Bootstrap Inference for Cointegrating Polynomial Regressions

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Abstract

This paper presents results concerning the performance of bootstrap algorithms for hypothesis tests in cointegrating polynomial regressions. A sieve bootstrap procedure is suggested to repeatedly draw from the autocorrelated innovation process driving the cointegrated system. Monte Carlo simulations show that the proposed procedure leads to smaller size distortions over the usual asymptotic approximations. A replication of the empirical EKC analysis in Wagner (2015) emphasizes that the improved size properties of the bootstrap tests can substantially alter the test decision in practice.

Keywords: Cointegration; Environmental Kuznets curve; Sieve bootstrap
JEL Classification: C12, C22

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1 Introduction

Empirical studies investigating the so-called environmental Kuznets curve (EKC) hypothesis, an inverted U-shaped relationship between economic activity and measures of pollution or emissions, rely on cointegrated polynomial regressions (CPR) and are often hampered by (very) small sample sizes. For example, Wagner (2015) studies the EKC hypothesis using yearly data for 19 early industrialized countries over the period 1870–2000. Since the data on carbon dioxide and sulphur dioxide emissions are usually not available at a higher frequency, researchers are often left with less than 200 observations to estimate their models. Although the OLS estimator has very high convergence rates for the coefficients in CPRs, to conduct proper inference, the FM-OLS estimator is needed but it builds on long-run (co)variance estimators with substantially slower convergence rates. Therefore, using asymptotic approximations for inference can potentially lead to size distortions and reduced power for hypothesis testing. Wagner and Hong (2016) show that inference based on the FM-OLS reduces size distortions compared with tests based on the asymptotically invalid OLS standard errors. However, substantial size distortions are still present for moderate sample sizes. Furthermore, the size-adjustments used in Wagner and Hong (2016) for simulated data cannot be straightforwardly applied by practitioners because the severity of the size distortions depend on the regressor endogeneity which is unknown in empirical applications.

One strategy to improve the small sample properties of hypothesis test is bootstrapping. Different methods are available to bootstrap time series regressions (see Palm et al., 2008, for a recent overview). Particularly, the sieve bootstrap has been successfully applied in regressions that involve (potentially) nonstationary processes, for example in the context of unit root testing (Chang and Park, 2003; Palm et al., 2008), or VAR modelling (Swensen, 2006). For cointegrating regressions, Li and Maddala (1997) show that bootstrapping can lead to substantial improvements over asymptotic approximations and Psaradakis (2001) proposes a sieve bootstrap that outperforms their block bootstrap approach. Further applications of the bootstrap in cointegrating regressions can be found in Chang et al. (2006), Shin and Hwang (2013) and Schild and Schweikert (2019).

In this paper, we propose a bootstrap algorithm for inference in CPRs based on the sieve bootstrap for the innovation process driving the cointegrated system. The tests' finite sample performance is evaluated in Monte Carlo simulations using the same parametrization as in Wagner and Hong (2016). Assuming that the number of bootstrap replications is fixed, every added Monte Carlo iteration contributes multiplicatively to the overall computational cost when evaluating such bootstrap methods. To avoid this inefficiency, we refer to the 'Warp-speed' bootstrap described in Giacomini et al. (2013). Our results show that the bootstrap has much smaller size distortions compared with asymptotic approximations proposed in the literature. Moreover, these size distortions are in the conservative direction which makes these tests easier to handle for practitioners. The size-adjusted power is comparable to the asymptotic approximation for tests of single hypothesis about the coefficient of the quadratic term. This does not hold for the corresponding tests about the coefficient of the linear term. Differences in relative performance can be explained by the different convergence rates of the FM-OLS estimator for linear and quadratic terms in the CPR.

To illustrate the importance of maintaining the nominal size for hypothesis tests in CPRs, we replicate the empirical study in Wagner (2015) and specifically focus on the significance test for the quadratic term. This test is often conducted in empirical studies to find out whether the functional form assumed under the EKC hypothesis provides an adequate fit to the data. Applying the bootstrap algorithm in addition to the asymptotic approximation shows that the picture is much more ambiguous for developed countries than previously believed.

The remainder of this paper is structured as follows: we briefly outline the CPR methodology and describe the bootstrap algorithm in Section 2. We discuss our simulation experiments in Section 3, present the empirical application in Section 4, and conclude in Section 5.

2 Methodology

The notation chosen for this section closely follows Wagner and Hong (2016). To simplify the exposition and without loss of generality, we focus on cointegrating polynomial regressions involving a single regressor variable x_t. Further, we restrict our discussion to CPRs with a constant and a linear trend:

$$y_t = c + \delta t + \beta_1 x_t + \cdots + \beta_q x_t^q + u_t, \qquad t = 1, \ldots, T, \tag{1}$$

where x_t is $I(1)$, $\Delta x_t = v_t$, q denotes the polynomial order, and u_t is a stationary error term. The $T \times (q+2)$ matrix $Z := (1, t, x_t, \ldots, x_t^q)$ contains the regressors and θ contains the corresponding coefficients. We further define the vector stochastic process $\{\xi_t\}_{t\in\mathbb{Z}} := \{(u_t, v_t)\}_{t\in\mathbb{Z}}$. Then, we can define its long-run covariance matrix

$$\Omega := \sum_{h=-\infty}^{\infty} E(\xi_0 \xi_h') = \begin{pmatrix} \Omega_{uu} & \Omega_{uv} \\ \Omega_{vu} & \Omega_{vv} \end{pmatrix}. \tag{2}$$

Analogously, we define the one-sided long-run covariance matrix

$$\Delta := \sum_{h=0}^{\infty} E(\xi_0 \xi_h') = \begin{pmatrix} \Delta_{uu} & \Delta_{uv} \\ \Delta_{vu} & \Delta_{vv} \end{pmatrix}, \tag{3}$$

and $\omega_{u\cdot v} = \Omega_{uu} - \Omega_{uv}\Omega_{vv}^{-1}\Omega_{vu}$.

The FM-OLS estimator applied to Equation (1) is based on two transformations. The first transformation involves the dependent variable and requires a consistent estimator for the long-run covariance matrix:

$$y_t^+ = y_t - \Delta x_t \hat{\Omega}_{vv}^{-1} \hat{\Omega}_{vu}. \tag{4}$$

For the second transformation, we need the estimator $\hat{\Delta}_{vu}^+ = \hat{\Delta}_{vu} - \hat{\Omega}_{uv}\hat{\Omega}_{vv}^{-1}\hat{\Delta}_{vv}$ to build the correction term

$$A^* = \hat{\Delta}_{vu}^+ \begin{bmatrix} 0 \\ 0 \\ T \\ 2\sum x_t \\ \vdots \\ q\sum x_t^{q-1} \end{bmatrix}. \tag{5}$$

Finally, the FM-OLS estimator is given by $\theta^+ = (Z'Z)^{-1}(Z'y^+ - A^*)$ and a consistent estimator of its variance is $\hat{\omega}_{u\cdot v}(Z'Z)^{-1}$. The corresponding t-statistics for single coefficient hypothesis have asymptotically standard normal distribution.

Next, we propose a bootstrap method for inference in CPRs. To test null hypotheses in the form of $H_0 : \beta_i = \beta_{i,0}$, $i \in \{1, \ldots, q\}$ against the two-sided alternative, we use the following sieve bootstrap algorithm:

1. Compute the residuals $\{\hat{u}_t^+ = (y_t - Z\theta_0^+, \Delta x_t)'\}_{t=1}^T$, where θ_0^+ is the FM-OLS estimator under the null hypothesis, i.e., the i-th element is substituted by the coefficient under the null hypothesis.

2. Estimate the VAR(p) model for $\{\hat{u}_t^+\}_{t=1}^T$ to obtain the coefficients $\Phi_1, \ldots, \Phi_p$ and save the residuals from this regression $\{\hat{e}_t\}_{t=1}^T$.

3. Center the estimated residuals $\{\hat{e}_t\}_{t=1}^T$ and draw a random sample from the centered residuals to obtain a bootstrap sample $\{e_t^*\}_{t=1}^T$. Construct the bootstrap noise series $\{u_t^*\}_{t=1}^T$ using the recursion

$$u_t^* = \sum_{j=1}^{p} \hat{\Phi}_j u_{t-j}^* + e_t^*, \tag{6}$$

   where the initial p values are given by $u_t^* = \hat{u}_t^+$.

4. Using the partition $u_t^* = (u_{1t}^*, u_{2t}^*)'$, generate the bootstrap replicate $\{x_t^*\}_{t=1}^T$ according to

$$x_t^* = x_{t-1}^* + u_{2t}^*, \qquad x_0^* = 0, \tag{7}$$

   and construct the matrix $Z^* = (1, t, x_t^*, \ldots, x_t^{*q})$. Build the bootstrap replicate $\{y_t^*\}_{t=1}^T$

according to

$$y_t^* = Z^* \theta_0^+ + u_{1t}^*. \tag{8}$$

5. Estimate θ^+ using the bootstrap sample $\{(x_t^*, y_t^*)\}_{t=1}^T$ and compute the associated t-statistic for $H_0 : \beta_i = \beta_{i,0}$.

6. Repeat Steps 3-5 sufficiently often to obtain a bootstrap sample of t-statistics.

7. Obtain a bootstrap estimate of the critical values at the α significance level from the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles of the empirical bootstrap distribution of t-statistics.

The bootstrap critical value can then be used as a bootstrap alternative to the usual asymptotically standard normal t-test approximations. Selecting the unknown lag order p in the sieve bootstrap can be based on the familiar Akaike information criterion (Psaradakis, 2001). We follow Schwert (1989) and let the maximum lag length from which the AIC selects the optimal value increase with the sample size. The long-run covariance matrices needed for the computation of the FM-OLS estimator and test statistics based upon it are obtained using the Bartlett and Quadratic Spectral (QS) kernels. For each kernel, we consider two bandwidth choices: (i) the data-dependent rule of Andrews (1991), and (ii) the sample size dependent rule of Newey and West (1994), i.e., $\lfloor 4(T/100)^{2/9} \rfloor$.

Additionally, we can test s linearly independent restrictions of the form

$$H_0 : R\theta = r, \tag{9}$$

where $R \in \mathbb{R}^{s \times (q+2)}$ with full rank s and $r \in \mathbb{R}^s$. This is accomplished by evaluating the Wald statistic

$$W = (R\hat{\theta}^+ - r)'[\hat{\omega}_{u \cdot v} R(Z'Z)^{-1} R']^{-1} (R\hat{\theta}^+ - r), \tag{10}$$

which is asymptotically χ_s^2 distributed under the null hypothesis. The bootstrapped equivalent, in principle, uses the same algorithm as outlined above, applying the linear restrictions in step 1 and computing the corresponding Wald statistic in step 5. The bootstrap critical value is then computed for the $(1 - \alpha)$-th quantile of the empirical bootstrap distribution of Wald statistics.

3 Finite sample performance

For our simulation study, we adopt the same data-generating process for the quadratic cointegrating polynomial regression model that is used in Wagner and Hong (2016):

$$y_t = c + \delta t + \beta_1 x_t + \beta_2 x_t^2 + u_t, \tag{11}$$

where $v_t = \Delta x_t$ and u_t are generated as:

$$\begin{aligned} u_t &= \rho_1 u_{t-1} + e_{1,t} + \rho_2 e_{2,t}, \qquad u_0 = 0, \\ v_t &= e_{2,t} + 0.5\, e_{2,t-1}, \end{aligned}$$

and $(e_{1,t}, e_{2,t})' \sim \mathcal{N}(0, I_2)$. The parameter ρ_1 controls the autocorrelation in the error term u_t and the parameter ρ_2 determines the degree of regressor endogeneity. The coefficient values are set to $c = \delta = 1$, $\beta_1 = 5$, and $\beta_2 = -0.3$.

First, we conduct experiments to determine the empirical size of the t-tests. Our results are reported in Table 1. The values for the (asymptotically invalid) OLS estimator and those of both asymptotic approximations, using bandwidth choice (i) and (ii), for the FM-OLS estimator are largely identical to the values reported in Table C4 in Wagner and Hong (2016). The small differences can be explained by the larger number of replications (10,000 instead of 5,000) we employ to ensure that these values can be compared with the 'Warp-speed' bootstrap results. While accounting for the serial correlation and endogeneity by the FM-OLS estimator improves the size properties in comparison to the standard OLS approach, substantial size distortions remain for higher levels of regressor endogeneity.

We find that our sieve bootstrap has better size properties than the asymptotic approximations. For example, in case of $T = 200$ and $\rho_1 = \rho_2 = 0.8$, the asymptotic approximations still exhibit size distortions of more than five times their nominal value. Instead, the bootstrap tests are almost exact when testing β_2 and they tend to be too conservative in case of β_1. For all parameter configurations, we find that the asymptotic approximations respond only minimally to an increased sample size from $T = 100$ to $T = 200$. A finding that is also reported in Wagner and Hong (2016). The same holds for the bootstrap tests which is not surprising since they are based on the FM-OLS estimator and its properties depend on nonparametric estimators of the long-run (co)variances with slow convergence rates. In case of β_2, we already obtain an empirical size that very close to its nominal size for $T = 100$. The pattern of relative performances are largely independent of kernel and bandwidth choices across parameterizations.

Second, we depict the size-adjusted power curves of single hypothesis tests for $\rho_1 = \rho_2 = 0.6$ in Figure 1. It turns out that the bootstrap has comparable size-adjusted power for β_2 (roughly at most within 10 percentage points for the analyzed range of values) but it performs much worse for β_1. This finding can be explained by the higher convergence rates of the FM-OLS estimator for β_2 making those estimates more reliable for our resampling algorithm. This feature of the CPRs also affects the power properties of the bootstrapped Wald tests depicted in Figure 2. The power properties of bootstrap and asymptotic approximation seem to converge for $T = 200$.[1] However, since the sample sizes in EKC studies are often very

[1]The detailed results are not reported in this paper, but can be obtained from the author upon request.

small, we suggest that particularly bootstrapping the t-test for hypotheses involving β_2 is an interesting option for practitioners. The size distortions are minimal at all levels of regressor endogeneity and, hence, size-adjustments are not necessarily needed to avoid Type-I errors. Moreover, its power is competitive in finite samples.

4 Empirical application

In the following, we replicate the empirical analysis of the EKC hypothesis in Wagner (2015). We consider the same 19 early industrialized countries over the period 1870–2000 and estimate the quadratic CPRs for each country using the FM-OLS estimator. Per capita carbon dioxide (CO2) and sulphur dioxide (SO2) emissions are chosen as the relevant measures of pollution and real per capita GDP is the measure for economic activity. All variables are log-transformed.

The coefficient estimates for the quadratic CPR are reported in Table 3 and the results only differ marginally from those reported in Table 4 and Table 14 in the supplementary material to the original article. However, comparing the p-values for the null hypothesis $\beta_2 = 0$, we find substantial differences. Using the t-test and the asymptotic approximation, the null hypothesis is rejected at the 1% significance level for 12 of 19 countries in case of CO2 and 17 of 19 countries in case of SO2 which, combined with the fact that the coefficient estimates have a negative sign, provides overwhelming evidence that the EKC hypothesis fits the data well. Bootstrapping those test statistics instead, only yields a rejection of the null hypothesis for 3 of 19 countries and 4 of 19 countries, respectively. Consequently, these results paint a much different picture and suggest that a linear curve also provides an accurate description of relationship between pollution and economic activity.

5 Conclusion

This paper demonstrates that using the sieve bootstrap algorithm can improve the finite sample size properties and the size-adjusted power of hypothesis tests in cointegrated polynomial regressions. While size distortions are reduced for the coefficients of the linear and quadratic terms, we only maintain the same size-adjusted power for tests involving the quadratic coefficient. We recommend that practitioners use these bootstrap tests to avoid the need for size-adjustments that depend on the (unknown) regressor endogeneity.

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Table 1: Empirical Null Rejection Probabilities (5% Significance Level) for single coefficient hypotheses

Panel A: t-tests for $H_0 : \beta_1 = 5$

$T = 100$

		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0476	.1053	.0923	.0355	.0333	.1210	.1096	.0390	.0408
0.3	.1241	.1184	.1215	.0319	.0318	.1164	.1179	.0319	.0348
0.6	.2770	.1466	.1603	.0300	.0323	.1378	.1473	.0261	.0322
0.8	.5193	.2743	.2926	.0287	.0249	.2625	.2822	.0241	.0257

$T = 200$

		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0493	.0833	.0747	.0345	.0265	.0878	.0885	.0355	.0321
0.3	.1286	.0900	.0962	.0302	.0247	.0839	.0890	.0298	.0258
0.6	.3073	.1220	.1309	.0291	.0270	.1142	.1207	.0304	.0247
0.8	.5643	.2706	.2840	.0345	.0283	.2584	.2744	.0378	.0286

Panel B: t-tests for $H_0 : \beta_2 = -0.3$

$T = 100$

		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0509	.1009	.0885	.0454	.0413	.1158	.1060	.0453	.0464
0.3	.1234	.1065	.1100	.0421	.0438	.1052	.1071	.0406	.0453
0.6	.2364	.1319	.1431	.0443	.0475	.1244	.1326	.0419	.0469
0.8	.3761	.1871	.1986	.0527	.0517	.1787	.1878	.0512	.0534

$T = 200$

		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0516	.0838	.0736	.0406	.0354	.0884	.0867	.0413	.0366
0.3	.1315	.0851	.0917	.0378	.0343	.0792	.0855	.0381	.0354
0.6	.2619	.1137	.1221	.0360	.0318	.1083	.1123	.0349	.0332
0.8	.4222	.1855	.1957	.0479	.0408	.1799	.1882	.0511	.0412

Note: We draw $R = 10,000$ replications from the DGP described in Equation (11) and apply the "Warp-speed" bootstrap algorithm described in Giacomini et al. (2013) to obtain bootstrap distributions. AND and NW denote the the data-dependent rule of Andrews (1991) and the sample size dependent rule of Newey and West (1994), respectively.

Table 2: Empirical Null Rejection Probabilities (5% Significance Level) for linearly independent restrictions

Wald tests for $H_0 : \beta_1 = 5, \beta_2 = -0.3$									
$T = 100$									
		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0509	.1280	.1075	.0388	.0380	.1503	.1327	.0395	.0450
0.3	.1672	.1482	.1546	.0308	.0345	.1461	.1482	.0301	.0385
0.6	.4042	.1987	.2180	.0434	.0492	.1864	.1976	.0420	.0469
0.8	.7446	.4204	.4456	.0734	.0789	.3983	.4275	.0643	.0803
$T = 200$									
		Bartlett Kernel				QS Kernel			
		Asymp.		Bootstrap		Asymp.		Bootstrap	
ρ_1, ρ_2	OLS	AND	NW	AND	NW	AND	NW	AND	NW
0.0	.0545	.0990	.0885	.0416	.0349	.1070	.1055	.0424	.0348
0.3	.1753	.1089	.1183	.0336	.0302	.0993	.1079	.0335	.0304
0.6	.4365	.1600	.1731	.0202	.0169	.1441	.1558	.0294	.0173
0.8	.7778	.4073	.4332	.0200	.0105	.3850	.4216	.0239	.0108

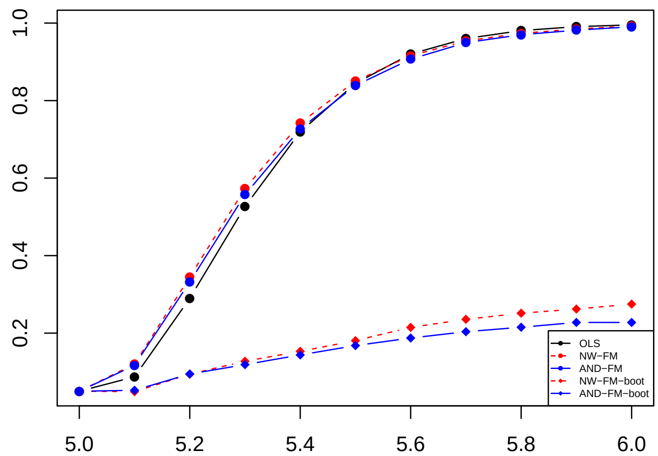
Note: We draw $R = 10,000$ replications from the DGP described in Equation (11) and apply the "Warp-speed" bootstrap algorithm described in Giacomini et al. (2013) to obtain bootstrap distributions. AND and NW denote the the data-dependent rule of Andrews (1991) and the sample size dependent rule of Newey and West (1994), respectively.

Table 3: FM-OLS estimation results of the CPR

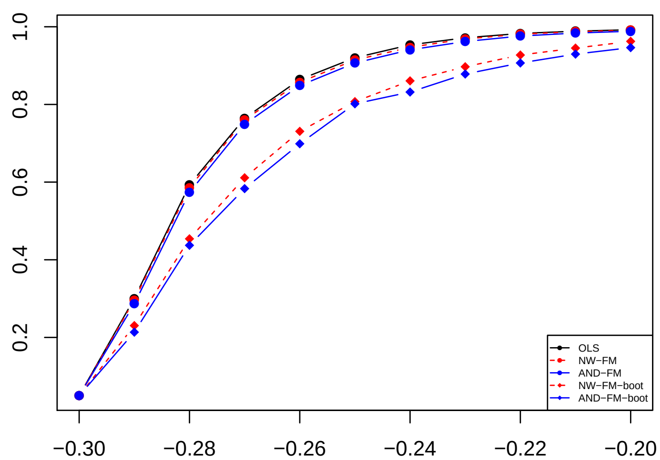
	Panel A: carbon dioxide emissions				
	δ	β_1	β_2	$p(\beta_2)_{\text{asymp}}$	$p(\beta_2)_{\text{boot}}$
Australia	0.035	4.534	-0.292	0.326	0.760
Austria	-0.015	8.377	-0.406	0.022	0.457
Belgium	-0.007	13.705	-0.727	0.000	0.009
Canada	0.002	15.703	-0.845	0.000	0.043
Denmark	-0.006	11.357	-0.565	0.000	0.024
Finland	-0.036	17.452	-0.832	0.000	0.049
France	-0.002	10.138	-0.542	0.000	0.002
Germany	-0.002	10.088	-0.548	0.000	0.040
Italy	-0.004	6.333	-0.275	0.086	0.555
Japan	0.014	13.603	-0.759	0.001	0.324
The Netherlands	0.002	10.217	-0.531	0.000	0.029
New Zealand	-0.002	1.919	-0.054	0.000	0.000
Norway	0.046	-6.894	0.339	0.027	0.558
Portugal	0.016	-2.921	0.210	0.254	0.675
Spain	0.010	5.931	-0.302	0.060	0.507
Sweden	-0.006	12.480	-0.645	0.000	0.242
Switzerland	-0.014	7.078	-0.303	0.054	0.566
UK	-0.005	7.747	-0.409	0.000	0.047
USA	-0.000	11.528	-0.600	0.000	0.029
	Panel B: sulphur dioxide emissions				
	δ	β_1	β_2	$p(\beta_2)_{\text{asymp}}$	$p(\beta_2)_{\text{boot}}$
Australia	0.019	-3.318	0.162	0.610	0.858
Austria	-0.024	26.364	-1.462	0.000	0.022
Belgium	-0.009	33.646	-1.850	0.000	0.010
Canada	0.009	23.532	-1.345	0.000	0.000
Denmark	-0.062	31.329	-1.552	0.000	0.035
Finland	0.003	27.739	-1.544	0.000	0.030
France	-0.001	19.854	-1.098	0.000	0.032
Germany	-0.006	19.570	-1.100	0.000	0.061
Italy	-0.030	13.509	-0.647	0.002	0.306
Japan	-0.003	18.028	-1.047	0.000	0.049
The Netherlands	-0.004	33.098	-1.839	0.000	0.009
New Zealand	0.010	23.484	-1.364	0.000	0.111
Norway	-0.012	22.938	-1.271	0.000	0.038
Portugal	0.009	1.643	-0.051	0.678	0.895
Spain	0.003	11.890	-0.664	0.000	0.225
Sweden	-0.040	39.246	-2.111	0.000	0.000
Switzerland	-0.077	31.084	-1.506	0.000	0.021
UK	-0.010	28.229	-1.545	0.000	0.029
USA	-0.013	17.806	-0.944	0.000	0.006

Note: The sample period is 1870–2000 with the exception of New Zealand where the sample starts in 1878. The long-run variance estimation is performed using the Bartlett kernel with the bandwidth chosen according to Newey and West (1994). $p(\beta_2)_{\text{asymp}}$ and $p(\beta_2)_{\text{boot}}$ denote the p-value for the hypothesis $\beta 2 = 0$, using the asymptotic approximation and the sieve bootstrap, respectively. 1,000 replications are used to compute the bootstrap sample of t-statistics.

Figure 1: These figures depict the size-corrected power curves of the t-test based on the OLS estimator (black), FM-OLS estimator with data-dependent bandwidth selection (blue) and sample size dependent bandwidth selection (red). The corresponding values for the bootstrap test are marked with a diamond.



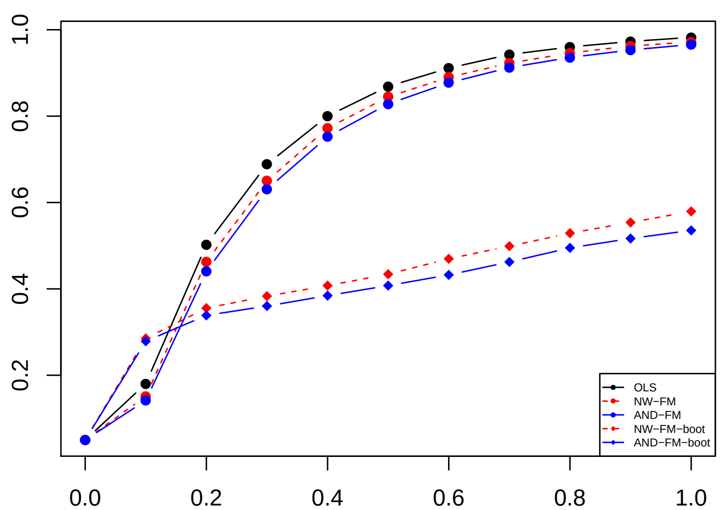
(a) Size-corrected power curves for the t-test of $\beta_1 = 5$ $(\rho_1, \rho_2 = 0.6)$



(b) Size-corrected power curves for the t-test of $\beta_2 = -0.3$ $(\rho_1, \rho_2 = 0.6)$

Figure 2: **Size-corrected power curves for the Wald test of** $\beta_1 = 5, \beta_2 = -0.3$ **(**$\rho_1, \rho_2 = 0.6$**)**

This figure depicts the size-corrected power curves of the Wald test based on the OLS estimator (black), FM-OLS estimator with data-dependent bandwidth selection (blue) and sample size dependent bandwidth selection (red). The corresponding bootstrap tests are marked with a diamond.



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