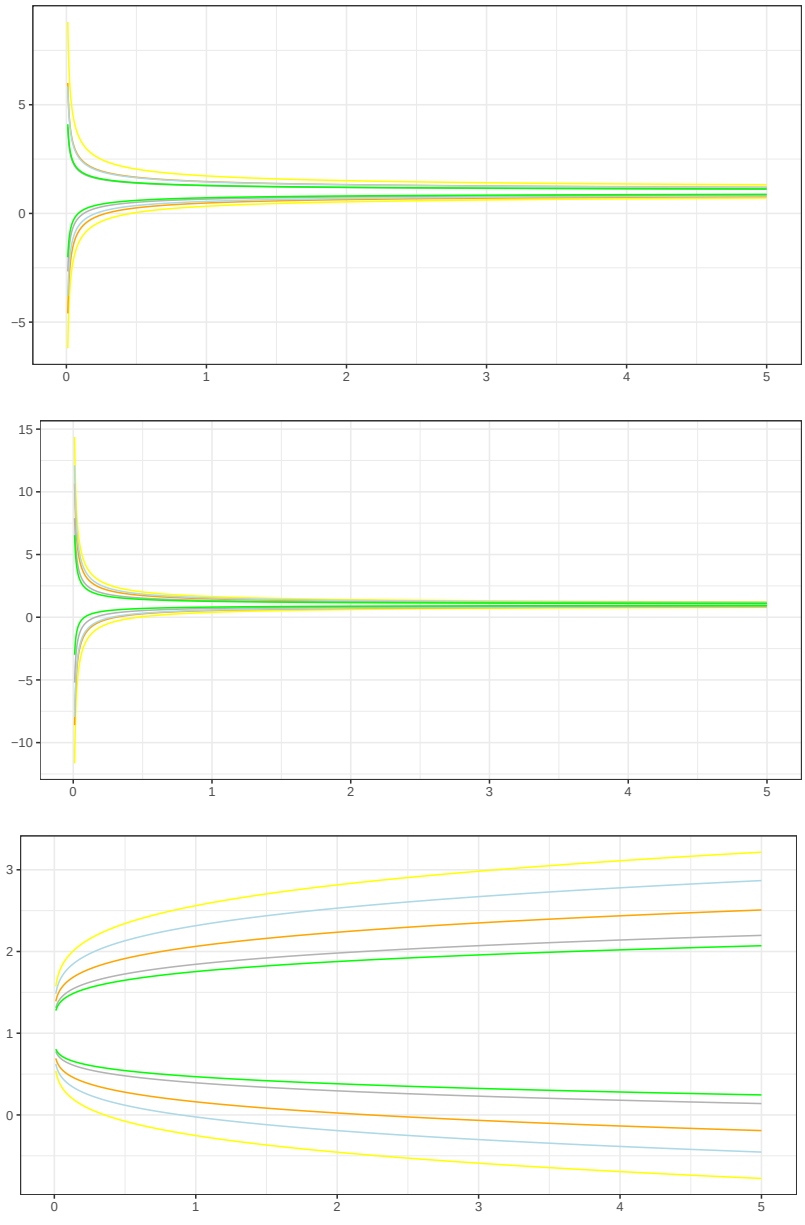

Table S.15: Panel analysis: tick-time information shares, liquidity measures and market characteristics

	tick-time IS		
	1-ns	10-ms	100-ms
avgDuration	−0.021**	−0.033***	−0.022*
	(0.008)	(0.008)	(0.012)
relTradevol	0.265***	0.414***	0.516***
	(0.045)	(0.065)	(0.065)
effectiveSpread	−0.167***	−0.099*	−0.067
	(0.047)	(0.053)	(0.056)
depthImbalanceRatio	−0.001	0.004*	0.001
	(0.002)	(0.002)	(0.002)
priceImpact	−0.0002	−0.002	0.014
	(0.009)	(0.009)	(0.026)
ReMeDI	0.060	0.028	0.043
	(0.071)	(0.054)	(0.065)
factor(market)N	0.057***	0.051***	0.043**
	(0.014)	(0.018)	(0.017)
factor(market)P	0.014	0.007	−0.001
	(0.010)	(0.006)	(0.008)
factor(market)T	−0.060***	−0.069***	−0.090***
	(0.014)	(0.017)	(0.018)
factor(market)Z	−0.007	0.005	0.009
	(0.006)	(0.016)	(0.020)
Observations	9,594	9,594	9,594
R^2	0.146	0.274	0.285
Adjusted R^2	0.142	0.271	0.282
F Statistic (df = 10; 9554)	163.077***	361.219***	380.400***
Note:			*p<0.1; **p<0.05; ***p<0.01

This table reports the results of a panel analysis, regressing the estimated daily information shares on covariates. The panel specification includes entity fixed effects (stock level) and market fixed effects (indicated with dummy variables and K as the reference group). Standard errors are clustered on stock level. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").