Integrated Variance Estimation for Assets Traded in Multiple Venues – Online Appendix

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A Proofs

A.1 Proofs of the main results

Lemma 1. Under Assumption 1 and Assumption 2, $T \to \infty$, $\delta \to 0$, and $T\delta^2 \to 0$, it holds that

$$\text{(i)} \qquad \sqrt{Tm} V_{Tm} W_{Tm}^{-1/2} \operatorname{vec}(\hat{\alpha}_\delta - \alpha_\delta) \xrightarrow{d} \mathcal{N}(0, I_{(N-1)^2 N^2}),$$

and

$$\text{(ii)} \qquad \hat{\alpha}_\perp \xrightarrow{p} \alpha_\perp.$$

Proof of Lemma 1. If β is known, we can rewrite the discretely sampled VECM as follows

$$\begin{aligned} \Delta P_{t_i} &= \Pi_\delta P_{t_{i-1}} + \epsilon_{t_i}, \qquad i = 1, \ldots, m, \quad t = 1, \ldots, T \\ &= \alpha_\delta \beta' P_{t_{i-1}} + \epsilon_{t_i} \end{aligned} \tag{A.1}$$

where ϵ_{t_i} is mean-zero Gaussian with covariance matrix $\Omega_{\delta,t_i} = \int_0^\delta \exp(u\Pi)\Omega(t_i - u)\exp(u\Pi')du$. Similar to Dorogovtsev (1978) and Prakasa Rao (1983), it can be shown that the least squares estimator α_δ is consistent under these conditions. Now, we derive the asymptotic distribution of $\hat{\alpha}_\delta$ under heteroskedasticity of unknown form.

According to Hafner and Herwartz (2009) it holds that

$$\lim_{T,m\to\infty}\frac{1}{Tm}\sum_{t=1}^{T}\sum_{i=1}^{m}E\left[(\beta' P_{t_{i-1}}P'_{t_{i-1}}\beta)\otimes(\epsilon_{t_i}\epsilon'_{t_i})\right]=W, \tag{A.2}$$

with some finite, positive definite matrix W. Under our assumptions for the cointegrated OU type process, it further holds that

$$\lim_{T,m\to\infty}\frac{1}{Tm}\sum_{t=1}^{T}\sum_{i=1}^{m}E\left[(\beta' P_{t_{i-1}}P'_{t_{i-1}}\beta)\right]=V, \tag{A.3}$$

for a non-singular V.

Then, using the consistent estimators

$$W_{Tm}=\frac{1}{Tm}\sum_{t=1}^{T}\sum_{i=1}^{m}(\beta' P_{t_{i-1}}P'_{t_{i-1}}\beta)\otimes(\epsilon_{t_i}\epsilon'_{t_i}), \tag{A.4}$$

and

$$V_{Tm}=\frac{1}{Tm}\sum_{t=1}^{T}\sum_{i=1}^{m}(\beta' P_{t_{i-1}}P'_{t_{i-1}}\beta)\otimes I_N, \tag{A.5}$$

we follow White (1980) and Hafner and Herwartz (2009), and obtain the following convergence result

$$\sqrt{Tm}V_{Tm}W_{Tm}^{-1/2}\ \mathrm{vec}(\hat{\alpha}_\delta-\alpha_\delta)\xrightarrow{d}\mathcal{N}(0,I_{(N-1)^2N^2}), \tag{A.6}$$

or equivalently

$$\sqrt{Tm}\ \mathrm{vec}(\hat{\alpha}_\delta-\alpha_\delta)\xrightarrow{d}\mathcal{N}(0,V_{Tm}^{-1}W_{Tm}V_{Tm}^{-1}). \tag{A.7}$$

This concludes the proof for part (i).

Without loss of generality, we choose $\hat{\alpha}_\perp=\alpha_\perp-\alpha_\delta(\hat{\alpha}'_\delta\alpha_\delta)^{-1}\hat{\alpha}'_\delta\alpha_\perp$, then $\hat{\alpha}'_\delta\hat{\alpha}_\perp=0$ and

$$\hat{\alpha}_\perp-\alpha_\perp=-\hat{\bar{\alpha}}_\delta(\hat{\alpha}_\delta-\alpha_\delta)'\alpha_\perp, \tag{A.8}$$

where $\hat{\bar{\alpha}}_\delta=\alpha_\delta(\hat{\alpha}'_\delta\alpha_\delta)^{-1}$. Using the property $\hat{\alpha}_\delta\xrightarrow{p}\alpha_\delta$, it follows from the Continuous Mapping Theorem that $\hat{\bar{\alpha}}_\delta\xrightarrow{p}\bar{\alpha}_\delta=\alpha_\delta(\alpha'_\delta\alpha_\delta)^{-1}$ and finally $\hat{\alpha}_\perp\xrightarrow{p}\alpha_\perp$.

□

Proof of Proposition 1. Item (i) follows directly from the stochastic trend implied by the Granger representation of (17). To appreciate this, note that

$$
\begin{aligned}
\widetilde{P}^*_{t_i} &= \alpha'_\perp \left(\sum_{j=1}^{t-1} \sum_{h=1}^{m} u_{j_h} + \sum_{h=1}^{i} u_{t_h} \right), \\
&= \alpha'_\perp \left(\sum_{j=1}^{t-1} \sum_{h=1}^{m} \epsilon_{j_h} + \sum_{h=1}^{i} \epsilon_{t_h} \right) + \left(\sum_{j=1}^{t-1} \sum_{h=1}^{m} \alpha'_\perp \left(\nu_{j_h} - \nu_{j_{h-1}} \right) + \sum_{h=1}^{i} \alpha'_\perp \left(\nu_{t_h} - \nu_{t_{h-1}} \right) \right) \\
&= P^*_{t_i} + \alpha'_\perp \nu_{t_i} - \alpha'_\perp \nu_{1_0}, \qquad \text{(A.9)}
\end{aligned}
$$

where $u_{t_{i-1}} = \epsilon_{t_i} + [I_N - (\alpha_\delta \beta' + I_N)L]\nu_{t_i}$. It then follows that

$$
\widetilde{P}^*_{t_i} = P^*_{t_i} + \alpha'_\perp \nu_{t_i} - \alpha'_\perp \nu_{1_0} = P^*_{t_i} + \tilde{\nu}_{t_i} - \alpha'_\perp \nu_{1_0} \qquad \text{(A.10)}
$$

where ν_{1_0} contains initial values. To show item (ii), we begin by noting that

$$
\mathrm{Var}(\alpha'_\perp \nu_{t_i}) = \mathrm{Var}(\tilde{\nu}_{t_i}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_{\perp,i} \alpha_{\perp,j} \Sigma_{\nu,ij}. \qquad \text{(A.11)}
$$

Since $\alpha_{\perp,n} \geq 0$ for all $n = 1, \ldots, N$, $\iota'_N \alpha_\perp = 1$, and Σ_ν is positive definite, applying the Cauchy-Schwarz inequality yields

$$
\begin{aligned}
\mathrm{Var}(\tilde{\nu}_{t_i}) &= \sum_i \alpha^2_{\perp,i} \Sigma_{\nu,ii} + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \alpha_{\perp,i} \alpha_{\perp,j} \Sigma_{\nu,ij} \\
&\leq \sum_i \alpha^2_{\perp,i} \Sigma_{\nu,ii} + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \alpha_{\perp,i} \alpha_{\perp,j} \sqrt{\Sigma_{\nu,ii} \Sigma_{\nu,jj}} \\
&\leq \left(\sum_{i=1}^{N} \alpha_{\perp,i} \sqrt{\Sigma_{\nu,ii}} \right)^2. \qquad \text{(A.12)}
\end{aligned}
$$

Because $\sum_{i=1}^{N} \alpha_{\perp,i} \sqrt{\Sigma_{\nu,ii}}$ is a convex combination of the of the square roots of the diagonal elements of Σ_ν, it then follows that

$$
\sum_{i=1}^{N} \alpha_{\perp,i} \sqrt{\Sigma_{\nu,ii}} \leq \sup_{n \in N} \sqrt{\Sigma_{\nu,nn}}. \qquad \text{(A.13)}
$$

Finally, (A.13) implies that

$$
\mathrm{Var}(\tilde{\nu}_{t_i}) \leq \left(\sup_{n \in N} \sqrt{\Sigma_{\nu,nn}} \right)^2 = \sup_{n \in N} \mathrm{Var}(\nu_{n,t_i}). \qquad \text{(A.14)}
$$

Proof of Theorem 1. Let $\mathbb{K} = (\text{vech}(\mathbb{K}_1), \ldots, \text{vech}(\mathbb{K}_N))'$ be a $N \times \frac{1}{2}N(N+1)$ matrix with $\mathbb{K}_N$ denoting a $N \times N$ matrix that has the (n,n)-th $n = 1, \ldots, N$ entry equal to one and all the remaining entries equal to zero, and $\mathbb{L}$ be a $\frac{1}{2}N(N+1) \times N^2$ elimination matrix such that, for any symmetric matrix A, $\text{vech}(A) = \mathbb{L}\,\text{vec}(A)$. First, write the diagonal elements of $\widehat{\langle P, P' \rangle}_{t-1:t}$ as $\mathbb{KL}\,\text{vec}\left(\widehat{\langle P, P' \rangle}_{t-1:t}\right)$. Next, write the difference between the market-specific estimates of the integrated variances and the integrated variance of the efficient price as $\mathbb{KL}\,\text{vec}\left(\widehat{\langle P, P' \rangle}_{t-1:t} - \beta_\perp \langle P^*, P^{*\prime} \rangle_{t-1:t} \beta_\perp'\right)$. It then follows that

$$
\begin{aligned}
\widehat{\langle P, P' \rangle}^{n,n}_{t-1:t} - \langle P^*, P^{*\prime} \rangle_{t-1:t} &= k'(n)\mathbb{KL}\,\text{vec}\left(\widehat{\langle P, P' \rangle}_{t-1:t} - \beta_\perp \langle P^*, P^{*\prime} \rangle_{t-1:t} \beta_\perp'\right) \\
&= k'(n)\mathbb{KL}\,\text{vec}\left(\widehat{\langle P, P' \rangle}_{t-1:t} - \beta_\perp \alpha_\perp' \langle P, P' \rangle_{t-1:t} \alpha_\perp \beta_\perp'\right) \\
&= k'(n)\mathbb{KL}\Big[\text{vec}\left(\widehat{\langle P, P' \rangle}_{t-1:t}\right) \\
&\quad - (\beta_\perp \alpha_\perp') \otimes (\beta_\perp \alpha_\perp')\,\text{vec}\left(\langle P, P' \rangle_{t-1:t}\right)\Big], \\
&= k'(n)\mathbb{KL}\left[I_{N^2} - (\beta_\perp \alpha_\perp') \otimes (\beta_\perp \alpha_\perp')\right] \\
&\quad \times \text{vec}\left(\langle P, P' \rangle_{t-1:t}\right) + O_p(m^{-\varphi}), \\
&= k'(n) \begin{pmatrix}
(1 - \alpha_{\perp,1}^2) \langle P, P' \rangle^{1,1}_{t-1:t} - \sum\limits_{\substack{i=1, \\ i \neq 1}}^{N} \alpha_{\perp,i}^2 \langle P, P' \rangle^{i,i}_{t-1:t} \\
(1 - \alpha_{\perp,2}^2) \langle P, P' \rangle^{2,2}_{t-1:t} - \sum\limits_{\substack{i=1, \\ i \neq 2}}^{N} \alpha_{\perp,i}^2 \langle P, P' \rangle^{i,i}_{t-1:t} \\
\vdots \\
(1 - \alpha_{\perp,N}^2) \langle P, P' \rangle^{N,N}_{t-1:t} - \sum\limits_{\substack{i=1, \\ i \neq N}}^{N} \alpha_{\perp,i}^2 \langle P, P' \rangle^{i,i}_{t-1:t}
\end{pmatrix} \\
&\quad - k'(n) \begin{pmatrix}
2 \sum\limits_{i=1}^{N-1} \sum\limits_{j=i+1}^{N} \alpha_{\perp,i} \alpha_{\perp,j} \langle P, P' \rangle^{i,j}_{t-1:t} \\
2 \sum\limits_{i=1}^{N-1} \sum\limits_{j=i+1}^{N} \alpha_{\perp,i} \alpha_{\perp,j} \langle P, P' \rangle^{i,j}_{t-1:t} \\
\vdots \\
2 \sum\limits_{i=1}^{N-1} \sum\limits_{j=i+1}^{N} \alpha_{\perp,i} \alpha_{\perp,j} \langle P, P' \rangle^{i,j}_{t-1:t}
\end{pmatrix} \\
&\quad + O_p(m^{-\varphi}).
\end{aligned} \tag{A.15}
$$

It then follows that (A.15) reduces to

$$\widehat{\langle P,P'\rangle}^{n,n}_{t-1:t} - \langle P^*,P^{*\prime}\rangle_{t-1:t} = (1-\alpha^2_{\perp,n})\,\langle P,P'\rangle^{n,n}_{t-1:t} - \sum_{i=1,i\neq n}^{N}\alpha^2_{\perp,i}\,\langle P,P'\rangle^{i,i}_{t-1:t} + \\ -2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_{\perp,i}\alpha_{\perp,j}\,\langle P,P'\rangle^{i,j}_{t-1:t} + O_p(m^{-\varphi}), \quad \text{(A.16)}$$

$n=1,\ldots,N$, which proves the theorem. Furthermore, if $\langle P,P'\rangle^{n,n}_{t-1:t} = \langle P,P'\rangle^{\bar{n},\bar{n}}_{t-1:t}$ for all $\bar{n}\neq n$ and $\bar{n}=1,\ldots N$ and the correlation between markets remain the same across all market combinations, then A.16 simplifies to

$$\widehat{\langle P,P'\rangle}^{n,n}_{t-1:t} - \langle P^*,P^{*\prime}\rangle_{t-1:t} = \widehat{\langle P,P'\rangle}^{n,n}_{t-1:t} - \rho\,(\alpha_{\perp,1}+\ldots+\alpha_{\perp,N})^2\,\widehat{\langle P,P'\rangle}^{n,n}_{t-1:t} + O_p(m^{-\varphi}).$$

It then follows that the

$$\lim_{\rho\to 1}\widehat{\langle P,P'\rangle}^{n,n}_{t-1:t} - \langle P^*,P^{*\prime}\rangle_{t-1:t} = O_p(m^{-\varphi}).$$

□

Proof of Theorem 2. Assumption 2 ensures that only one common stochastic trend (the efficient price) exists, i.e., we restrict our analysis to the case that α, β are $N\times(N-1)$ matrices. This implies that $\alpha_\perp$, $\beta_\perp$ are $N\times 1$ vectors and $RV^{(m)}_{GRT,t}$ are scalar random variables defined by

$$\begin{aligned} RV^{(m)}_{GRT,t} &= \sum_{i=(t-1)/\delta}^{t/\delta}(\hat{\alpha}'_\perp\hat{\epsilon}_{t_i})^2, \qquad t=1,\ldots,T \quad \text{(A.17)} \\ &= \sum_{i=(t-1)/\delta}^{t/\delta}(\hat{\alpha}'_\perp\Delta P_{t_i})^2 \\ &= \sum_{i=(t-1)/\delta}^{t/\delta}(\alpha'_\perp\Delta P_{t_i})^2 + O_p((Tm)^{-1/2}), \end{aligned}$$

where $\hat{\epsilon}_{t_i} = \Delta P_{t_i} - \hat{\Pi}_\delta P_{t_{i-1}}$ and $\hat{\alpha}'_\perp\iota_N = 1$. The last equality follows from Lemma 1 (ii) and Slutsky's Theorem. We further note that

$$\lim_{m\to\infty}\sum_{i=1}^{m}E(\epsilon_{t_i}\epsilon'_{t_i}) = \int_{t-1}^{t}\Omega(u)du = \lim_{m\to\infty}\sum_{i=1}^{m}E(\Delta P_{t_i}\Delta P'_{t_i}), \quad \text{(A.18)}$$

defines the daily quadratic variation of the process.

Since $\hat{\alpha}_{\perp}' \Delta P_{t_i}$ is scalar variable, we can use the univariate results in Barndorff-Nielsen and Shephard (2004a) to derive the estimators asymptotic distribution. Analogously to the proof of Theorem 1 in Barndorff-Nielsen and Shephard (2004a), it then follows that

$$m^{1/2}\left(\sum_{i=(t-1)/\delta}^{t/\delta}(\hat{\alpha}_{\perp}'\Delta P_{t_i})^2 - \int_{t-1}^{t}\alpha_{\perp}'\Omega(u)\alpha_{\perp}du\right) \xrightarrow{st} \mathcal{MN}\left(0, 2\int_{t-1}^{t}\alpha_{\perp}'\Omega(u)\alpha_{\perp}\alpha_{\perp}'\Omega(u)\alpha_{\perp}du\right),$$

for $m \to \infty$. The convergence is stable in law as defined in Rényi (1963) and Aït-Sahalia and Jacod (2014) and the distribution is mixed normal with random variance $2\int_{t-1}^{t}\alpha_{\perp}'\Omega(u)\alpha_{\perp}\alpha_{\perp}'\Omega(u)\alpha_{\perp}du$. Since the estimator has another unknown quantity as its variance, this result is not statistically feasible. Alternatively, one can use the convergence result for the realized quarticity (Barndorff-Nielsen and Shephard, 2002)

$$\frac{1}{3\delta}\sum_{i=(t-1)/\delta}^{t/\delta}(\hat{\alpha}_{\perp}'\Delta P_{t_i})^4 \xrightarrow{p} \int_{t-1}^{t}\alpha_{\perp}'\Omega(u)\alpha_{\perp}\alpha_{\perp}'\Omega(u)\alpha_{\perp}du, \tag{A.19}$$

and state the standard central limit theorem

$$m^{1/2}\frac{\left(\sum_{i=(t-1)/\delta}^{t/\delta}(\hat{\alpha}_{\perp}'\Delta P_{t_i})^2 - \int_{t-1}^{t}\alpha_{\perp}'\Omega(u)\alpha_{\perp}du\right)}{\sqrt{\frac{2}{3\delta}\sum_{i=(t-1)/\delta}^{t/\delta}(\hat{\alpha}_{\perp}'\Delta P_{t_i})^4}} \xrightarrow{d} \mathcal{N}(0,1). \tag{A.20}$$

□

Proof of Theorem 3. The jump-robust estimators $RBPV_{GRT,t}^{(m)}$ are scalar random variables defined by

$$\begin{aligned} RBPV_{GRT,t}^{(m)} &= \frac{\pi}{2}\sum_{i=(t-1)/\delta}^{t/\delta-1}|\hat{\alpha}_{\perp}'\hat{\epsilon}_{t_i}||\hat{\alpha}_{\perp}'\hat{\epsilon}_{t_{i+1}}|, \qquad t=1,\ldots,T \qquad \text{(A.21)} \\ &= \frac{\pi}{2}\sum_{i=(t-1)/\delta}^{t/\delta-1}|\hat{\alpha}_{\perp}'\Delta P_{t_i}||\hat{\alpha}_{\perp}'\Delta P_{t_{i+1}}| \\ &= \frac{\pi}{2}\sum_{i=(t-1)/\delta}^{t/\delta-1}|\alpha_{\perp}'\Delta P_{t_i}||\alpha_{\perp}'\Delta P_{t_{i+1}}| + O_p((Tm)^{-1/2}), \end{aligned}$$

where $\hat{\epsilon}_{t_i} = \Delta P_{t_i} - \hat{\Pi}_\delta P_{t_{i-1}}$ and $\hat{\alpha}'_\perp \iota_N = 1$. The last equality follows from the fact that the exact discretized OU type process with jumps is given by

$$\begin{aligned} \Delta P_{t_i} &= \Pi_\delta P_{t_{i-1}} + \epsilon_{t_i} + \iota_N \Delta J_{t_i}, \qquad i = 1, \ldots, m, \quad t = 1, \ldots, T, \\ &= \Pi_\delta P_{t_{i-1}} + \tilde{\epsilon}_{t_i}, \end{aligned} \tag{A.22}$$

where ΔJ_{t_i} is the jump size for a jump between t_{i-1} and t_i. This implies that Lemma 1 still holds and the discretely sampled least squares estimator of $\alpha_\perp$ remains consistent in the presence of simultaneous jumps.

We can now turn to the central limit theorem outlined in Barndorff-Nielsen et al. (2006). Considering that we have shown that the random variables $\hat{\alpha}'_\perp \Delta P_{t_i}$ are uncorrelated, it follows from Theorem 2 in Barndorff-Nielsen et al. (2006) that

$$\begin{aligned} m^{1/2} &\left(\frac{\pi}{2} \sum_{i=(t-1)/\delta}^{t/\delta - 1} |\hat{\alpha}'_\perp \Delta P_{t_i}||\hat{\alpha}'_\perp \Delta P_{t_{i+1}}| - \int_{t-1}^{t} \alpha'_\perp \Omega(u) \alpha_\perp du \right) \\ &\xrightarrow{st} \mathcal{MN}\left(0, (2+\theta) \int_{t-1}^{t} \alpha'_\perp \Omega(u) \alpha_\perp \alpha'_\perp \Omega(u) \alpha_\perp du \right), \end{aligned} \tag{A.23}$$

for $\theta = \pi^2/4 + \pi - 5$ and $m \to \infty$. The statistically feasible result using the realized quarticity is given by

$$m^{1/2} \frac{\left(\frac{\pi}{2} \sum_{i=1}^{m} |\hat{\alpha}'_\perp \Delta P_{t_i}||\hat{\alpha}'_\perp \Delta P_{t_{i+1}}| - \int_{t-1}^{t} \alpha'_\perp \Omega(u) \alpha_\perp du \right)}{\sqrt{\frac{(2+\theta)}{3\delta} \sum_{i=1}^{m} (\hat{\alpha}'_\perp \Delta P_{t_i})^4}} \xrightarrow{d} \mathcal{N}(0, 1). \tag{A.24}$$

□

Proof of Theorem 4. Firstly, we begin by acknowledging the findings of Dias et al. (2022a), who established the consistency of the IV estimator for $\alpha_\perp$. Weak law of large numbers and Lemma 2 in Dias et al. (2022a) yield that Z_{t_i} is a set of valid, $\text{plim}_{T,m\to\infty} \frac{1}{TM} \sum_{t=1}^{T} \sum_{i=1}^{m} Z_{t_{i-\bar{q}-\kappa}} \nu'_{t_i} = o_p(1)$, and relevant, $\text{plim}_{T,m\to\infty} \frac{1}{TM} \sum_{t=1}^{T} \sum_{i=1}^{m} Z_{t_{i-\bar{q}-\kappa}} \tilde{P}'_{t_{i-1}} \beta = O_p(1)$, instruments for all $\underline{\kappa} \leq \kappa \leq \bar{\kappa}$. Next, re-write the IV estimator of $\boldsymbol{\alpha}_\delta$ in (19) as

$$\widehat{\boldsymbol{\alpha}}_{\delta,\text{IV}} - \boldsymbol{\alpha}_\delta = \left[\frac{\widetilde{P}'_\beta (I_N \otimes Z)}{Tm} (TmW) \frac{(I_N \otimes Z)' \widetilde{P}_\beta}{Tm} \right]^{-1} \frac{\widetilde{P}'_\beta (I_N \otimes Z)}{Tm} (TmW) \frac{(I_N \otimes Z)' u}{Tm}.$$

As $T, m \to \infty$ and $T\delta^2 \to 0$, relevance of the instruments ensures that $\text{plim}_{T,m\to\infty}(Tm)^{-1}\widetilde{P}'_\beta(I_N \otimes Z) = O_p(1)$ and $\text{plim}_{T,m\to\infty} TmW = O_p(1)$, whereas validity yields $\text{plim}_{T,m\to\infty}(Tm)^{-1}(I_N \otimes Z)'u = o_p(1)$. These imply that $\widehat{\boldsymbol{\alpha}}_{\delta,\text{IV}} - \boldsymbol{\alpha}_\delta = O_p((Tm)^{-1/2})$. Finally, following the steps in item (ii) in Lemma 1, the Continuous Mapping theorem ensures that $\widehat{\alpha}_{\text{IV},\perp} = \alpha_\perp$ as $T, m \to \infty$ and $T\delta^2 \to 0$. Consequently, $\Delta\overline{\widetilde{P}^*}_{\text{IV},t_i} - \Delta\overline{\widetilde{P}^*}_{t_i} = O_p((Tm)^{-1/2})$, with $\Delta\overline{\widetilde{P}^*}_{t_i} = \sum_{j=1}^{k_m} g\left(\frac{j}{k_m}\right)\left(\Delta\widetilde{P}^*_{t_{i+j}} - \Delta\widetilde{P}^*_{t_{i-1+j}}\right)$ and $\Delta\widetilde{P}^*_{t_i} = \alpha'_\perp\Delta\widetilde{P}_{t_i}$. Secondly, note that Assumption 4 implies that the market microstructure noise is $\bar{q}$-dependent, meaning that it satisfies the dependence structure in Hautsch and Podolskij (2013). According to Lemma 1 in Hautsch and Podolskij (2013), it then follows that $\frac{1}{\sqrt{n}\theta\psi_2}\sum_{i=1}^{m-k_m}|\Delta\overline{\widetilde{P}^*}_{t_i}|^2 \xrightarrow{p} \int_{t-1}^{t} \alpha'_\perp\Omega(u)\alpha_\perp du + \frac{\psi_1}{\theta^2\psi_2}\widehat{\rho}^2$, as $T, m \to \infty$ and $T\delta^2 \to 0$. Thirdly, applying Theorem 3.1 in Jacod et al. (2009) (see also Theorem 1 in Hautsch and Podolskij, 2013) to $C^{(m)}_{GRT,t}(\bar{q})$, we deduce that, as $T \to \infty$, $\delta \to 0$, and $T\delta^2 \to 0$,

$$m^{1/4}\left(C^{(m)}_{GRT,t}(\bar{q}) - \int_{t-1}^{t} \alpha'_\perp\Omega(u)\alpha_\perp du\right) \xrightarrow{st} \mathcal{MN}\left(0, \Gamma_t(\bar{q})\right),$$

where $\Gamma_t(\bar{q}) = \int_{t-1}^{t}\gamma_u^2(\bar{q})du$ and

$$\gamma_u^2(\bar{q}) = \frac{4}{\psi_2^2}\left(\Phi_{22}\theta\sigma_u^4 + 2\Phi_{12}\frac{\rho^2\sigma_u^2}{\theta} + \Phi_{11}\frac{\rho^4}{\theta^3}\right),$$

where $\Phi_{11} = 1/6$, $\Phi_{12} = 1/96$, and $\Phi_{22} = 151/80640$. As $T \to \infty$, $\delta \to 0$, and $T\delta^2 \to 0$ the feasible version of the central limit theorem follows directly from Hautsch and Podolskij's 2013 Proposition 1:

$$m^{1/4}\frac{\left(C^{(m)}_{GRT,t}(\bar{q}) - \int_{t-1}^{t} \alpha'_\perp\Omega(u)\alpha_\perp du\right)}{\sqrt{\Gamma_t^{(m)}(\bar{q})}} \xrightarrow{d} \mathcal{N}\left(0, 1\right), \tag{A.25}$$

where

$$\begin{aligned}\Gamma_t^{(m)}(\bar{q}) = \; & \tfrac{4\Phi_{22}}{3\theta\psi_2^4}\sum_{i=1}^{m-k_m}(\Delta\overline{\widetilde{P}^*}_{\text{IV},t_i})^4 + \tfrac{8\rho_m^2}{\sqrt{m}\theta^2}\left(\tfrac{\Phi_{12}}{\psi_2^3} - \tfrac{\Phi_{22}\psi_1}{\psi_2^4}\right)\sum_{i=1}^{m-2k_m}(\Delta\overline{\widetilde{P}^*}_{\text{IV},t_i}) \\ & + \tfrac{4\rho_m^4}{\theta^3}\left(\tfrac{\Phi_{11}}{\psi_2^2} - 2\tfrac{\Phi_{12}\psi_1}{\psi_2^3} + \tfrac{\Phi_{22}\psi_1^2}{\psi_2^4}\right).\end{aligned} \tag{A.26}$$

More specifically, while both results hinge on the $\bar{q}$-dependence of the market microstructure noise (Assumption 4), the feasible version of the central limit theorem is based on the consistency of $\Gamma_t^{(m)}(\bar{q})$.

$\square$

A.2 Supplementary results for the cointegrated OU process with constant covariance

Assuming that the contemporaneous covariance matrix is constant, i.e., $\Omega_{\delta,t_i} = \Omega_\delta$ for $i = 1, \ldots, m$ and $t = 1, \ldots, T$, the autocovariance function of ΔP_{t_i} for lag h is given by

$$\begin{aligned} \gamma(h) &= Cov(\Delta P_{t_i}, \Delta P_{t_{i-h}}) \\ &= \Omega_\delta \beta (I_r + \alpha_\delta' \beta)^{h-1} \alpha_\delta' + \sum_{i=0}^{\infty} \alpha_\delta (I_r + \beta' \alpha_\delta)^i \beta' \Omega_\delta \beta (I_r + \alpha_\delta' \beta)^{h+i} \alpha_\delta'. \end{aligned} \tag{A.27}$$

Under the stability conditions for the cointegrated OU process expressed in Assumption 2, the autocovariances decay for $h \to \infty$. However, it is illustrative to analyse a more restrictive scenario in which $N = 2$ and $\beta' \alpha_\delta = -1$, representing a two-market setting with full adjustment of equilibrium errors during one intra-day sampling interval. It is easy to see that in this stylized example, the autocovariance function is zero for all $h > 1$. If one market is dominant such that its corresponding adjustment coefficient is zero, the returns of the dominated market follows an MA(1) process with a negative first-order autocorrelation. According to Bandi and Russell (2008), the MA(1) market microstructure model is suggested as a valid approximation in the case of equities when considering transaction prices and/or quotes posted on multiple exchanges. They use information provided by multiple exchanges to estimate the noise moments but do not exploit the fact that prices for assets traded on multiple exchanges are cointegrated. Hansen and Lunde (2006) and Andersen et al. (2022) show that the high-frequency return dynamics are often inconsistent with an MA(1) representation and argue for a more general specification of the noise process.

The asymptotic distribution of our proposed $RV_{GRT,t}^{(m)}$ estimator is derived under the assumption of stochastic volatility in the main text. The corresponding result for the constant covariance case, $\Omega_{\delta,t_i} = \Omega_\delta$, for $t = 1, \ldots, T$, $i = 1, \ldots, m$, is stated next.

Corollary 1. Under Assumption 2 and constant covariance matrix Ω_δ, it holds that

$$m^{1/2} \frac{\left(\frac{1}{m} \sum_{i=1}^{m} \left[\hat{\alpha}_{\delta,\perp}' \hat{\epsilon}_{t_i} \right]^2 - \alpha_\perp' \Omega_\delta \alpha_\perp \right)}{\sqrt{2 \alpha_\perp' \Omega_\delta \alpha_\perp \alpha_\perp' \Omega_\delta \alpha_\perp}} \xrightarrow{d} \mathcal{N}(0,1).$$

for $T \to \infty$, $\delta \to 0$, and $T\delta^2 \to 0$.

Proof of Corollary 1. As in the proof of Theorem 2, we restrict our analysis to the case that α, β are $N \times (N-1)$ matrices. This implies that $\alpha_{\delta,\perp}$, $\beta_\perp$ are $N \times 1$ vectors. Now, $RV^{(m)}_{GRT,t} = RV^{(m)}_{GRT}$ for each t and it can be defined by

$$RV^{(m)}_{GRT} \quad = \quad \frac{1}{m}\sum_{i=1}^{m}\left[\hat{\alpha}'_{\delta,\perp}\hat{\epsilon}_{t_i}\right]^2 m, \tag{A.28}$$

where $\hat{\epsilon}_{t_i} = \Delta P_{t_i} - \hat{\Pi}_\delta P_{t_{i-1}}$ and $\hat{\alpha}'_{\delta,\perp}\iota = 1$.

Since $\epsilon_{t_i} \sim \mathcal{N}(0, \Omega_\delta)$ and $\hat{\Pi}_\delta \xrightarrow{p} \Pi_\delta$ for $T \to \infty$, $\delta \to 0$, and $T\delta^2 \to 0$, we have $\hat{\epsilon}_{t_i} \xrightarrow{d} \epsilon_{t_i}$. Further, we have $\hat{\alpha}_{\delta,\perp} \xrightarrow{p} \alpha_\perp$ according to Lemma 1 (ii). Now, we can apply Slutsky's Theorem to obtain

$$\hat{\alpha}'_{\delta,\perp}\hat{\epsilon}_{t_i} \xrightarrow{d} \alpha'_\perp\epsilon_{t_i} \sim \mathcal{N}(0, \alpha'_\perp\Omega_\delta\alpha_\perp). \tag{A.29}$$

Applying the CLT (see, for example, DasGupta, 2008, Theorem 3.7 and Example 3.6), we have

$$m^{1/2}\left(\frac{1}{m}\sum_{i=1}^{m}\left[\hat{\alpha}'_{\delta,\perp}\hat{\epsilon}_{t_i}\sqrt{m}\right]^2 - \alpha'_\perp\Omega_\delta\alpha_\perp m\right) \xrightarrow{d} \mathcal{N}\left(0, E(\alpha'_\perp\epsilon_{t_i}\sqrt{m})^4 - [\alpha'_\perp\Omega_\delta\alpha_\perp m]^2)\right). \tag{A.30}$$

Using the properties of the normal distribution, it follows that

$$E(\alpha'_\perp\epsilon_{t_i}\sqrt{m})^4 = 3[\alpha'_\perp\Omega_\delta\alpha_\perp m]^2, \tag{A.31}$$

and this implies

$$m^{1/2}\frac{\left(\frac{1}{m}\sum_{i=1}^{m}\left[\hat{\alpha}'_{\delta,\perp}\hat{\epsilon}_{t_i}\right]^2 - \alpha'_\perp\Omega_\delta\alpha_\perp\right)}{\sqrt{2\alpha'_\perp\Omega_\delta\alpha_\perp\alpha'_\perp\Omega_\delta\alpha_\perp}} \xrightarrow{d} \mathcal{N}\left(0,1\right), \tag{A.32}$$

for $m \to \infty$ which concludes the proof.

□

B Additional theoretical results

B.1 A jump- and additive noise-robust estimator

It is possible to further strengthen the pre-averaging estimator in (22) in the main paper to account for jumps and design an estimator that is robust to fragmentation, additive noise, and jumps. To achieve this, we compute Hautsch and Podolskij's 2013 preaveraged realized bipower variation estimator:

$$PRBPV_{GRT,t}^{(m)} = \sum_{i=1}^{m-2k_m} |\Delta\overline{\widetilde{P}^{*}}_{\mathrm{IV},t_i}||\Delta\overline{\widetilde{P}^{*}}_{\mathrm{IV},t_{i+1}}|. \tag{B.1}$$

The estimator in (B.1) still needs to be biascorrected in order to obtain a consistent estimate of the integrated variance:

$$BT_{GRT,t}^{(m)} = \frac{2}{\sqrt{m}\theta\pi\psi_2} PRBPV_{GRT,t}^{(m)} - \frac{\psi_1}{2m\theta^2\psi_2}\sum_{i=1}^{m}|\Delta\widetilde{P}^{*}_{\mathrm{IV},t_i}|^2. \tag{B.2}$$

The noise- and jump-robust two-step $BT_{GRT,t}^{(m)}$ estimator consistently estimates the integrated variance of the efficient price, such that under the price process in (D.9) and Assumption 4, $BT_{GRT,t}^{(m)} \xrightarrow{p} \int\limits_{t-1}^{t} \alpha_{\perp}'\Omega(u)\alpha_{\perp}du$. Similarly to the pre-averaging estimator, finite-sample adjustments can be applied to improve the finite-sample performance of the $BT_{GRT,t}^{(m)}$ estimator, namely replacing the constants ψ_1 and ψ_2 with their empirical counterparts and standardizing the estimator by $m/(m-2k_m+2)$ to account for the true number of summands.[1]

B.2 Disentangling fragmentation noise and additive microstructure noise

We consider the contaminated prices given by the additive noise model

$$\tilde{P}_{t_i} = P_{t_i} + \nu_{t_i}, \tag{B.3}$$

where ν_{t_i} is i.i.d. noise with covariance matrix $\Sigma_{nu} = diag(\sigma_{\nu}^2)$ to simplify the analysis.

As shown by Johansen (1995), we can compute the covariance matrix of the dis-

[1] While we do not provide a CLT for the noise- and jump-robust estimator, such a result can be derived using the existing results in Podolskij and Vetter (2009) and Christensen et al. (2014).

cretely sampled processes ΔP_{t_i}, i.e., the prices that are contaminated by fragmentation noise but not contaminated by additive noise, as follows

$$Var(\Delta P_{t_i}) = \alpha_\delta \Sigma_{\beta\beta} \alpha_\delta' + \Omega_{\delta,t_i}, \tag{B.4}$$

where

$$\Sigma_{\beta\beta} = \sum_{i=0}^{\infty} (I_r + \beta' \alpha_\delta)^i \beta' \Omega_{\delta,t_i} \beta (I_r + \alpha_\delta' \beta)^i. \tag{B.5}$$

In contrast, the variance of the returns of the efficient price $P_{t_i}^*$ is given by

$$Var(\Delta P_{t_i}^*) = \alpha_\perp{}' \Omega_{\delta,t_i} \alpha_\perp. \tag{B.6}$$

The price series are generated according to our baseline stochastic volatility specification with additive microstructure noise. We consider both 5-minute and 1-minute sampling. Now, in our simulation experiment, we compute the daily RV for each price series under different configurations of $\alpha_\perp$ and divide the measure by the number of intraday observations to obtain $\widehat{Var}(\Delta P_{j,t_i})$. Then, we decompose the estimate into fragmentation noise and microstructure noise based on our theoretical measures. Analogously to Theorem 1, this is accomplished by first subtracting the integrated variance from the quadratic variance in each market, $\widehat{Var}(\Delta P_{j,t_i}) - Var(\Delta P_{t_i}^*)$ with $j = 1, \ldots, N$, which leaves the compound noise component $\varsigma_{frag+MSN}^{(j)}$ that includes both fragmentation noise and additive microstructure noise. Since we also know the microstructure noise variance in our simulation, σ_ν^2, we can complete the variance decomposition. Depending on the specified $\alpha_\perp$, we obtain different relative contributions of fragmentation noise and additive noise..

Next, we try to estimate the components by applying the ReMeDI estimator for the microstructure noise variance (Li and Linton, 2022). The results in Table B.1 reveal that the additive noise variance σ_ν^2 can be estimated sufficiently well after first cleaning the prices from fragmentation noise, i.e., computing $\hat{\varsigma}_{GRT,MSN}$ based on $\hat{\alpha}_\perp' \Delta \tilde{P}_{t_i}$. If the estimator is directly applied to the price series, it appears to target the compound noise variance, which includes both fragmentation and additive noise.

B.3 Volatility signature plots

We generate contaminated prices with the additive noise model (i.i.d. noise with diagonal covariance matrix) at the 1-second frequency. We set the noise variance to $7.5 \cdot 10^{-7}$

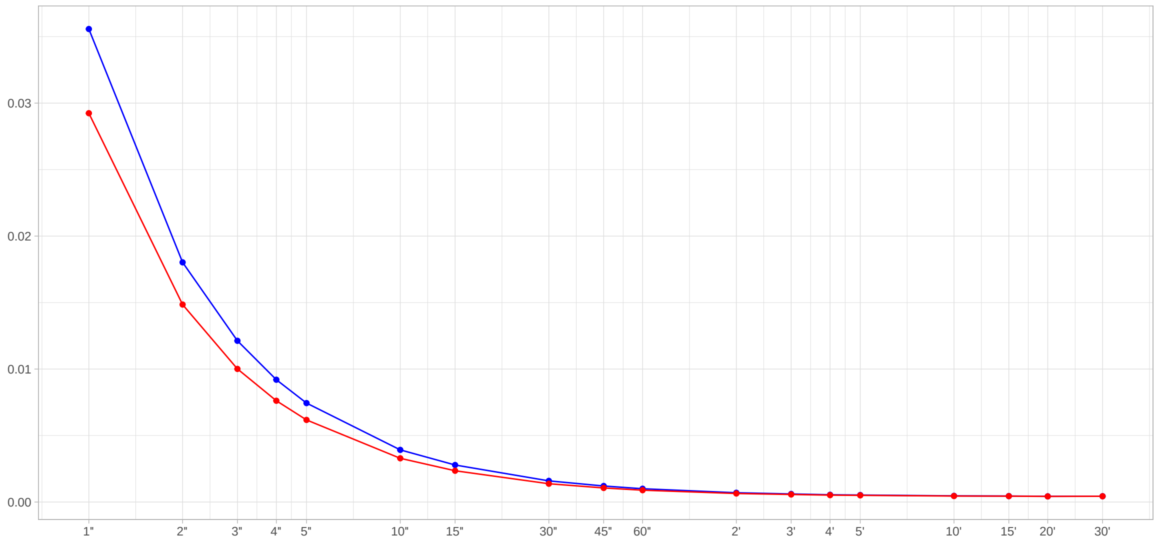
Table B.1: Noise variance decomposition in the stochastic volatility simulation ($\rho = 0$)

$\alpha_\perp$	$RV^{(1)}$	$RV^{(2)}$	$\varsigma^{(1)}_{frag+MSN}$	$\varsigma^{(2)}_{frag+MSN}$	σ^2_ν	$\hat{\varsigma}_{GRT,MSN}$	$\hat{\varsigma}^{(1)}_{frag+MSN}$	$\hat{\varsigma}^{(2)}_{frag+MSN}$
			Panel A: 5-min sampling					
$(0.9, 0.1)'$	5.124	5.127	2.576	2.580	2	3.324	4.636	4.648
$(0.8, 0.2)'$	5.125	5.121	3.013	3.009	2	2.844	4.641	4.643
$(0.7, 0.3)'$	5.128	5.116	3.326	3.314	2	2.525	4.645	4.639
$(0.6, 0.4)'$	5.131	5.111	3.516	3.496	2	2.352	4.648	4.638
$(0.5, 0.5)'$	5.135	5.107	3.582	3.555	2	2.314	4.655	4.635
			Panel B: 1-min sampling					
$(0.9, 0.1)'$	3.106	3.120	2.195	2.210	2	2.524	3.043	3.054
$(0.8, 0.2)'$	3.106	3.120	2.351	2.365	2	2.046	3.043	3.054
$(0.7, 0.3)'$	3.106	3.119	2.462	2.475	2	1.724	3.043	3.053
$(0.6, 0.4)'$	3.106	3.119	2.529	2.541	2	1.556	3.043	3.053
$(0.5, 0.5)'$	3.107	3.119	2.551	2.563	2	1.540	3.044	3.053

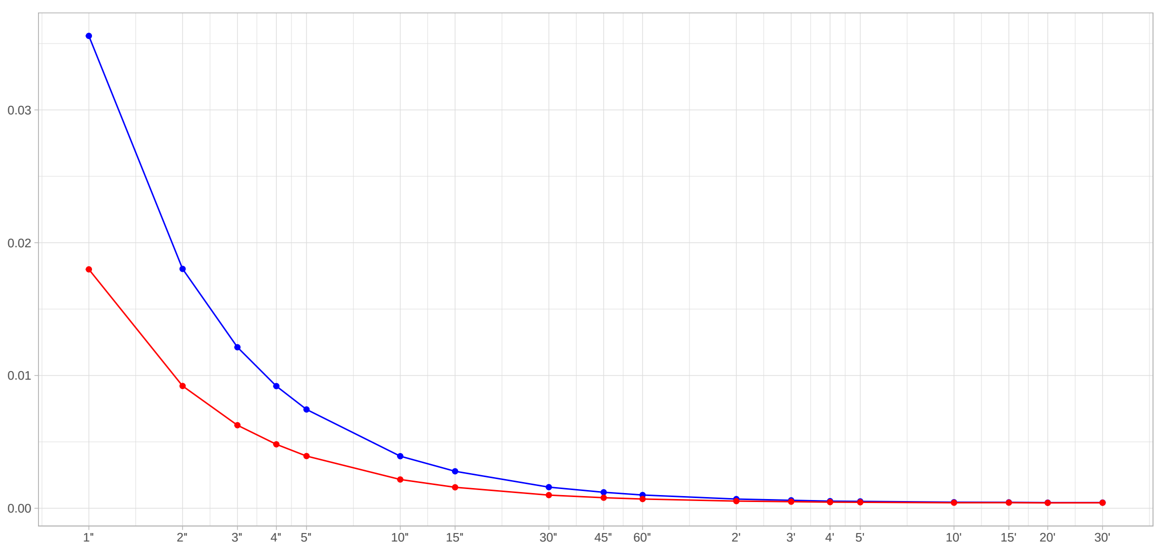
This table reports the averages ($\times 10^6$) over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0$. The superscript (1) and (2) denote the estimators for the price series $P_{1,t}$ and $P_{2,t}$, respectively. $\varsigma^{(j)}_{frag+MSN} = \widehat{Var}(\Delta P_{j,t_i}) - Var(\Delta P^*_{t_i})$ is the theoretical noise variance including fragmentation noise and additive noise. $\hat{\varsigma}_{GRT,MSN}$ denotes the ReMeDI estimator applied to $\alpha_\perp{}'\Delta\tilde{P}_{t_i}$. $\hat{\varsigma}^{(j)}_{frag+MSN}$ denotes the ReMeDI estimator applied to the individual price series.

which is slightly larger than the estimated noise variance obtained from the ReMeDI estimator at the 1-second interval in our empirical application. First, we estimate the $\alpha_\perp$ weights using the instrumental variable estimator to achieve consistency in the presence of additive microstructure noise. Next, we compute the daily realized variance for the first market and for the defragmented returns at different stages of temporal aggregation up to a 30-minutes sampling frequency. The resulting volatility signature plots (see Figure B.1 for a high noise to signal ratio and Figure B.2 for a low noise to signal ratio) show that the additional prices variation due to fragmentation is relatively small if the leading market receives a high weight (upper panel: $\alpha_\perp = (0.9, 0.1)'$). In contrast, the difference increases if both markets receive equal weights (lower panel: $\alpha_\perp = (0.5, 0.5)'$) which implies that both markets adjust to price signals originating in the other market. In both cases, the GRT-based volatility estimates are less affected by additive microstructure noise which aligns well with our theoretical result summarized in Proposition 1 in the main paper.

Figure B.1: Simulated volatility signature plots (noise ratio: 0.001). The blue line indicates the RV estimates for the first market and the red line indicates the RV estimates for the defragmented returns.

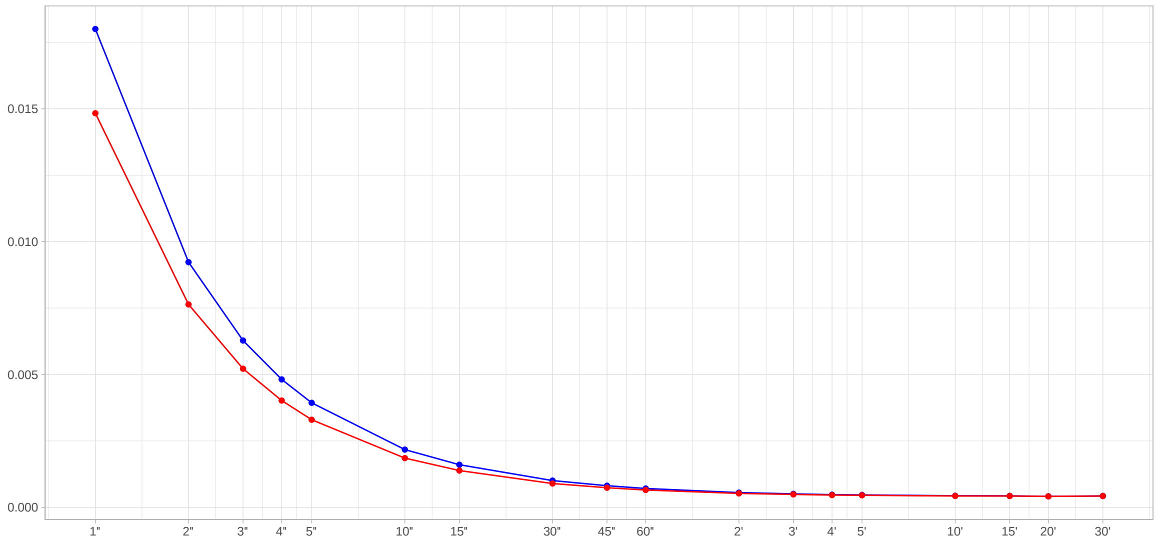


(a) $\alpha_\perp = (0.9, 0.1)'$

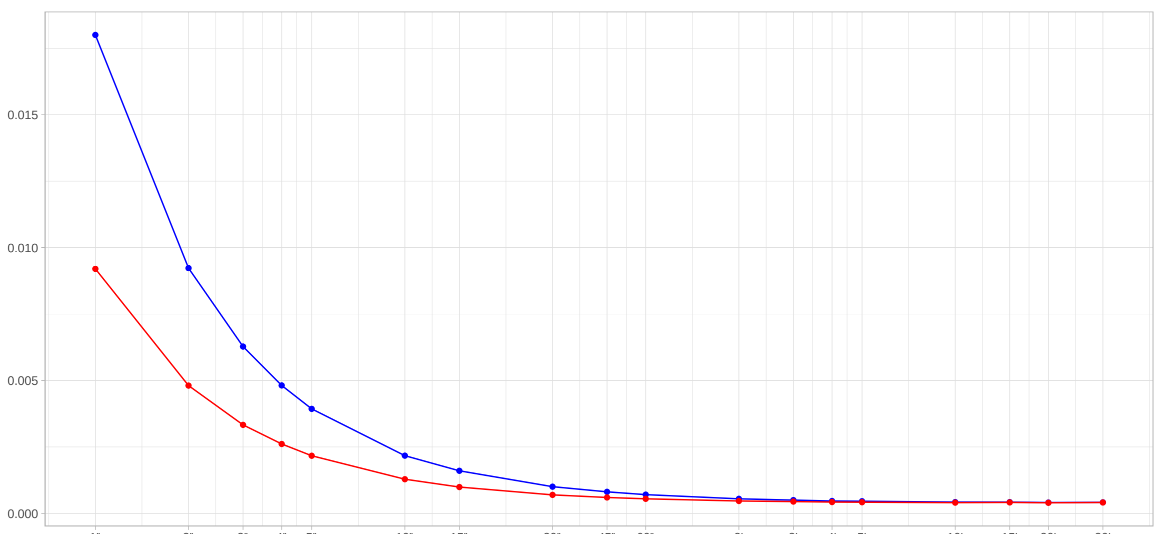


(b) $\alpha_\perp = (0.5, 0.5)'$

Figure B.2: Simulated volatility signature plots (noise ratio: 0.0005). The blue line indicates the RV estimates for the first market and the red line indicates the RV estimates for the defragmented returns.



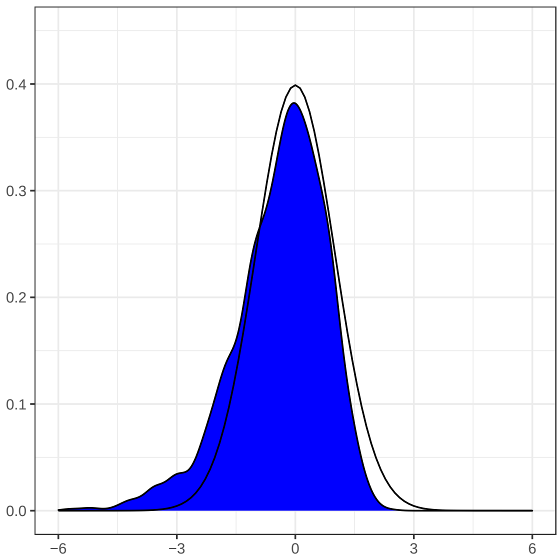
(a) $\alpha_{\perp} = (0.9, 0.1)'$



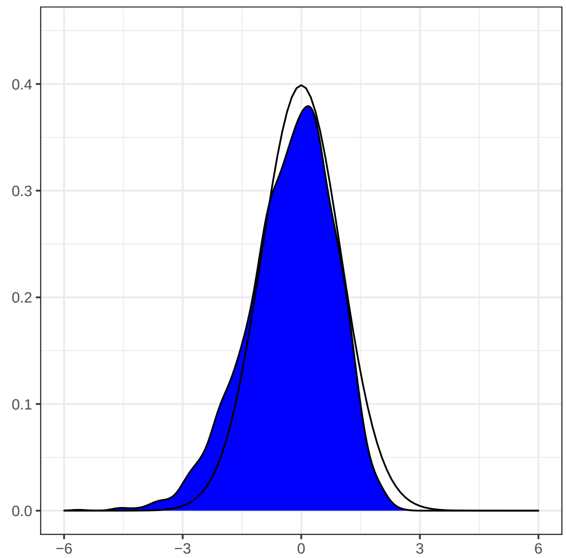
(b) $\alpha_{\perp} = (0.5, 0.5)'$

B.4 Asymptotic approximation: plots

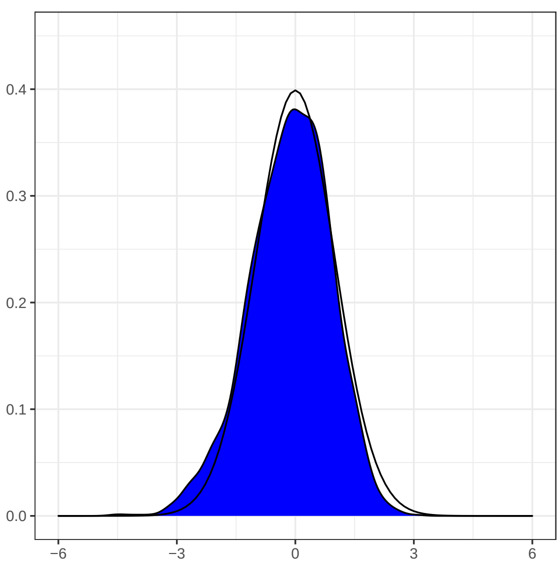
Figure B.3: Asymptotic distribution of $RV_{GRT}^{(m)}$ with a constant covariance matrix, $T = 2,500$, $\alpha = (-5, 5)'$



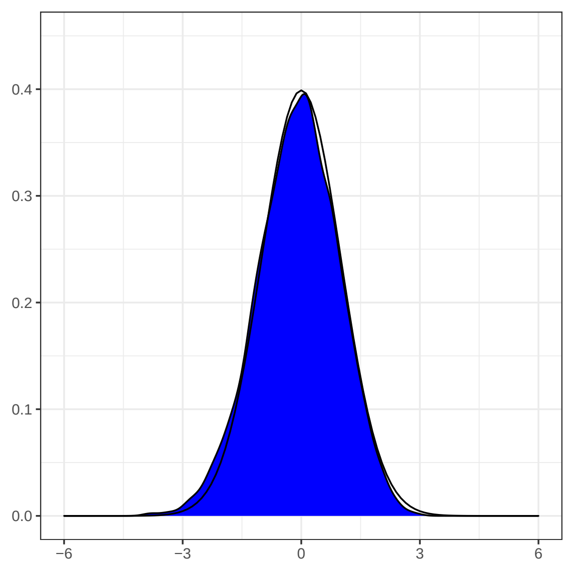
(a) $m = 39$, mean: -0.380, var: 1.334



(b) $m = 78$, mean: -0.262, var: 1.199

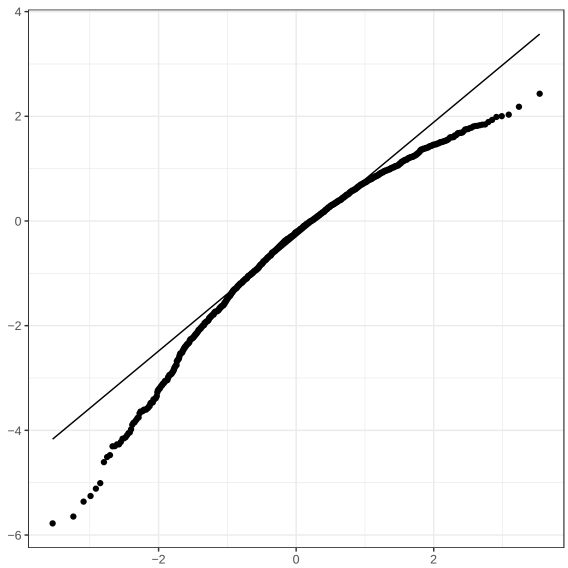


(c) $m = 390$, mean: -0.139, var: 1.047

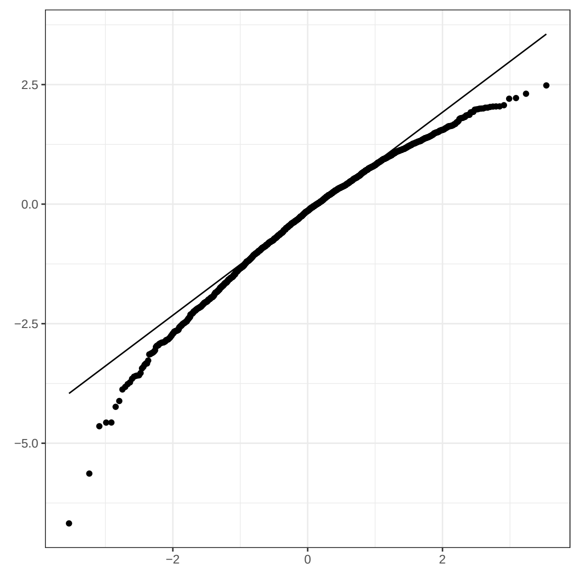


(d) $m = 780$, mean: -0.083, var: 1.032

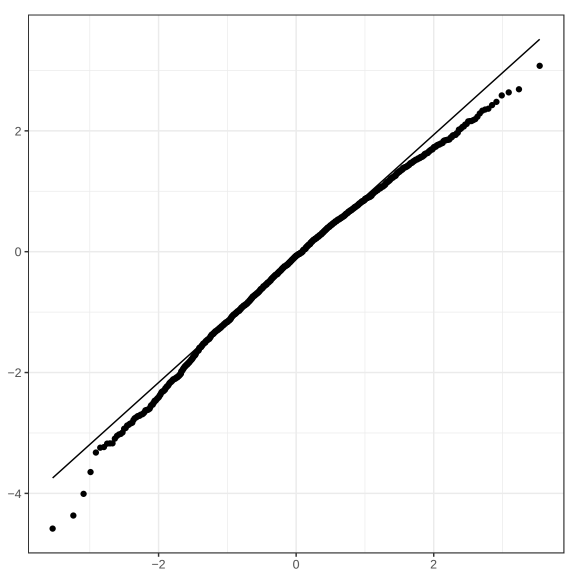
Figure B.4: QQ-plots for $RV_{GRT}^{(m)}$ with a constant covariance matrix, $T = 2,500$, $\alpha = (-5, 5)'$



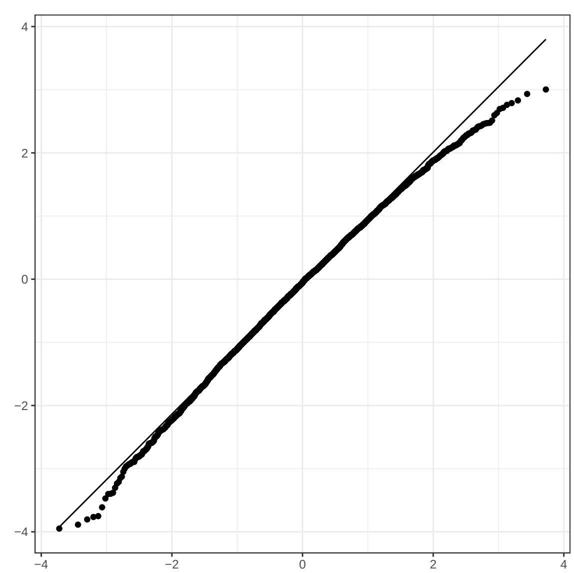
(a) $m = 39$, mean: -0.380, var: 1.334



(b) $m = 78$, mean: -0.262, var: 1.199

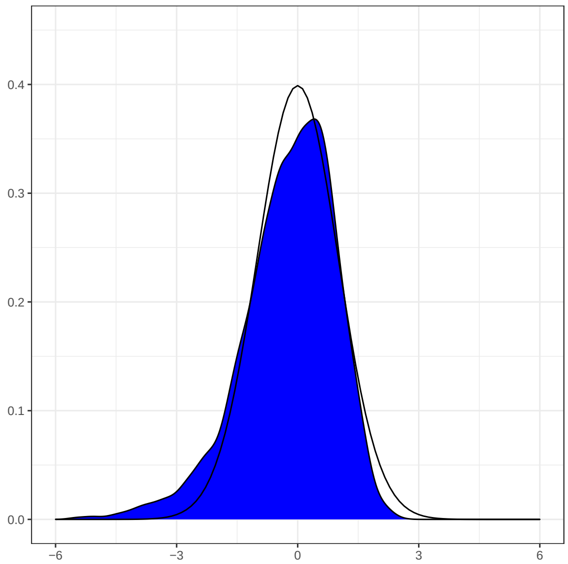


(c) $m = 390$, mean: -0.139, var: 1.047

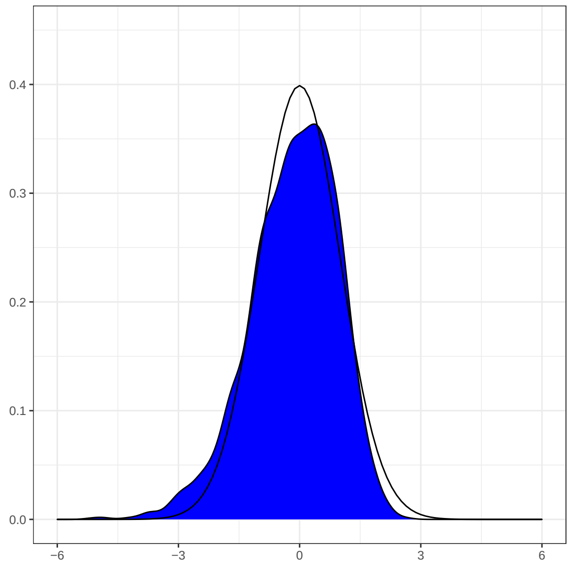


(d) $m = 780$, mean: -0.083, var: 1.032

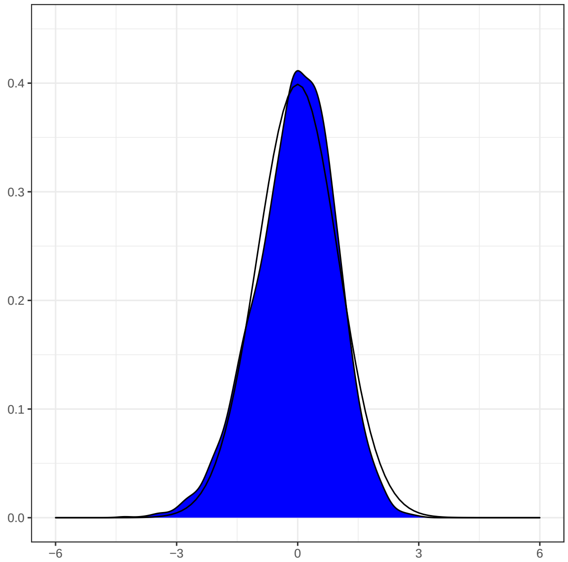
Figure B.5: Asymptotic distribution of $RV_{GRT,t}^{(m)}$ with stochastic volatility, $R = 2,500$, $\alpha = (-5, 5)'$



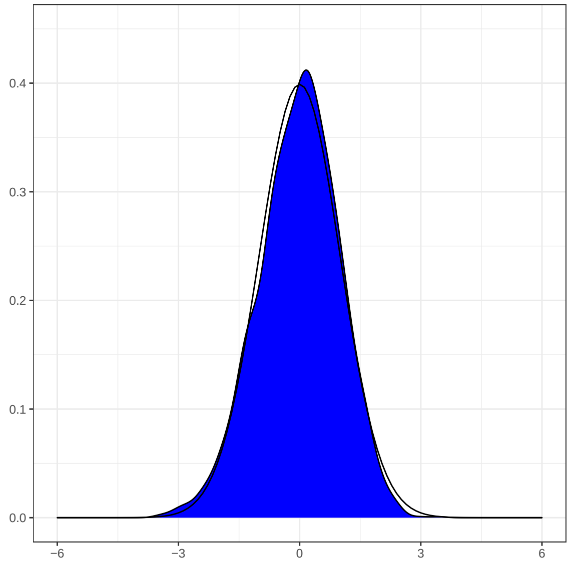
(a) $m = 39$, $T = 100$, $T\delta^2 = 0.0657$
mean: -0.232, var: 1.395



(b) $m = 78$, $T = 200$, $T\delta^2 = 0.0329$
mean: -0.150, var: 1.153

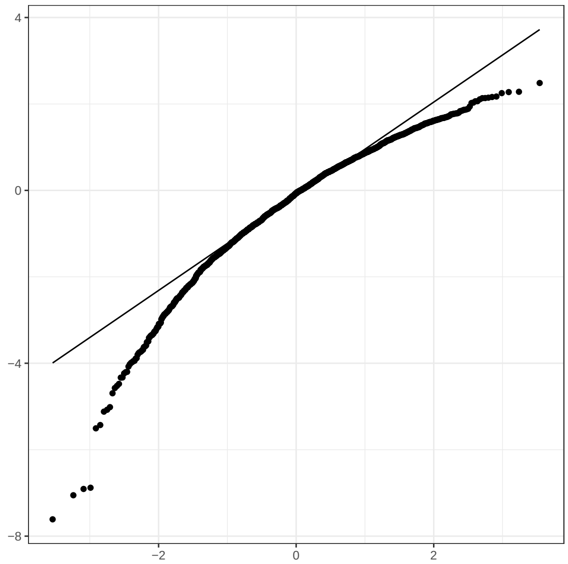


(c) $m = 390$, $T = 400$, $T\delta^2 = 0.0026$
mean: -0.053, var: 0.964

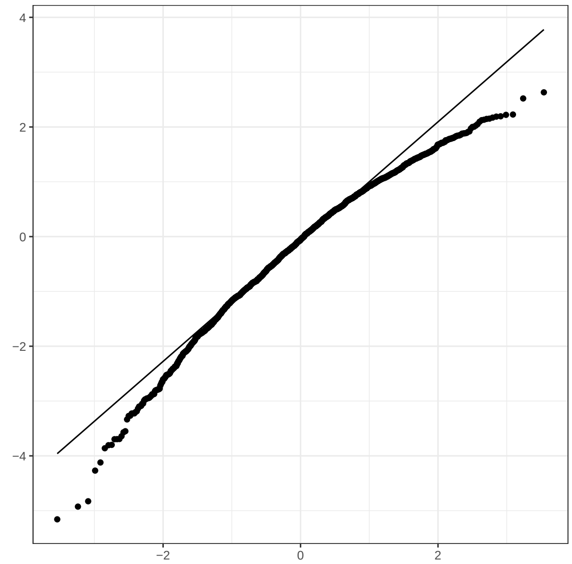


(d) $m = 780$, $T = 800$, $T\delta^2 = 0.0013$
mean: -0.017, var: 0.972

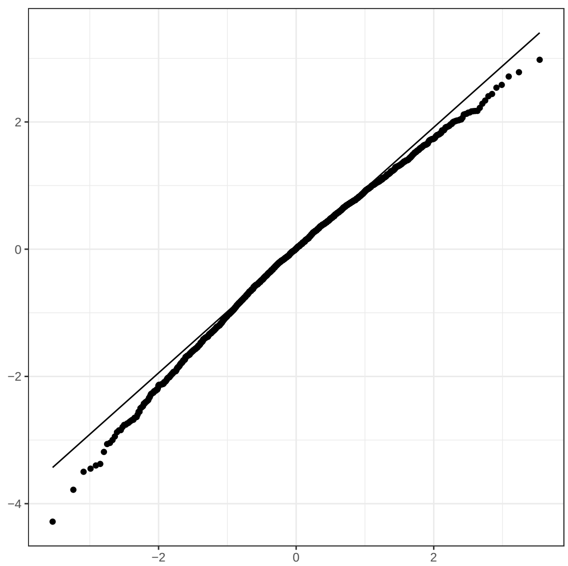
Figure B.6: QQ-plots for $RV_{GRT,t}^{(m)}$ with stochastic volatility, $R = 2,500$, $\alpha = (-5, 5)'$



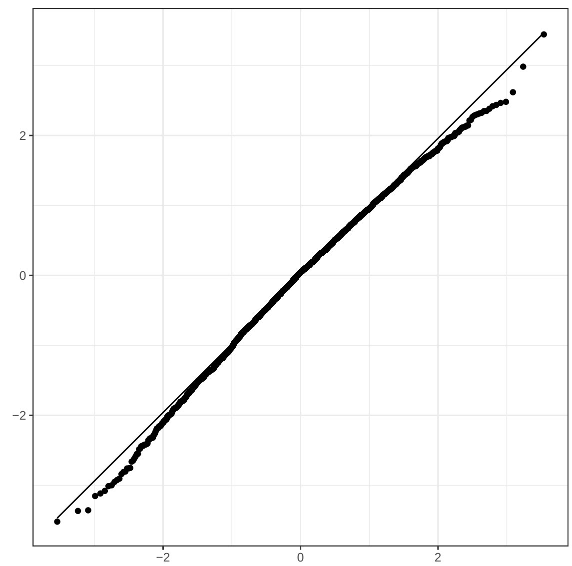
(a) $m = 39$, $T = 100$, $T\delta^2 = 0.0657$
mean: -0.232, var: 1.395



(b) $m = 78$, $T = 200$, $T\delta^2 = 0.0329$
mean: -0.150, var: 1.153



(c) $m = 390$, $T = 400$, $T\delta^2 = 0.0026$
mean: -0.053, var: 0.964



(d) $m = 780$, $T = 800$, $T\delta^2 = 0.0013$
mean: -0.017, var: 0.972

Table B.2: Estimated mean and variance of the standardized $RV^{(m)}_{GRT,t}$

Panel A: mean

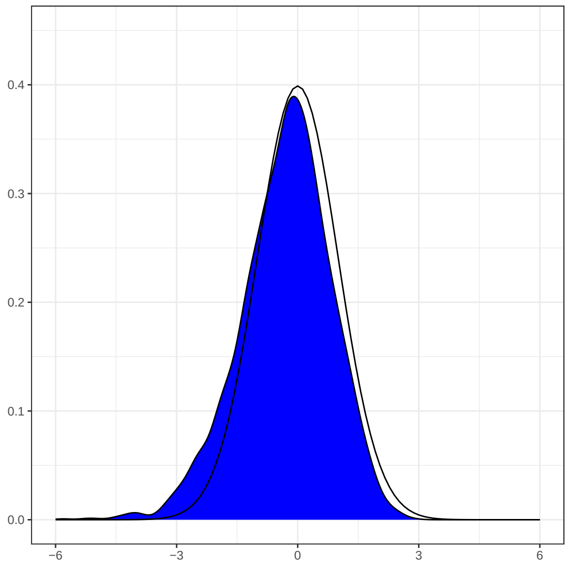
δ \ T	50	100	200	400	800
1/39	-0.237	-0.232	-0.235	-0.236	-0.234
1/78	-0.128	-0.135	-0.150	-0.147	-0.151
1/390	-0.049	-0.046	-0.050	-0.053	-0.047
1/780	0.098	0.125	-0.017	0.006	-0.017
1/2340	1.062	0.027	0.149	0.116	0.014

Panel B: variance

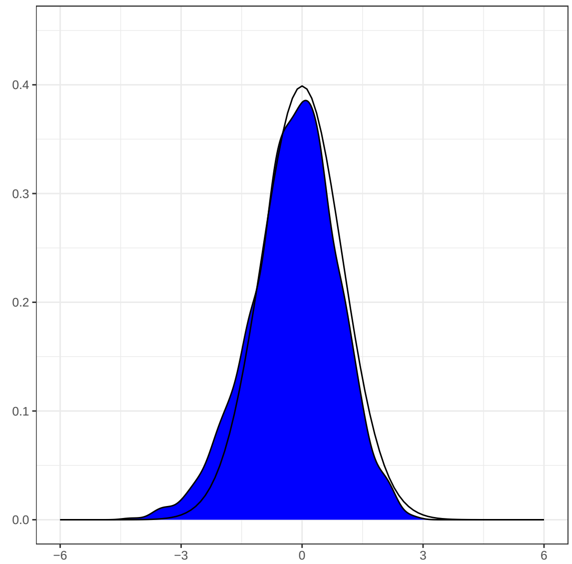
δ \ T	50	100	200	400	800
1/39	1.403	1.395	1.400	1.402	1.398
1/78	1.137	1.142	1.153	1.150	1.154
1/390	0.962	0.961	0.964	0.964	0.964
1/780	0.959	0.956	0.972	0.970	0.972
1/2340	0.893	0.970	0.961	0.963	0.971

Note: This table reports result based on 2,500 draws from the stochastic volatility two-market model with $\rho = 0$.

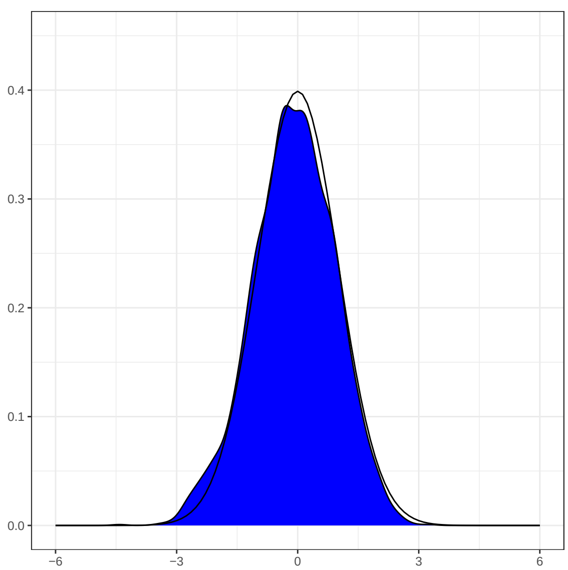
Figure B.7: Asymptotic distribution of $RBPV^{(m)}_{GRT,t}$, $R = 2,500$, $\alpha = (-5, 5)'$



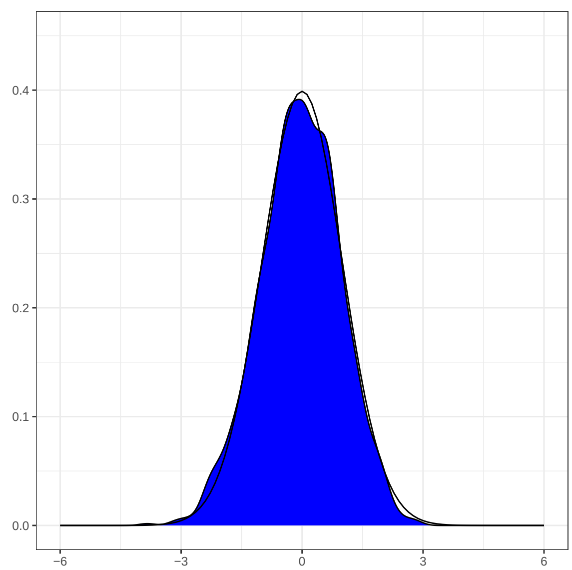
(a) $m = 39$, $T = 100$, $T\delta^2 = 0.0657$
mean: -0.298, var: 1.243



(b) $m = 78$, $T = 200$, $T\delta^2 = 0.0329$
mean: -0.196, var: 1.148

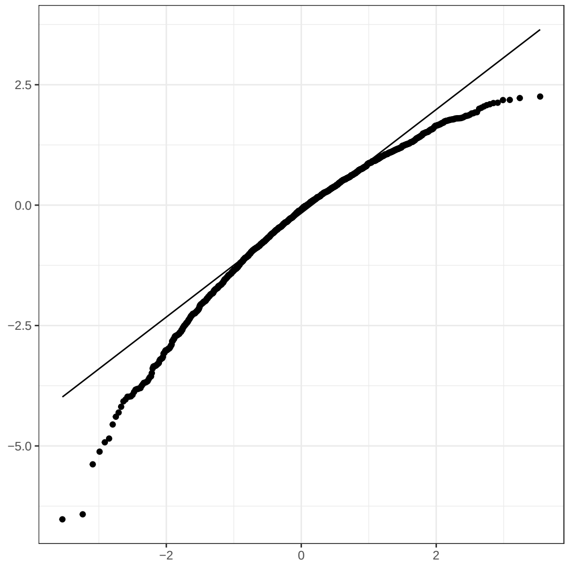


(c) $m = 390$, $T = 400$, $T\delta^2 = 0.0026$
mean: -0.095, var: 1.031

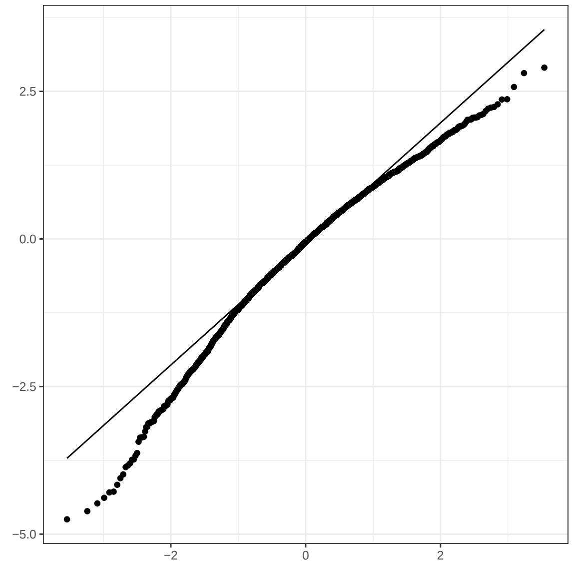


(d) $m = 780$, $T = 800$, $T\delta^2 = 0.0013$
mean: -0.048, var: 0.982

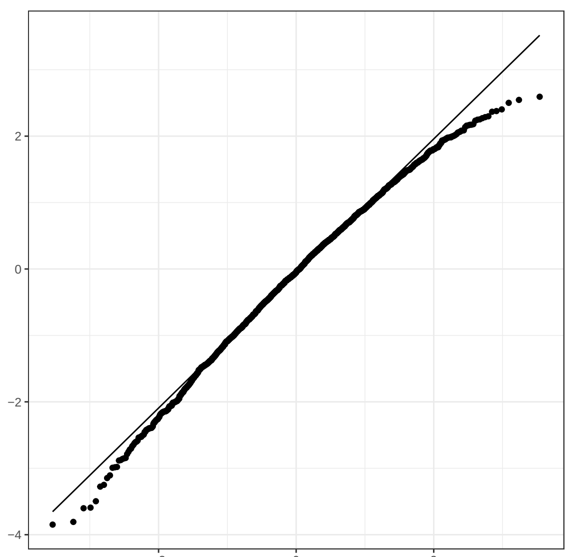
Figure B.8: QQ-plots for $RBPV^{(m)}_{GRT,t}$, $R = 2,500$, $\alpha = (-5, 5)'$



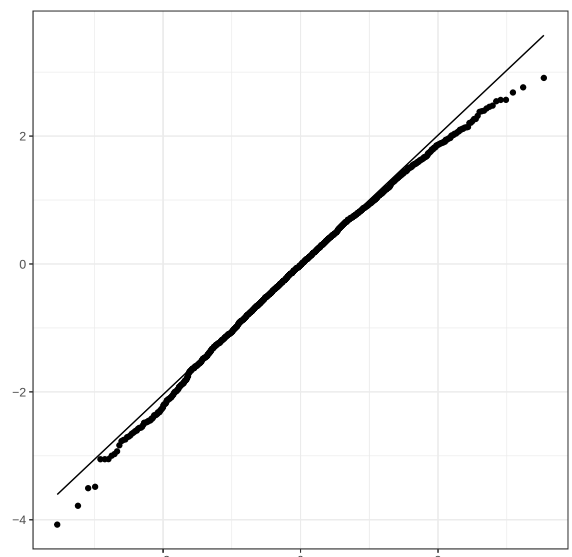
(a) $m = 39$, $T = 100$, $T\delta^2 = 0.0657$
mean: -0.298, var: 1.243



(b) $m = 78$, $T = 200$, $T\delta^2 = 0.0329$
mean: -0.196, var: 1.148

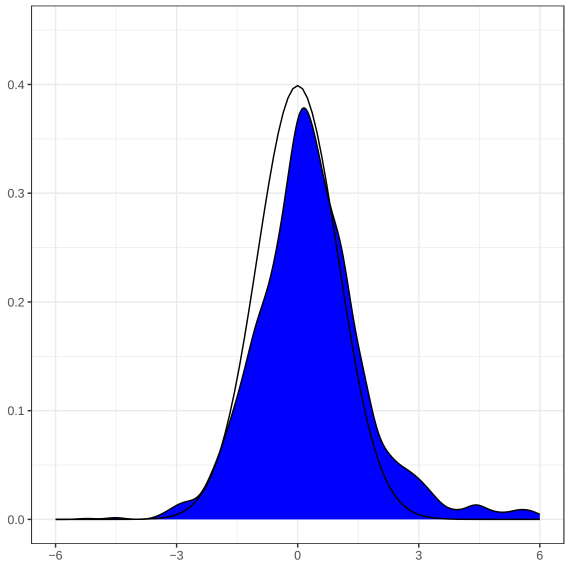


(c) $m = 390$, $T = 400$, $T\delta^2 = 0.0026$
mean: -0.095, var: 1.031

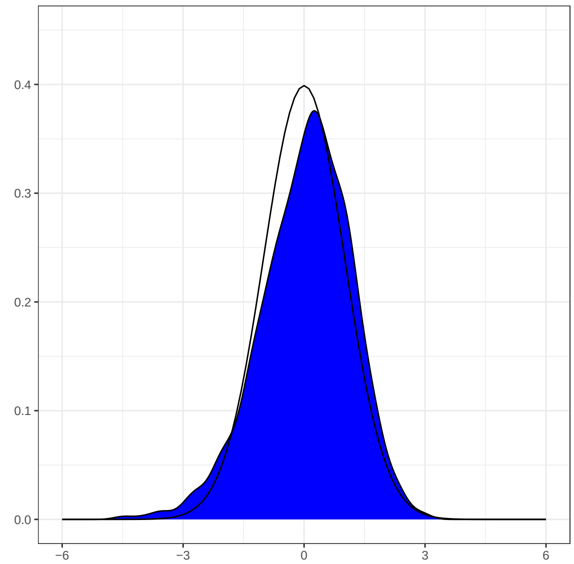


(d) $m = 780$, $T = 800$, $T\delta^2 = 0.0013$
mean: -0.048, var: 0.982

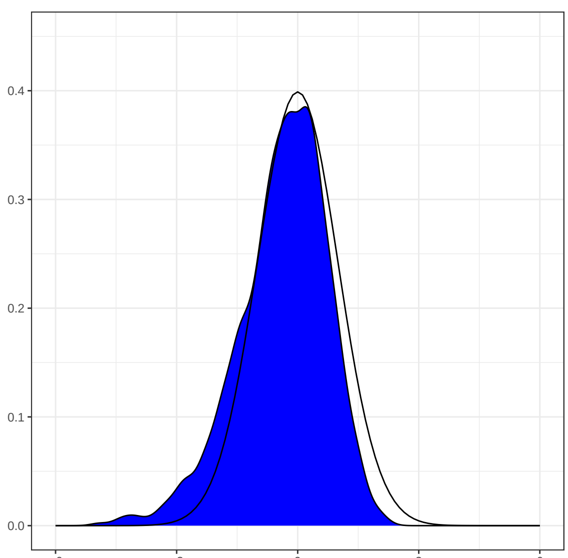
Figure B.9: Asymptotic distribution of the bias-corrected pre-averaged estimator $C_{GRT,t}^{(m)}$, $R = 2,500$, $\alpha = (-5, 5)'$, OLS estimator



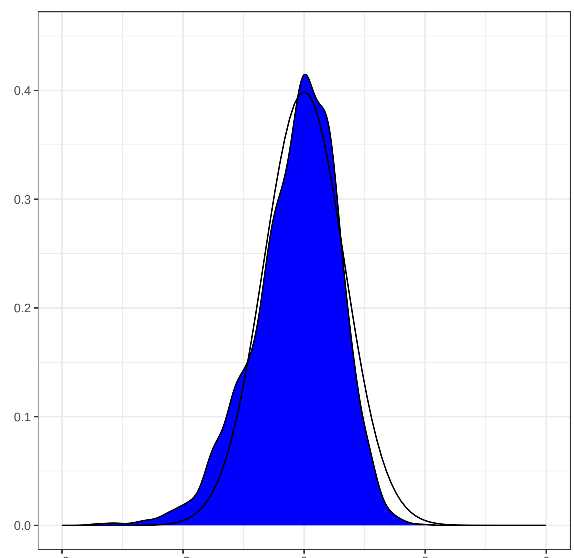
(a) $m = 39$, $T = 100$, $T\delta^2 = 0.0657$
mean: 0.445, var: 3.267



(b) $m = 78$, $T = 200$, $T\delta^2 = 0.0329$
mean: 0.049, var: 1.256

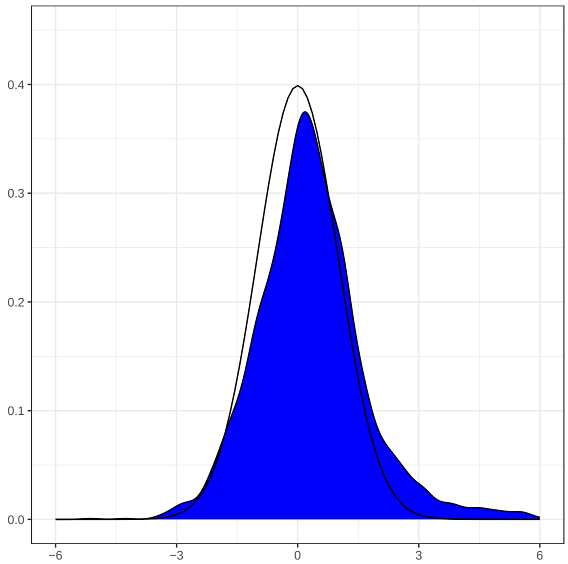


(c) $m = 390$, $T = 400$, $T\delta^2 = 0.0026$
mean: -0.379, var: 1.229

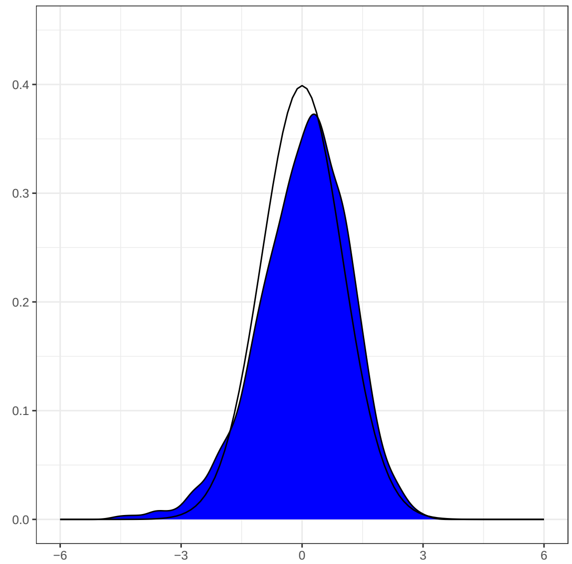


(d) $m = 780$, $T = 800$, $T\delta^2 = 0.0013$
mean: -0.205, var: 1.119

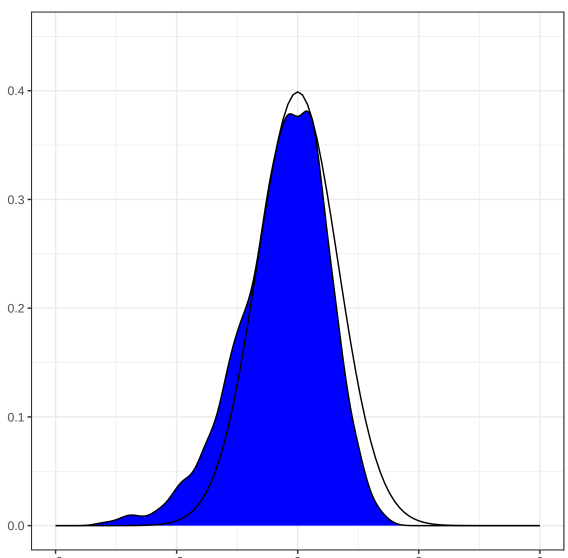
Figure B.10: Asymptotic distribution of the bias-corrected pre-averaged estimator $C_{GRT,t}^{(m)}$, $R = 2,500$, $\alpha = (-5, 5)'$, DGP with MSN, IV estimator



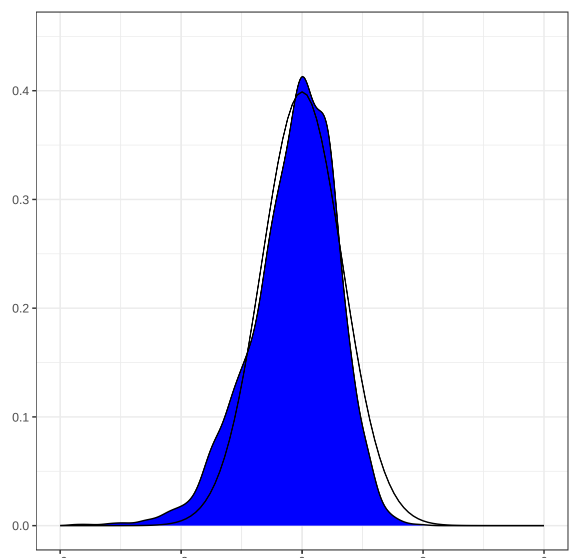
(a) $m = 39$, $T = 100$, $T\delta^2 = 0.0657$
mean: 0.427, var: 3.246



(b) $m = 78$, $T = 200$, $T\delta^2 = 0.0329$
mean: 0.050, var: 1.255

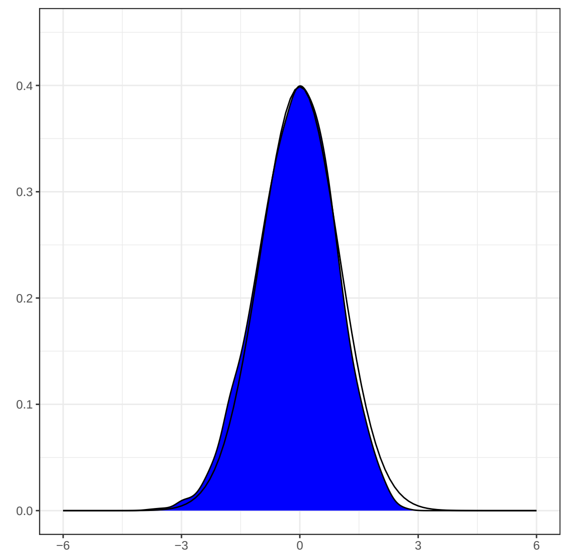


(c) $m = 390$, $T = 400$, $T\delta^2 = 0.0026$
mean: -0.379, var: 1.231



(d) $m = 780$, $T = 800$, $T\delta^2 = 0.0013$
mean: -0.203, var: 1.117

Figure B.11: Asymptotic distribution of the bias-corrected pre-averaged estimator $C_{GRT,t}^{(m)}$, $T = 2500$, DGP with MSN, IV estimator



$m = 23400$, mean: -0.100, var: 0.956

Table B.3: Estimated mean and variance of the standardized bias-corrected pre-averaged estimator $C^{(m)}_{GRT,t}$, OLS estimator

Panel A: mean					
δ \ T	50	100	200	400	800
1/39	0.427	0.411	0.412	0.405	0.413
1/78	0.049	0.063	0.071	0.048	0.048
1/390	-0.373	-0.342	-0.367	-0.379	-0.380
1/780	-0.128	-0.200	-0.203	-0.202	-0.205
1/2340	-0.172	-0.211	-0.272	-0.274	-0.270

Panel B: variance					
δ \ T	50	100	200	400	800
1/39	2.966	2.678	2.683	2.700	2.884
1/78	1.254	1.244	2.196	1.256	1.255
1/390	1.232	1.219	1.230	1.233	1.234
1/780	1.093	1.116	1.117	1.117	1.118
1/2340	1.124	1.136	1.153	1.154	1.153

Note: This table reports result based on 2,500 draws from the stochastic volatility two-market model with $\rho = 0$.

Table B.4: Estimated mean and variance of the standardized bias-corrected pre-averaged estimator $C^{(m)}_{GRT,t}$, IV estimator

Panel A: mean

δ \ T	50	100	200	400	800
1/39	0.436	0.427	0.436	0.426	0.400
1/78	0.048	0.069	0.050	0.049	0.050
1/390	-0.376	-0.321	-0.346	-0.379	-0.379
1/780	-0.030	-0.168	-0.205	-0.204	-0.203
1/2340	0.034	0.095	-0.010	-0.244	-0.267

Panel B: variance

δ \ T	50	100	200	400	800
1/39	2.854	3.246	3.893	3.086	2.819
1/78	1.256	1.896	1.255	1.256	1.254
1/390	1.232	1.210	1.220	1.233	1.234
1/780	1.061	1.106	1.118	1.118	1.117
1/2340	1.068	1.051	1.080	1.145	1.152

Note: This table reports result based on 2,500 draws from the stochastic volatility two-market model with $\rho = 0$.

C Monte Carlo simulation

In our simulation experiments, we evaluate the relative performance of our GRT-based class of estimators across various data-generating processes (DGPs), including those with constant and stochastic volatility processes, small ($N = 2$) and large ($N = 5$) number of exchanges, jump component, and additive market microstructure noise. We start by defining the set of alternative univariate estimators we employ in our analysis.[2] The first estimator we consider is the standard RV estimator, which is the efficient estimator in the absence of both additive market microstructure and fragmentation noises. Next, we focus on estimators that are robust to different types of additive market microstructure noise contamination. Specifically, we consider two realized kernel estimators proposed by Hansen and Lunde (2006), denoted by RV^{AC} and RV^{NW}. These two estimators are unbiased in the presence of endogenous noise. Additionally, we consider two variants of the generalized realized kernel estimators proposed by Barndorff-Nielsen et al. (2008, 2011) constructed using the modified Tukey-Hanning (RK^{MTH}) and Parzen (RK^{P}) kernel functions, as well as the Hautsch and Podolskij's 2013 noise-robust pre-averaging estimator. The tuning parameter θ in the pre-averaging estimator is set identically to that used in the GRT-based estimator introduced in Section 2.4.2 in the main paper. Finally, we consider the jump-robust realized bipower variation measure proposed by Barndorff-Nielsen and Shephard (2004c) as this is the most widely used estimator of the integrated variance in the presence of jumps.

C.1 Constant volatility: reduced-form models

The first DGP is based on De Jong's 2002 two-market price discovery model (see also Hasbrouck, 1995, for a more comprehensive discussion on how this simple DGP captures cointegration in market microstructure models). This simple price discovery model illustrates how the presence of learning induces an error correction mechanism, which in turn leads to fragmentation noise. Consequently, additive noise-robust estimators are generally inconsistent for the integrated variance (see Theorem 1). We generate 1-minute data ($m = 390$, NYSE trading hours from 9:30 to 16:00) over $T = 1826$ trading

[2] Subsection D.1 in the Online Appendix provides a detailed description of the alternative estimators used in the Monte Carlo exercises.

days (five years) according to the bivariate VECM,

$$\begin{bmatrix} \Delta P_{1,t_i} \\ \Delta P_{2,t_i} \end{bmatrix} = \begin{bmatrix} \alpha_{\delta,1} \\ \alpha_{\delta,2} \end{bmatrix} (P_{1,t_{i-1}} - P_{2,t_{i-1}}) + \begin{bmatrix} \epsilon_{1,t_i} \\ \epsilon_{2,t_i} \end{bmatrix}, \quad i = 1, \ldots, m, \quad t = 1, \ldots, T, \quad \text{(C.1)}$$

where the error term $\epsilon_{t_i} = (\epsilon_{1,t_i}, \epsilon_{2,t_i})'$ is mean-zero i.i.d. Gaussian with constant covariance matrix

$$\Omega_\delta = \begin{pmatrix} \tau \cdot 1.3 \cdot 10^{-7} & 10^{-8} \\ 10^{-8} & 1.3 \cdot 10^{-7} \end{pmatrix}, \quad \text{(C.2)}$$

and τ is a scaling factor for the first markets' innovation variance. The covariance matrix Ω_δ and the values of α_δ are chosen in accordance with our empirical application.

We investigate the performance of the GRT-based RV estimator computed using prices sampled at the 1-minute interval under two alternative specifications of (C.1). The first configuration (DGP I) sets $\alpha = (-0.5, 0.5)'$, which yields $\alpha_{\delta=1/390} \approx (-0.00128, 0.00128)'$ and $\alpha_\perp = (0.5, 0.5)'$. The latter implies a high level of fragmentation noise, as both markets contribute equally to the price discovery process. In line with our empirical exercise, we set $\tau = 1.25$ to have slightly unequal variance levels in the idiosyncratic innovation processes across the two markets.[3] Finally, the quadratic variation of P^* and P in DGP I are given by

$$\langle P^*, P^{*\prime} \rangle_{t-1:t} = 3.055 \cdot 10^{-5} \quad \text{and} \quad \langle P, P' \rangle_{t-1:t} = \begin{pmatrix} 6.357 \cdot 10^{-5} & 3.879 \cdot 10^{-6} \\ 3.879 \cdot 10^{-6} & 5.086 \cdot 10^{-5} \end{pmatrix}. \quad \text{(C.3)}$$

The left panel of Table C1 presents the results for DGP I.[4] We report the average bias across all replications, along with the corresponding root mean squared error (RMSE) in parentheses for each estimator. More specifically, the first set of results corresponds to the multivariate GRT-based RV estimator, whereas the second and third sets display the results from the alternative univariate estimators computed using prices from markets one and two, respectively: the RV, PRV, RV^{AC}, RV^{NW}, RK^{MTH}, and RK^{P} estimators. In summary, our two-step RV estimator outperforms all competing estimators in terms of both bias and RMSE, thereby confirming our theoretical results:

[3] The variance of the innovation terms can be estimated from the residuals of the VECM when applied to empirical data. For our sample of DJIA stocks, these variance ratios range from 1.014 (CSCO) to 1.471 (TRV).

[4] Additional simulation results for mixed frequencies are reported in Table D2 in the Online Appendix. In short, results remain unchanged when the noise-robust estimators are computed using prices sample at higher frequencies.

estimators that satisfy Assumption 3 are not robust to fragmentation noise and hence deliver inconsistent estimates of $\langle P^*, P^{*\prime}\rangle_{t-1:t}$ (Theorem 1), whereas our two-step estimator successfully estimates the integrated variance (Theorem 2). More specifically, we find that the univariate RV estimator is a consistent estimator of the respective diagonal element of the quadratic variation of P and, therefore, fails to consistently estimate the integrated variance of the efficient price. The kernel-based estimators unsuccessfully attempt to control for some of the fragmentation noise, and their estimates remain largely biased irrespective of the lag truncation we adopt.

The second parametrization (DGP II) sets $\alpha = (-0.9, 0.1)'$ with the corresponding 1-minute adjustment coefficients and their orthogonal complement given by $\alpha_{\delta=1/390} = (-0.0023, 0.0003)'$ and $\alpha_\perp = (0.1, 0.9)'$, respectively. The latter implies a low level of fragmentation noise, as the second market is almost entirely responsible for impounding information to the efficient price. We set $\tau = 1$ to ensure the same variance level of the idiosyncratic innovation process for both markets at the 1-minute sampling interval. The integrated variance of the efficient price and the quadratic variation of P then read

$$\langle P^*, P^{*\prime}\rangle_{t-1:t} = 4.238 \cdot 10^{-5} \quad \text{and} \quad \langle P, P'\rangle_{t-1:t} \; = \begin{pmatrix} 5.085 \cdot 10^{-5} & 3.900 \cdot 10^{-6} \\ 3.900 \cdot 10^{-6} & 5.085 \cdot 10^{-5} \end{pmatrix}. \quad \text{(C.4)}$$

The right panel of Table C1 presents the results. Similarly to the first specification, the GRT-based estimator largely outperforms the competing estimators in terms of both bias and efficiency. The bias associated with the RV_{GRT} estimator is close to zero, which reinforces the theoretical properties established in Theorem 2. In contrast, the alternative univariate estimators exhibit substantial biases across the board. Given the lower level of fragmentation noise relative to DGP I, both the bias and RMSE measures are significantly smaller than those reported under DGP I. More specifically, although the realized kernel estimators appear to mitigate some of the fragmentation noise as the lag truncation parameter increases, the estimates based on both market prices remain severely biased even when the lag truncation is set to 20. Overall, our simulation findings confirm that the noise-robust univariate estimators are not able to separate the fragmentation noise from the efficient price process if both markets contribute to the price discovery process.[5]

[5] As highlighted in Theorem 1, if one market is solely responsible for the price discovery process - i.e., there is no fragmentation noise – then the univariate estimator based on prices from the leading exchange is consistent for the integrated variance. Results are available from the authors upon request.

Table C1: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU process

		DGP I			DGP II	
		P^*			P^*	
RV_{GRT}	**0.001 (0.217)**	–	–	**-0.167 (0.335)**	–	–
		P_1			P_1	
RV_1	3.289 (3.322)	–	–	0.843 (0.923)	–	–
PRV_1	2.597 (2.870)	–	–	0.308 (1.030)	–	–
	$H = 3$	$H = 8$	$H = 20$	$H = 3$	$H = 8$	$H = 20$
RV_1^{AC}	3.273 (3.497)	3.237 (3.775)	3.172 (4.311)	0.843 (1.298)	0.834 (1.773)	0.830 (2.508)
RV_1^{NW}	3.282 (3.373)	3.261 (3.456)	3.220 (3.654)	0.844 (1.051)	0.837 (1.244)	0.828 (1.622)
RK_1^{MTH}	3.281 (3.391)	3.272 (3.449)	3.241 (3.586)	0.843 (1.089)	0.842 (1.215)	0.833 (1.490)
RK_1^{P}	3.283 (3.402)	3.266 (3.469)	3.232 (3.640)	0.845 (1.108)	0.840 (1.259)	0.830 (1.585)
		P_2			P_2	
RV_2	2.026 (2.059)	–	–	0.839 (0.914)	–	–
PRV_2	1.490 (1.772)	–	–	0.316 (1.013)	–	–
	$H = 3$	$H = 8$	$H = 20$	$H = 3$	$H = 8$	$H = 20$
RV_2^{AC}	2.021 (2.231)	2.031 (2.531)	1.958 (3.092)	0.842 (1.267)	0.865 (1.747)	0.824 (2.569)
RV_2^{NW}	2.028 (2.116)	2.020 (2.210)	1.996 (2.422)	0.845 (1.040)	0.844 (1.233)	0.835 (1.619)
RK_2^{MTH}	2.032 (2.142)	2.018 (2.191)	2.011 (2.343)	0.849 (1.085)	0.839 (1.197)	0.842 (1.473)
RK_2^{P}	2.031 (2.149)	2.014 (2.212)	2.008 (2.400)	0.849 (1.101)	0.837 (1.240)	0.842 (1.569)

Note: The table reports the estimated average bias ($\times 10^5$) over the full sampling period with the RMSE ($\times 10^5$) shown in parentheses. Data are generated from DGP I (left panel), with $\alpha = (-0.5, 0.5)'$, $\alpha_\perp = (0.5, 0.5)'$, and $\tau = 1.25$; and DGP II (right panel), with $\alpha = (-0.9, 0.1)'$, $\alpha_\perp = (0.1, 0.9)'$ and $\tau = 1$. H denotes the bandwidth used in both the realized kernel and Hansen and Lunde's 2006 noise-robust estimators. The estimators with the lowest RMSE are marked in boldface. The subscripts in the second and third panels indicate that the realized measures are computed using prices from market one and two, respectively. Finally, the true values, scaled up by 10^5, are: for DGP I, $\langle P^*, P^{*\prime}\rangle_{t-1:t} = 3.055$, $\langle P, P'\rangle^{1,1}_{t-1:t} = 6.357$ and $\langle P, P'\rangle^{2,2}_{t-1:t} = 5.086$; and for DGP II $\langle P^*, P^{*\prime}\rangle_{t-1:t} = 4.238$, $\langle P, P'\rangle^{1,1}_{t-1:t} = 5.085$ and $\langle P, P'\rangle^{2,2}_{t-1:t} = 5.085$.

C.2 Stochastic volatility process

This experiment examines how the properties of our GRT-based RV estimator vary with the level of fragmentation noise under a realistic set of assumptions about the price process. Specifically, we relax the constant volatility assumption used in the previous DGP and parameterize the market-specific spot variances as following a single-factor stochastic volatility process with serially correlated daily integrated variances. Moreover, we allow for the leverage effect, i.e., correlation between the innovations of the volatility and price processes.

We assume a two-market setting ($N = 2$), where both price processes share the same parametrization of the stochastic volatility processes (Dias et al., 2021). More specifically, we simulate prices from the cointegrated OU type process in (1) in the main paper at a 1-second sampling interval over $T = 1826$. The contemporaneous correlation between the innovation processes is constant and takes values $\rho \in \{0.5, 0.7, 0.9\}$.[6] The market-specific spot variances follow a single-factor stochastic volatility process as in Huang and Tauchen (2005)The spot variances evolve according to the process

$$\sigma_j^2(t) = \exp(\varsigma_0 + \varsigma_1 V_j(t)), \tag{C.5}$$

with

$$dV_j(t) = \gamma_V V_j(t)dt + dB_j(t), \;\; j = 1, 2. \tag{C.6}$$

We follow Barndorff-Nielsen et al. (2008) and choose $\varsigma_0 = 0$, $\varsigma_1 = 0.125$, $\gamma_V = -0.025$, $corr(dW_j(t), dB_j(t)) = -0.30$, and $corr(dB_1(t), dB_2(t)) = 0.95$.[7] The daily integrated market-specific variances follow the AR(1) process,

$$\ln \sigma_{j,t}^2 = \varrho_0 + \varrho_1 \ln \sigma_{j,t-1}^2 + \varsigma \upsilon_{j,t}, \quad j = 1, 2, \quad t = 1, \ldots, T, \tag{C.7}$$

where the volatility innovations $\upsilon_t = (\upsilon_{1,t}, \upsilon_{2,t})'$ are generated by a two-dimensional Gaussian white noise process with constant correlation of 0.95 and unit variances. The autoregressive parameter is set to 0.98, while the coefficients ϱ_0 and ς are calibrated

[6] We report our main results for $\rho = 0.7$, a value chosen to reflect a moderately high level of contemporaneous correlation, relative to the average of 0.56 observed in our empirical application. The results for alternative specifications of the contemporaneous correlation parameter are reported in Tables D4 and D5 in Subsection D.3 of the Online Appendix.

[7] The results are not sensitive to the leverage correlation parameter and the correlation between the innovations of both volatility processes. Additional results are reported in Subsection D.3 of the Online Appendix.

such that the expected variance is $7.5 \cdot 10^{-4}$ and thereby corresponds approximately to the values observed in our empirical application. Finally, to ensure a fair comparison, we compute the GRT-based RV estimators and its univariate counterparts using prices sampled at the 1-minute frequency, while all noise-robust estimators are computed with prices sampled at the 1-second interval.

The upper and lower panels in Table C2 report the bias and RMSE, respectively, for all estimators. In summary, our RV_{GRT} estimator outperforms all competing univariate estimators in terms of both bias and RMSE. This result holds regardless of the specification of $\alpha_{\perp}$. Moreover, we find that the relative performance of the GRT-based estimator improves as the level of fragmentation noise increases. Consistent with Theorem 1, we find even stronger support for our multivariate estimator when the contemporaneous correlation is lower (e.g., $\rho = 0.5$). Additionally, estimators that satisfy Assumption 3 fail to consistently estimate the integrated variance in the presence of fragmentation noise. Finally, Appendix D in the Online Appendix presents additional simulations in which an *ad hoc* equal-weighted estimator is considered. In line with Theorem 2, estimating the drift substantially pays off in terms of both bias and RMSE measures.

Table C2: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (mixed sampling frequencies)

	Bias												
$\alpha_\perp$	$RV_{GRT}^{(390)}$	$RV_1^{(390)}$	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	$RV_2^{(390)}$	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
$(0.9, 0.1)'$	**0.400**	0.618	0.543	0.659	0.655	0.631	0.640	0.637	0.560	0.605	0.607	0.709	0.682
$(0.8, 0.2)'$	**0.272**	1.124	1.048	1.166	1.161	1.129	1.133	1.142	1.065	1.112	1.113	1.207	1.174
$(0.7, 0.3)'$	**0.165**	1.486	1.410	1.529	1.524	1.486	1.485	1.505	1.427	1.474	1.476	1.564	1.527
$(0.6, 0.4)'$	**0.077**	1.705	1.629	1.748	1.743	1.702	1.698	1.724	1.646	1.694	1.695	1.779	1.739
$(0.5, 0.5)'$	**0.009**	1.780	1.704	1.824	1.819	1.776	1.770	1.799	1.721	1.769	1.771	1.853	1.811
	RMSE												
$\alpha_\perp$	$RV_{GRT}^{(390)}$	$RV_1^{(390)}$	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	$RV_2^{(390)}$	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
$(0.9, 0.1)'$	1.565	1.768	1.769	1.314	**1.211**	3.812	5.160	2.162	2.194	1.441	1.383	4.261	5.349
$(0.8, 0.2)'$	**1.423**	2.379	2.349	2.138	2.074	4.140	5.389	2.697	2.677	2.108	2.078	4.565	5.554
$(0.7, 0.3)'$	**1.331**	2.910	2.866	2.757	2.707	4.467	5.628	3.174	3.127	2.662	2.644	4.864	5.771
$(0.6, 0.4)'$	**1.284**	3.255	3.204	3.141	3.097	4.695	5.798	3.482	3.422	3.008	2.995	5.068	5.922
$(0.5, 0.5)'$	**1.274**	3.386	3.330	3.283	3.242	4.782	5.861	3.583	3.519	3.122	3.110	5.135	5.967

The upper panel reports the average bias ($\times 10^5$) across all replications, whereas the lower panel displays the RMSE ($\times 10^5$). The results are based on 1,000 simulations from the two-market stochastic volatility model with $\rho = 0.7$. We report results for the 1-minute GRT-based RV estimator, $RV_{GRT}^{(390)}$, alongside the 1-minute univariate RV estimator and several alternative noise-robust estimators computed at a 1-second frequency: the pre-averaging estimator (PRV); the noise-robust estimators RV^{AC} and RV^{NW} proposed by Hansen and Lunde (2006); and two variants of the realized kernel estimators, RK^{MTH} and RK^{P}. The subscripts 1 and 2 indicate that the realized measures are computed using prices from market one and two, respectively. The estimators with the lowest RMSE are marked in boldface.

C.3 Jump-diffusion model with additive microstructure noise

This DGP extends the baseline model simulated in Subsection C.2 by incorporating additive market microstructure noise and a jump component to market prices. More specifically, we examine the performance of the 1-minute GRT-based realized variance, 1-minute realized bipower variation, pre-averaging, and pre-averaged realized bipower estimators vis-à-vis their univariate estimators counterparts across settings with different levels of informational fragmentation and market microstructure noise.

We assume a five-market setting ($N = 5$) to mirror the structure of our empirical analysis. The prices are generated according to the jump-diffusion model in in (D.9). The jumps are generated by $dJ(t) = \sum_{i=1}^{N(t)} Y_i$, where $N(t) \sim \text{Poisson}(\lambda t)$ is a compound Poisson process that determines the number of jumps, λ is the jump intensity parameter, and and $Y_i \sim N(0, \sigma_J^2)$ denotes the normally distributed jump sizes. The choice of λ is guided by Huang and Tauchen (2005) who entertain several values based on estimates reported in empirical studies. We adopt $\lambda = 0.014$ as the jump intensity estimated by Andersen et al. (2002) for the daily S&P 500 cash index.[8] The jump magnitudes are specified such that jumps do not dominate the trajectory of the prices. We follow Li and Linton (2022) and set $\sigma_J^2 = 7.5 \cdot 10^{-5}$, ensuring that the variance of the jump component corresponds roughly to 10% of the integrated variance.[9] Naturally, setting either a higher jump intensity parameter or larger jump magnitudes leads to the jump-robust estimators eventually dominating their competitors. The market-specific spot variance are again generated according to the single-factor stochastic volatility process as in the previous subsection with daily integrated variances following an AR(1) process.[10]

To align the simulation closely with our empirical analysis, we set $T = 1826$ and the adjustment coefficient matrix in line with the estimates of α for IBM – a stock with a medium trading activity in the DJIA. We consider two versions of the adjustment matrix. The first specification accounts for high level of fragmentation noise with $\alpha_\perp = (0.2, 0.2, 0.2, 0.2, 0.2)'$, whereas the second version sets $\alpha_\perp = (0.125, 0.125, 0.125, 0.125, 0.5)'$, corresponding to a moderate level of fragmentation noise. Prices on all five exchanges

[8] We also report the results for a two-market system setting with $\lambda = 0.082$ to evaluate the performance of our GRT-based estimators in a high jump intensity environment. Overall, the results remain largely unchanged (see Subsection D.5 in the Online Appendix for details).

[9] Table E.7 in the Online Appendix reports the proportion of the quadratic variation attributed to jumps which amounts to 12% on average across all stocks of the DJIA.

[10] Alternatively, jumps could be introduced in the volatility process, or simultaneously in both the price and volatility processes (the so-called co-jumping, as in Todorov and Tauchen, 2011). Since jumps in the volatility process affect all realized measures similarly, we do not find qualitatively different results when allowing for co-jumping (see Subsection D.9 in the Online Appendix for details).

are contaminated with additive market microstructure noise satisfying Assumption 4. More specifically, we simulate ν_{t_i} as a VMA(1) process with $\varpi_{n,1} = 0.5$ for $n = 1, \ldots, 5$. The variance of the market microstructure noise is constant across all markets and set to $7.5 \cdot 10^{-7}$, yielding an average noise-to-signal ratio of 0.001. This value is conservative relative to the estimates obtained from our empirical analysis and is also consistent with previous studies (see, among others, Hansen and Lunde, 2006; Li and Linton, 2022). The correlation between the market microstructure noise across markets is set to 0.5. Finally, we simulate non-synchronous trading using independent Poisson sampling schemes, resulting in an average of 20,000 observations per day. Prices across markets are synchronized using the refresh-time method and then rounded to the third decimal place. The RV and RBPV estimators are computed using data sampled at 1-minute intervals, whereas the remaining estimators are computed using all available observations. Consequently, this DGP incorporates the combined effects of fragmentation noise, jumps, and additive market microstructure noise, as commonly modeled in the single-market setting.

Panels A and B in Table C3 report the results for high and moderate levels of fragmentation noise, respectively. Each panel contains two sub-panels presenting the bias and RMSE results. Overall, the main takeaways from this simulation are as follows. First, our results are consistent with the previous simulation exercises and Theorems 3 and 4: the GRT-based estimators outperform their univariate counterparts for all classes of estimators except of the pre-averaged realized bipower variation estimator. Second, the noise-robust GRT-based pre-averaged RV estimator performs strongly in terms of both bias and RMSE, confirming the results in Theorem 4. For example, it achieves the lowest bias and RMSE among all competitors under the moderate fragmentation noise specification. This finding underscores the importance of formally addressing additive market microstructure noise alongside fragmentation noise (Proposition 1). Third, the GRT-based realized kernel exhibits strong performance in terms of bias, but has a RMSE three times larger than GRT-based pre-averaging estimator, reflecting its slower convergence rate. Finally, the 1-minute GRT-based RV strikes a good balance between bias and RMSE, confirming the well-known robustness of the RV class of estimators when applied to lower sampling frequencies (see, for instance, Liu et al., 2015).

Table C3: Simulation Results: Bias and relative efficiency of GRT-based estimators in a five market system (mixed sampling frequencies)

Panel A: $\alpha_\perp = (0.2, 0.2, 0.2, 0.2, 0.2)'$

	Bias							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**1.354**	**1.327**	**-0.049**	-3.501	**-0.008**	**7.241**	**0.000**	**0.016**
1	3.817	3.676	1.442	-2.894	1.597	14.475	0.702	0.420
2	4.106	3.907	1.692	-2.797	1.812	14.539	1.397	1.194
3	4.079	3.879	1.670	-2.805	1.812	14.538	1.459	1.274
4	4.129	3.924	1.716	**-2.788**	1.916	14.561	1.443	1.211
5	4.086	3.912	1.683	-2.800	1.797	14.541	1.376	1.177

	RMSE							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**1.776**	**1.864**	**0.837**	5.027	**1.448**	**7.254**	**1.953**	**2.671**
1	4.320	4.244	2.122	4.223	3.370	14.590	2.344	2.865
2	4.732	4.617	2.537	4.069	3.507	14.658	3.083	3.595
3	4.689	4.545	2.573	4.081	3.553	14.660	3.164	3.690
4	4.854	4.730	2.657	**4.031**	3.665	14.701	3.043	3.468
5	4.660	4.512	2.486	4.095	3.558	14.662	3.085	3.631

Panel B: $\alpha_\perp = (0.125, 0.125, 0.125, 0.125, 0.5)'$

	Bias							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**1.562**	**1.537**	**-0.003**	-3.636	**0.052**	**8.210**	**0.029**	**0.045**
1	3.563	3.408	1.200	-3.145	1.380	14.221	0.502	0.237
2	3.854	3.650	1.442	-3.051	1.599	14.288	1.183	1.009
3	3.816	3.609	1.420	-3.059	1.572	14.289	1.244	1.089
4	3.873	3.65	1.466	-3.042	1.706	14.310	1.227	1.022
5	3.878	3.706	1.473	**-3.039**	1.588	14.289	1.350	1.269

	RMSE							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**1.988**	**2.078**	**0.874**	5.227	**1.628**	**8.224**	**2.096**	**2.864**
1	4.034	3.955	1.842	4.573	3.191	14.312	2.272	2.860
2	4.432	4.334	2.234	4.424	3.271	14.384	2.935	3.531
3	4.378	4.259	2.277	4.436	3.370	14.387	2.988	3.601
4	4.541	4.411	2.356	4.385	3.423	14.423	2.867	3.378
5	4.408	4.281	2.220	**4.428**	3.249	14.383	3.182	3.925

This table reports the average bias and RMSE (scaled by $\times 10^4$) for several RV estimators over the full sampling period. Prices are simulated from (D.9) and contaminated with additive market microstructure noise simulated from a $VMA(1)$ process, with noise ratio set to 0.001. Non-synchronous trading and price rounding are then imposed. Finally, prices are aggregated using the refresh time method, resulting in an average of 20,000 observations per day. The columns denote the alternative estimators used in this simulation: the 1-minute realized variance estimator ($RV^{(390)}$), realized bipower variation ($RBPV$), the pre-averaging estimator (PRV), pre-averaged realized bipower variation ($PBRV$), the noise-robust estimators RV^{AC} and RV^{NW} proposed by Hansen and Lunde (2006), and two variants of the realized kernel estimators, RK^{MTH} and RK^{P}. The rows correspond to either the GRT-based estimators (GRT) or the univariate estimators that use prices sampled from a single market, ($n = 1, \ldots, 5$). For each class of estimator, the lowest bias and RMSE are marked in boldface.

D Supplementary simulation results

This section complements the discussion in Section C and provides a detailed description of the univariate estimators used to assess the relative performance of the GRT-based class of estimators, together with additional results for $N = 2$ exchanges, constant and stochastic volatility models, alternative cross-market correlations, small T, jumps without additive market microstructure noise, no jumps with additive noise only, co-jumping, and time-varying α.

D.1 Univariate estimators applied to single market data

In this section, we provide a description of the univariate estimators that are used to assess the relative performance of the GRT-based estimator in the main paper. We use the notation $r_{n,t_i} = P_{n,t_i} - P_{n,t_{i-1}}$ for intra-day returns of market n, $n = 1, \ldots, N$, sampled equidistantly in calendar time at a fixed interval δ.

The first class of estimators we consider is the RV estimator (Andersen et al., 2001):

$$RV_{n,t} = \sum_{i=1}^{m} r_{n,t_i}^2, \quad n = 1, \ldots, N. \tag{D.1}$$

This estimator simply adds up the squared intra-day returns at a given sampling interval δ to measure the integrated variance of any market n for day t.[11] The RV estimator consistently estimates market n's quadratic variation, i.e. $RV_{n,t} \xrightarrow{p} \langle P, P' \rangle_{t-1:t}^{n,n}$ as $m \to \infty$ (Barndorff-Nielsen and Shephard, 2002). Alternatively, the realized covariance (RCov) estimator, denoted as $RCov_t = \sum_{i=1}^{m} \Delta P_{t_i} \Delta P'_{t_i}$, consistently estimates $\langle P, P' \rangle_{t-1:t}$ (Barndorff-Nielsen and Shephard, 2004b). Both the univariate and multivariate realized variance estimators attain the optimal convergence rate $m^{1/2}$, but they are biased and inconsistent estimators for the integrated variance in the presence of market microstructure noise. Additionally, in view of Theorem 1, if $\alpha_\perp \neq k(n)$ and $\rho \neq 0$, both the RV and RCov estimators are inconsistent estimators of the integrated variance.

In our second class of estimators, we examine two kernel-based univariate methods proposed by Hansen and Lunde (2006). These estimators capture the effects of serial correlation in high frequency returns induced by additive market microstructure noise.

[11] Bandi and Russell (2008) discuss the choice of optimal sampling frequency for the simple RV estimator as a bias-variance trade-off.

More specifically, they read

$$RV_{n,t}^{AC} = \sum_{i=1}^{m} r_{n,t_i}^2 + 2\sum_{h=1}^{H} \frac{m}{(m-h)} \sum_{i=1}^{m-h} r_{n,t_i} r_{n,t_{i+h}}, \tag{D.2}$$

and

$$RV_{n,t}^{NW} = \sum_{i=1}^{m} r_{n,t_i}^2 + 2\sum_{h=1}^{H} \left(1 - \frac{h}{(H+1)}\right) \sum_{i=1}^{m-h} r_{n,t_i} r_{n,t_{i+h}}, \tag{D.3}$$

where H is a frequency-dependent truncation parameter for the covariance terms. The first term on the right side of (D.2) and (D.3) is the classical RV estimator and the second term is a bias correction. The first estimator scales the h-th autocorrelation by $m/(m-h)$ to compensate for the 'missing' autocovariance terms at the end of each day. The upward scaling has the drawback that it increases the variance of the estimator. The optimal bandwidth for the estimators to balance bias and variance is given by $H = \lceil (4m/100)^{2/9} \rceil$. Both estimators are robust to the presence of endogenous noise in the sense that they are unbiased for this general type of noise.

Finally, we extend our analysis to the generalized realized kernel estimator of Barndorff-Nielsen et al. (2008, 2011). The realized kernel estimator for trading day t is given by:

$$RK_{n,t} = \sum_{i=1}^{m} r_{n,t_i}^2 + \sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)(\eta_h + \eta_{-h}), \quad \eta_h = \sum_{i=1}^{m-h} r_{n,t_i} r_{n,t_{i+h}}, \tag{D.4}$$

where $k(\cdot)$ is either the modified Tukey-Hanning kernel or Parzen kernel function. For our simulations and the empirical application, we denote the realized kernel estimator with modified Tukey-Hanning kernel by RK^{MTH} and with Parzen kernel by RK^{P}. The optimal choice of the bandwidth H is discussed in Barndorff-Nielsen et al. (2009). Barndorff-Nielsen et al. (2008) show that realized kernel estimators are unbiased and consistent. Estimators based on 'flat-top' kernel functions like the Bartlett or modified Tukey-Hanning converge at a faster rate than those based on the 'non-flat-top' Parzen kernel, but are not guaranteed to be non-negative. However, both kernel choices are robust to time-dependent and endogenous noise. An analysis of the finite sample performance of realized kernels is provided by Bandi and Russell (2011).[12]

Additionally, the pre-averaging estimator according to Hautsch and Podolskij (2013) is implemented as an alternative to the realized kernel estimators. We define a sequence

[12]See also Liu et al. (2015) for a discussion about the performance of alternative realized measures estimators.

of integers that satisfies $k_m = \theta m^{\frac{1}{2}} + o(m^{-\frac{1}{4}})$ for some $\theta > 0$ and use the typical weighting function

$$g(x) = x \wedge (1-x) = \min(x, 1-x). \tag{D.5}$$

The preaveraged returns are then given by

$$\bar{r}_{n,t_i} = \frac{1}{n} \sum_{j=1}^{k_m} g\left(\frac{j}{k_m}\right) \left(P_{n,t_{i+j}} - P_{n,t_{i-1+j}}\right). \tag{D.6}$$

The preaveraging estimator for trading day t using data observed in market n is defined as

$$PRV_{n,t} = \sum_{i=0}^{m-k_m} \bar{r}_{n,t_i}^2.$$

Diminishing weights reduce the impact of noisy observations so that the estimator is consistent in the presence of additive microstructure noise.

Finally, we consider the jump-robust realized bipower variation measure proposed by Barndorff-Nielsen and Shephard (2004c). It is the most widely used estimator of the integrated variance in the presence of jumps and can be expressed by

$$RBPV_{n,t} = \frac{\pi}{2} \sum_{i=1}^{m-1} |r_{n,t_i}||r_{n,t_{i+1}}|. \tag{D.7}$$

The idea behind this estimator is the there will at most be a single jump within two adjacent intervals. This isolated jump will be dampened by the multiplication by a small adjacent return. As the number of intraday observations grows this is sufficient to render the jump contribution negligible.

D.2 Two-market system: constant covariance

Table D1: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU process (1-min, $\tau = 1.5$)

		DGP I	
		$\hat{P}^*$	
RV_{GRT}	**0.021 (0.242)**	–	–
		P_1	
RV_1	4.240 (4.277)	–	–
PRV_1	3.406 (3.708)	–	–
	$H = 3$	$H = 8$	$H = 20$
RV_1^{AC}	4.219 (4.470)	4.171 (4.778)	4.085 (5.379)
RV_1^{NW}	4.230 (4.333)	4.204 (4.422)	4.150 (4.638)
RK_1^{MTH}	4.229 (4.353)	4.217 (4.415)	4.177 (4.564)
RK_1^{P}	4.231 (4.365)	4.210 (4.437)	4.166 (4.625)
		P_2	
RV_2	1.709 (1.747)	–	–
PRV_2	1.177 (1.519)	–	–
	$H = 3$	$H = 8$	$H = 20$
RV_2^{AC}	1.706 (1.951)	1.720 (2.290)	1.657 (2.914)
RV_2^{NW}	1.712 (1.816)	1.706 (1.927)	1.687 (2.176)
RK_2^{MTH}	1.716 (1.844)	1.703 (1.905)	1.700 (2.082)
RK_2^{P}	1.715 (1.853)	1.700 (1.930)	1.697 (2.148)

Note: The table reports the estimated average bias ($\times 10^5$) over the full sampling period with the RMSE ($\times 10^5$) given in parentheses. Data are drawn from DGP I with $\alpha = (-0.5, 0.5)'$ and $\tau = 1.5$. H denotes the lag truncation parameter used for realized kernel estimators. The estimators with the lowest bias and RMSE are marked in boldface.

Table D2: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU process (mixed frequencies)

		DGP I			DGP II	
		P^*			P^*	
RV_{GRT}	**0.001 (0.222)**	–	–	**-0.045 (0.307)**	–	–
		P_1			P_1	
RV_1	3.280 (3.311)	–	–	0.836 (0.911)	–	–
PRV_1	3.222 (3.257)	–	–	0.791 (0.878)	–	–
	$H=3$	$H=8$	$H=20$	$H=3$	$H=8$	$H=20$
RV_1^{AC}	3.291 (3.295)	3.281 (3.289)	3.285 (3.307)	0.843 (0.852)	0.835 (0.857)	0.839 (0.894)
RV_1^{NW}	3.291 (3.292)	3.287 (3.290)	3.286 (3.293)	0.843 (0.846)	0.840 (0.847)	0.839 (0.858)
RK_1^{MTH}	3.291 (3.292)	3.289 (3.292)	3.285 (3.290)	0.843 (0.847)	0.841 (0.848)	0.838 (0.852)
RK_1^{P}	3.291 (3.293)	3.288 (3.291)	3.285 (3.291)	0.843 (0.847)	0.840 (0.849)	0.838 (0.855)
		P_2			P_2	
RV_2	2.027 (2.060)	–	–	0.847 (0.923)	–	–
PRV_2	1.982 (2.018)	–	–	0.803 (0.888)	–	–
	$H=3$	$H=8$	$H=20$	$H=3$	$H=8$	$H=20$
RV_2^{AC}	2.021 (2.025)	2.021 (2.031)	2.032 (2.054)	0.840 (0.849)	0.841 (0.863)	0.852 (0.904)
RV_2^{NW}	2.023 (2.024)	2.022 (2.025)	2.026 (2.034)	0.842 (0.846)	0.841 (0.849)	0.846 (0.864)
RK_2^{MTH}	2.024 (2.026)	2.022 (2.025)	2.023 (2.029)	0.843 (0.848)	0.841 (0.848)	0.842 (0.856)
RK_2^{P}	2.024 (2.025)	2.022 (2.025)	2.024 (2.031)	0.843 (0.847)	0.841 (0.849)	0.843 (0.860)

Note: The table reports the estimated average bias ($\times 10^5$) over the full sampling period with the RMSE ($\times 10^5$) shown in parentheses. Data are generated from DGP I (left panel), with $\alpha = (-0.5, 0.5)'$, $\alpha_\perp = (0.5, 0.5)'$, and $\tau = 1.25$; and DGP II (right panel), with $\alpha = (-0.9, 0.1)'$, $\alpha_\perp = (0.1, 0.9)'$ and $\tau = 1$. H denotes the bandwidth used in both the realized kernel and Hansen and Lunde's 2006 noise-robust estimators. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals while the RV estimators are aggregated to 1-minute intervals. The estimators with the lowest RMSE are marked in boldface. The subscripts in the second and third panels indicate that the realized measures are computed using prices from market one and two, respectively. Finally, the true values, scaled up by 10^5, are: for DGP I, $\langle P^*, P^{*\prime}\rangle_{t-1:t} = 3.086$, $\langle P, P'\rangle^{1,1}_{t-1:t} = 6.436$ and $\langle P, P'\rangle^{2,2}_{t-1:t} = 5.152$; and for DGP II $\langle P^*, P^{*\prime}\rangle_{t-1:t} = 4.282$, $\langle P, P'\rangle^{1,1}_{t-1:t} = 5.184$ and $\langle P, P'\rangle^{2,2}_{t-1:t} = 5.136$.

Table D3: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU process (5-min)

	DGP I			DGP II		
		P^*			P^*	
RV_{GRT}	**-0.052 (0.493)**	–	–	**-0.106 (0.671)**	–	–
		P_1			P_1	
RV_1	3.285 (3.444)	–	–	0.849 (1.190)	–	–
PRV_1	2.099 (2.770)	–	–	-0.066 (1.469)	–	–
	$H = 3$	$H = 8$	$H = 20$	$H = 3$	$H = 8$	$H = 20$
RV_1^{AC}	3.168 (4.160)	2.890 (5.088)	2.556 (7.046)	0.819 (2.339)	0.698 (3.542)	0.672 (5.685)
RV_1^{NW}	3.198 (3.616)	3.079 (3.902)	2.909 (4.510)	0.811 (1.589)	0.763 (2.114)	0.730 (3.003)
RK_1^{MTH}	3.169 (3.685)	3.124 (3.922)	2.967 (4.415)	0.787 (1.710)	0.783 (2.085)	0.733 (2.815)
RK_1^{P}	3.167 (3.723)	3.100 (4.001)	2.926 (4.608)	0.789 (1.768)	0.774 (2.207)	0.725 (3.061)
		P_2			P_2	
RV_2	2.037 (2.194)	–	–	0.825 (1.157)	–	–
PRV_2	1.107 (1.819)	–	–	-0.074 (1.455)	–	–
	$H = 3$	$H = 8$	$H = 20$	$H = 3$	$H = 8$	$H = 20$
RV_2^{AC}	1.930 (2.884)	1.779 (3.801)	1.380 (5.733)	0.763 (2.288)	0.685 (3.508)	0.464 (5.835)
RV_2^{NW}	1.993 (2.396)	1.912 (2.706)	1.734 (3.279)	0.803 (1.556)	0.757 (2.091)	0.655 (2.972)
RK_2^{MTH}	2.001 (2.489)	1.936 (2.705)	1.810 (3.178)	0.810 (1.689)	0.769 (2.057)	0.698 (2.777)
RK_2^{P}	1.995 (2.524)	1.917 (2.787)	1.768 (3.351)	0.807 (1.748)	0.757 (2.182)	0.674 (3.018)

Note: The table reports the estimated average bias ($\times 10^5$) over the full sampling period with the RMSE ($\times 10^5$) shown in parentheses. Data are generated from DGP I (left panel), with $\alpha = (-0.5, 0.5)'$, $\alpha_\perp = (0.5, 0.5)'$, and $\tau = 1.25$; and DGP II (right panel), with $\alpha = (-0.9, 0.1)'$, $\alpha_\perp = (0.1, 0.9)'$ and $\tau = 1$. H denotes the bandwidth used in both the realized kernel and Hansen and Lunde's 2006 noise-robust estimators. The estimators with the lowest RMSE are marked in boldface. The subscripts in the second and third panels indicate that the realized measures are computed using prices from market one and two, respectively. Finally, the true values, scaled up by 10^5, are: for DGP I, $\langle P^*, P^{*\prime} \rangle_{t-1:t} = 3.086$, $\langle P, P' \rangle^{1,1}_{t-1:t} = 6.436$ and $\langle P, P' \rangle^{2,2}_{t-1:t} = 5.152$; and for DGP II $\langle P^*, P^{*\prime} \rangle_{t-1:t} = 4.282$, $\langle P, P' \rangle^{1,1}_{t-1:t} = 5.184$ and $\langle P, P' \rangle^{2,2}_{t-1:t} = 5.136$.

D.3 Two-market system: stochastic volatility

Table D4: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (mixed frequencies, $\rho = 0.5$)

$\alpha_\perp$	$RV_{GRT}^{(390)}$	$RV_1^{(390)}$	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	$RV_2^{(390)}$	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias								
$(0.9, 0.1)'$	**0.569**	1.048	0.972	1.090	1.085	1.055	1.060	1.075	0.998	1.034	1.038	1.181	1.148
$(0.8, 0.2)'$	**0.376**	1.888	1.812	1.931	1.926	1.883	1.878	1.915	1.838	1.875	1.879	2.009	1.966
$(0.7, 0.3)'$	**0.219**	2.489	2.412	2.533	2.528	2.476	2.464	2.516	2.438	2.477	2.481	2.601	2.551
$(0.6, 0.4)'$	**0.098**	2.851	2.774	2.895	2.891	2.833	2.816	2.878	2.800	2.840	2.843	2.958	2.903
$(0.5, 0.5)'$	**0.014**	2.974	2.897	3.019	3.014	2.953	2.935	3.001	2.923	2.963	2.967	3.079	3.022
					Panel B: RMSE								
$(0.9, 0.1)'$	**1.675**	2.270	2.247	2.005	1.936	4.079	5.349	2.661	2.632	2.018	1.997	4.621	5.716
$(0.8, 0.2)'$	**1.414**	3.521	3.472	3.434	3.391	4.891	5.960	3.831	3.752	3.341	3.340	5.409	6.306
$(0.7, 0.3)'$	**1.246**	4.501	4.443	4.476	4.442	5.631	6.553	4.769	4.670	4.345	4.350	6.117	6.873
$(0.6, 0.4)'$	**1.162**	5.109	5.047	5.111	5.080	6.118	6.957	5.351	5.244	4.957	4.966	6.579	7.252
$(0.5, 0.5)'$	**1.143**	5.325	5.260	5.334	5.305	6.291	7.099	5.548	5.437	5.164	5.173	6.733	7.372

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.5$. We report results for the 1-minute GRT-based RV estimator, $RV_{GRT}^{(390)}$, alongside the 1-minute univariate RV estimator and several alternative noise-robust estimators computed at a 1-second frequency: the pre-averaging estimator (PRV); the noise-robust estimators RV^{AC} and RV^{NW} proposed by Hansen and Lunde (2006); and two variants of the realized kernel estimators, RK^{MTH} and RK^{P}. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D5: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (mixed frequencies, $\rho = 0.9$)

$\alpha_{\perp}$	$RV_{GRT}^{(390)}$	$RV_1^{(390)}$	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	$RV_2^{(390)}$	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias								
$(0.9, 0.1)'$	0.181	0.188	**0.113**	0.229	0.224	0.207	0.221	0.184	0.108	0.181	0.179	0.217	0.207
$(0.8, 0.2)'$	**0.126**	0.360	0.285	0.401	0.396	0.376	0.388	0.356	0.279	0.353	0.351	0.386	0.374
$(0.7, 0.3)'$	**0.078**	0.483	0.408	0.525	0.520	0.498	0.508	0.479	0.403	0.477	0.475	0.508	0.494
$(0.6, 0.4)'$	**0.035**	0.559	0.484	0.600	0.596	0.572	0.581	0.555	0.478	0.552	0.550	0.582	0.567
$(0.5, 0.5)'$	**-0.002**	0.587	0.511	0.628	0.623	0.599	0.607	0.583	0.506	0.58	0.578	0.609	0.593
					Panel B: RMSE								
$(0.9, 0.1)'$	1.188	1.491	1.526	**0.747**	0.559	3.676	5.072	1.800	1.882	1.115	0.996	3.977	5.143
$(0.8, 0.2)'$	1.148	1.571	1.586	0.952	**0.813**	3.715	5.094	1.824	1.885	1.152	1.041	3.992	5.145
$(0.7, 0.3)'$	1.121	1.669	1.668	1.137	**1.025**	3.759	5.122	1.864	1.909	1.216	1.124	4.014	5.155
$(0.6, 0.4)'$	**1.106**	1.749	1.737	1.269	1.172	3.795	5.144	1.891	1.924	1.258	1.162	4.029	5.162
$(0.5, 0.5)'$	**1.101**	1.795	1.777	1.337	1.247	3.815	5.155	1.887	1.914	1.255	1.160	4.030	5.160

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.9$. We report results for the 1-minute GRT-based RV estimator, $RV_{GRT}^{(390)}$, alongside the 1-minute univariate RV estimator and several alternative noise-robust estimators computed at a 1-second frequency: the pre-averaging estimator (PRV); the noise-robust estimators RV^{AC} and RV^{NW} proposed by Hansen and Lunde (2006); and two variants of the realized kernel estimators, RK^{MTH} and RK^{P}. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D6: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (1-min, $\rho = 0.5$)

$\alpha_{\perp}$	RV_{GRT}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias								
$(0.9, 0.1)'$	**0.578**	6.733	-1.072	6.726	6.73	6.289	5.959	6.803	-0.963	6.803	6.803	6.544	6.205
$(0.8, 0.2)'$	**0.455**	11.974	4.105	11.954	11.964	11.172	10.595	12.025	4.197	12.012	12.019	11.410	10.826
$(0.7, 0.3)'$	**0.341**	15.721	7.806	15.691	15.706	14.661	13.905	15.753	7.881	15.730	15.741	14.882	14.122
$(0.6, 0.4)'$	**0.238**	17.973	10.029	17.936	17.955	16.756	15.890	17.986	10.087	17.956	17.971	16.960	16.092
$(0.5, 0.5)'$	**0.144**	18.730	10.775	18.691	18.711	17.456	16.550	18.723	10.816	18.692	18.708	17.643	16.738
					Panel B: RMSE								
$(0.9, 0.1)'$	**2.996**	4.478	7.334	5.936	4.867	18.123	24.059	4.790	7.627	6.258	5.193	18.273	24.225
$(0.8, 0.2)'$	**2.630**	6.841	7.602	7.861	7.095	18.626	24.234	7.002	7.867	8.077	7.284	18.781	24.404
$(0.7, 0.3)'$	**2.352**	8.670	8.345	9.486	8.867	19.185	24.502	8.754	8.560	9.632	8.980	19.336	24.675
$(0.6, 0.4)'$	**2.171**	9.800	8.970	10.522	9.971	19.595	24.713	9.832	9.138	10.618	10.032	19.734	24.887
$(0.5, 0.5)'$	**2.101**	10.193	9.214	10.887	10.357	19.741	24.780	10.182	9.338	10.941	10.374	19.863	24.955

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.5$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D7: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (1-min, $\rho = 0.7$)

$\alpha_\perp$	RV_{GRT}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias								
$(0.9, 0.1)'$	**0.442**	4.041	-3.732	4.041	4.041	3.779	3.576	4.076	-3.656	4.088	4.082	3.950	3.722
$(0.8, 0.2)'$	**0.364**	7.187	-0.624	7.179	7.183	6.710	6.358	7.211	-0.558	7.214	7.213	6.871	6.495
$(0.7, 0.3)'$	**0.292**	9.436	1.597	9.422	9.429	8.804	8.344	9.449	1.653	9.446	9.448	8.955	8.473
$(0.6, 0.4)'$	**0.226**	10.788	2.932	10.770	10.779	10.062	9.535	10.789	2.977	10.783	10.786	10.202	9.655
$(0.5, 0.5)'$	**0.166**	11.243	3.379	11.224	11.233	10.482	9.931	11.232	3.415	11.225	11.228	10.612	10.042
					Panel B: RMSE								
$(0.9, 0.1)'$	**2.940**	3.491	7.577	5.238	3.982	18.001	24.062	3.852	7.828	5.581	4.348	18.145	24.207
$(0.8, 0.2)'$	**2.710**	4.678	7.325	6.087	5.051	18.153	24.06	4.883	7.551	6.337	5.283	18.287	24.200
$(0.7, 0.3)'$	**2.536**	5.676	7.365	6.878	5.984	18.341	24.111	5.784	7.553	7.054	6.126	18.460	24.246
$(0.6, 0.4)'$	**2.424**	6.316	7.484	7.411	6.592	18.485	24.159	6.355	7.630	7.528	6.668	18.586	24.288
$(0.5, 0.5)'$	**2.380**	6.551	7.551	7.611	6.817	18.539	24.173	6.534	7.653	7.678	6.837	18.620	24.296

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D8: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (1-min, $\rho = 0.9$)

$\alpha_\perp$	RV_{GRT}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias								
$(0.9, 0.1)'$	**0.303**	1.348	-6.392	1.355	1.352	1.270	1.193	1.353	-6.359	1.371	1.362	1.335	1.234
$(0.8, 0.2)'$	**0.272**	2.400	-5.353	2.404	2.402	2.249	2.122	2.401	-5.324	2.416	2.408	2.311	2.160
$(0.7, 0.3)'$	**0.243**	3.152	-4.611	3.154	3.153	2.949	2.785	3.149	-4.585	3.162	3.155	3.007	2.820
$(0.6, 0.4)'$	**0.217**	3.603	-4.165	3.604	3.604	3.368	3.183	3.597	-4.142	3.608	3.603	3.423	3.214
$(0.5, 0.5)'$	**0.194**	3.755	-4.016	3.756	3.756	3.509	3.314	3.745	-3.996	3.756	3.750	3.56	3.343
					Panel B: RMSE								
$(0.9, 0.1)'$	**2.867**	2.876	8.054	4.854	3.459	17.974	24.131	3.279	8.228	5.167	3.834	18.018	24.112
$(0.8, 0.2)'$	**2.783**	3.074	7.848	4.972	3.623	17.975	24.095	3.355	7.990	5.216	3.899	18.001	24.065
$(0.7, 0.3)'$	**2.720**	3.279	7.727	5.100	3.798	17.987	24.075	3.449	7.833	5.277	3.980	17.994	24.035
$(0.6, 0.4)'$	**2.680**	3.438	7.670	5.203	3.935	18.000	24.065	3.507	7.739	5.315	4.031	17.988	24.015
$(0.5, 0.5)'$	**2.663**	3.525	7.666	5.260	4.011	18.010	24.063	3.500	7.696	5.310	4.024	17.978	24.002

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.9$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D9: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (1-min, $\rho = 0.7$, $corr(dW(t), dB_j(t)) = 0.6$)

$\alpha_\perp$	RV_{GRT}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^P	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^P
Panel A: Bias													
$(0.9, 0.1)'$	**-0.352**	4.040	-3.792	3.981	4.011	3.779	3.637	4.083	-3.744	4.029	4.056	3.802	3.646
$(0.8, 0.2)'$	**-0.233**	7.187	-0.681	7.120	7.154	6.726	6.447	7.218	-0.643	7.156	7.187	6.738	6.445
$(0.7, 0.3)'$	**-0.088**	9.437	1.543	9.364	9.400	8.837	8.463	9.456	1.571	9.389	9.423	8.837	8.450
$(0.6, 0.4)'$	**0.082**	10.789	2.881	10.713	10.751	10.111	9.685	10.797	2.899	10.727	10.762	10.100	9.661
$(0.5, 0.5)'$	**0.279**	11.244	3.332	11.167	11.205	10.549	10.112	11.240	3.340	11.169	11.205	10.527	10.077
Panel B: RMSE													
$(0.9, 0.1)'$	**2.967**	3.517	7.572	5.250	4.000	17.678	23.724	3.858	7.792	5.491	4.313	17.995	23.862
$(0.8, 0.2)'$	**2.720**	4.707	7.308	6.084	5.062	17.843	23.746	4.898	7.492	6.240	5.249	18.133	23.858
$(0.7, 0.3)'$	**2.534**	5.704	7.343	6.868	5.992	18.045	23.822	5.803	7.483	6.957	6.094	18.311	23.915
$(0.6, 0.4)'$	**2.423**	6.344	7.459	7.397	6.598	18.203	23.897	6.376	7.555	7.434	6.638	18.448	23.976
$(0.5, 0.5)'$	**2.399**	6.578	7.526	7.596	6.821	18.270	23.936	6.556	7.582	7.586	6.809	18.497	24.007

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D10: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility (1-min, $\rho = 0.7$, $corr(dB_1(t), dB_2(t)) = 0.5$)

$\alpha_\perp$	RV_{GRT}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^P	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^P
Panel A: Bias													
$(0.9, 0.1)'$	**0.455**	4.070	-3.702	4.064	4.067	3.807	3.596	4.114	-3.599	4.115	4.115	3.986	3.722
$(0.8, 0.2)'$	**0.375**	7.241	-0.570	7.227	7.234	6.761	6.399	7.273	-0.477	7.267	7.270	6.930	6.517
$(0.7, 0.3)'$	**0.301**	9.508	1.668	9.487	9.498	8.871	8.401	9.528	1.751	9.516	9.522	9.030	8.510
$(0.6, 0.4)'$	**0.231**	10.870	3.013	10.845	10.857	10.137	9.600	10.879	3.085	10.862	10.870	10.286	9.700
$(0.5, 0.5)'$	**0.167**	11.327	3.463	11.301	11.314	10.560	9.998	11.324	3.525	11.306	11.315	10.698	10.089
Panel B: RMSE													
$(0.9, 0.1)'$	**2.971**	3.546	7.566	5.267	4.029	17.889	23.940	6.165	9.070	7.400	6.502	18.670	24.476
$(0.8, 0.2)'$	**2.736**	4.818	7.352	6.185	5.178	18.064	23.951	6.515	8.588	7.690	6.833	18.701	24.395
$(0.7, 0.3)'$	**2.559**	5.920	7.468	7.068	6.211	18.286	24.024	6.929	8.369	8.039	7.226	18.776	24.374
$(0.6, 0.4)'$	**2.445**	6.688	7.692	7.717	6.944	18.476	24.103	7.167	8.238	8.241	7.453	18.817	24.359
$(0.5, 0.5)'$	**2.399**	7.091	7.896	8.065	7.331	18.589	24.157	7.110	8.082	8.187	7.395	18.778	24.319

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

Table D11: Simulation Results: Bias and relative efficiency of RV_{GRT} and equal-weighted RV estimator for the cointegrated OU type of process with stochastic volatility (1-min, $\rho = 0.7$

$\alpha_\perp$	RV_{GRT}	RV_{equal}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias									
$(0.1, 0.9)'$	**0.442**	-7.189	4.041	-3.732	4.041	4.041	3.779	3.576	4.076	-3.656	4.088	4.082	3.950	3.722
$(0.2, 0.8)'$	**0.364**	-4.048	7.187	-0.624	7.179	7.183	6.710	6.358	7.211	-0.558	7.214	7.213	6.871	6.495
$(0.3, 0.7)'$	**0.292**	-1.805	9.436	1.597	9.422	9.429	8.804	8.344	9.449	1.653	9.446	9.448	8.955	8.473
$(0.4, 0.6)'$	**0.226**	-0.459	10.788	2.932	10.770	10.779	10.062	9.535	10.789	2.977	10.783	10.786	10.202	9.655
$(0.5, 0.5)'$	0.166	**-0.010**	11.243	3.379	11.224	11.233	10.482	9.931	11.232	3.415	11.225	11.228	10.612	10.042
					Panel B: RMSE									
$(0.1, 0.9)'$	**2.940**	4.498	3.491	7.577	5.238	3.982	18.001	24.062	3.852	7.828	5.581	4.348	18.145	24.207
$(0.2, 0.8)'$	**2.710**	3.222	4.678	7.325	6.087	5.051	18.153	24.06	4.883	7.551	6.337	5.283	18.287	24.200
$(0.3, 0.7)'$	**2.536**	2.574	5.676	7.365	6.878	5.984	18.341	24.111	5.784	7.553	7.054	6.126	18.46	24.246
$(0.4, 0.6)'$	2.424	**2.385**	6.316	7.484	7.411	6.592	18.485	24.159	6.355	7.63	7.528	6.668	18.586	24.288
$(0.5, 0.5)'$	2.380	**2.366**	6.551	7.551	7.611	6.817	18.539	24.173	6.534	7.653	7.678	6.837	18.620	24.296

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

D.4 Two-market system: small T

Table D12: Stochastic volatility simulation with a shorter estimation period ($\rho = 0.7$, $T = 100$, 1-minute sampling frequency)

$\alpha_\perp$	RV_{GRT}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^P	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^P
				Panel A: Bias									
$(0.9, 0.1)'$	**0.926**	3.726	-3.323	3.649	3.688	3.762	3.592	3.808	-3.338	3.685	3.746	3.361	2.521
$(0.8, 0.2)'$	**1.066**	6.599	-0.483	6.515	6.557	6.452	6.149	6.670	-0.506	6.54	6.605	6.042	5.070
$(0.7, 0.3)'$	**1.260**	8.651	1.548	8.563	8.607	8.378	7.981	8.712	1.515	8.578	8.645	7.958	6.893
$(0.6, 0.4)'$	**1.508**	9.884	2.768	9.793	9.838	9.538	9.086	9.934	2.726	9.797	9.866	9.109	7.990
$(0.5, 0.5)'$	**1.810**	10.296	3.178	10.205	10.250	9.934	9.464	10.336	3.126	10.199	10.267	9.496	8.360
				Panel B: RMSE									
$(0.9, 0.1)'$	2.925	**2.432**	5.360	3.520	2.716	11.777	15.573	2.683	5.417	3.768	2.976	12.105	15.713
$(0.8, 0.2)'$	**2.491**	3.260	5.193	4.108	3.461	11.900	15.584	3.409	5.230	4.293	3.631	12.182	15.656
$(0.7, 0.3)'$	**2.164**	3.951	5.220	4.657	4.108	12.044	15.629	4.037	5.231	4.791	4.217	12.288	15.647
$(0.6, 0.4)'$	**1.988**	4.391	5.299	5.026	4.528	12.153	15.669	4.434	5.282	5.119	4.593	12.365	15.646
$(0.5, 0.5)'$	**1.974**	4.551	5.345	5.163	4.681	12.193	15.678	4.556	5.299	5.223	4.710	12.380	15.627

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over $T = 100$ trading days. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

D.5 Two-market system: jumps only

Table D13: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and jumps ($\lambda = 0.014$, 1-min sampling frequency)

$\alpha_{\perp}$	RV_{GRT}	$RBPV_{GRT}$	PRV_1	RV_1	$RBPV_1$	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	$RBPV_2$	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias											
$(0.9, 0.1)'$	1.245	**0.236**	4.743	3.723	-2.780	4.642	4.635	3.598	2.968	4.832	3.807	-2.745	4.634	4.651	3.754	3.050
$(0.8, 0.2)'$	1.179	0.182	7.699	6.679	**0.140**	7.561	7.572	6.352	5.583	7.778	6.753	0.166	7.542	7.577	6.498	5.655
$(0.7, 0.3)'$	1.136	**0.141**	9.812	8.792	2.227	9.646	9.672	8.320	7.451	9.880	8.855	2.243	9.617	9.666	8.457	7.513
$(0.6, 0.4)'$	1.115	**0.120**	11.082	10.062	3.481	10.900	10.933	9.502	8.573	11.139	10.114	3.487	10.860	10.917	9.629	8.625
$(0.5, 0.5)'$	1.115	**0.120**	11.508	10.488	3.901	11.320	11.356	9.897	8.947	11.554	10.530	3.898	11.270	11.330	10.015	8.990
					Panel B: RMSE											
$(0.9, 0.1)'$	4.922	**2.847**	5.328	3.344	7.544	9.462	6.771	15.717	20.366	5.581	3.780	7.577	9.535	6.947	16.144	21.075
$(0.8, 0.2)'$	4.824	**2.658**	6.059	4.301	7.385	9.878	7.335	15.824	20.339	6.263	4.636	7.436	9.954	7.493	16.244	21.017
$(0.7, 0.3)'$	4.762	**2.531**	6.727	5.132	7.443	10.29	7.878	15.977	20.381	6.885	5.393	7.495	10.358	8.012	16.384	21.025
$(0.6, 0.4)'$	4.733	**2.464**	7.175	5.669	7.551	10.583	8.255	16.105	20.442	7.294	5.876	7.590	10.636	8.362	16.495	21.050
$(0.5, 0.5)'$	4.736	**2.462**	7.342	5.866	7.609	10.698	8.398	16.169	20.491	7.424	6.026	7.624	10.727	8.475	16.539	21.063

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

Table D14: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and jumps ($\lambda = 0.082$, 1-min sampling frequency)

$\alpha_{\perp}$	RV_{GRT}	$RBPV_{GRT}$	PRV_1	RV_1	$RBPV_1$	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	$RBPV_2$	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
						Panel A: Bias										
$(0.9, 0.1)'$	6.245	**0.337**	10.579	4.661	1.742	10.879	10.758	10.979	10.404	10.497	4.698	1.391	10.467	10.457	10.003	9.225
$(0.8, 0.2)'$	6.318	**0.426**	14.023	8.106	5.145	14.279	14.181	14.191	13.452	13.929	8.129	4.784	13.855	13.867	13.206	12.269
$(0.7, 0.3)'$	6.331	**0.456**	16.484	10.567	7.576	16.709	16.626	16.484	15.625	16.378	10.578	7.204	16.272	16.300	15.490	14.437
$(0.6, 0.4)'$	6.283	**0.418**	17.963	12.045	9.035	18.168	18.095	17.857	16.922	17.843	12.044	8.653	17.719	17.756	16.854	15.731
$(0.5, 0.5)'$	6.174	**0.316**	18.458	12.540	9.523	18.656	18.586	18.311	17.344	18.326	12.526	9.129	18.194	18.235	17.299	16.148
						Panel B: RMSE										
$(0.9, 0.1)'$	12.333	**3.603**	12.750	4.198	13.601	16.196	13.889	23.541	29.203	12.821	4.593	13.795	16.382	13.964	23.121	28.357
$(0.8, 0.2)'$	12.269	**3.376**	13.388	5.349	13.606	16.699	14.474	23.814	29.274	13.413	5.608	13.693	16.784	14.468	23.249	28.294
$(0.7, 0.3)'$	12.225	**3.215**	13.975	6.36	13.760	17.165	15.016	24.077	29.371	13.961	6.526	13.761	17.176	14.95	23.405	28.288
$(0.6, 0.4)'$	12.196	**3.119**	14.377	7.019	13.914	17.488	15.389	24.258	29.44	14.329	7.117	13.856	17.447	15.279	23.515	28.284
$(0.5, 0.5)'$	12.181	**3.089**	14.529	7.261	13.984	17.609	15.529	24.318	29.451	14.449	7.302	13.891	17.535	15.388	23.536	28.248

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

D.6 Two-market system: MSN only

Table D15: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and additive microstructure noise (noise ratio = 0.001, $\rho_\nu = 0.5$, 1-min sampling frequency)

$\alpha_\perp$	RV_{GRT}	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias													
$(0.9, 0.1)'$	52.049	-7.366	**-0.087**	8.578	-0.053	0.089	62.557	-3.355	4.382	14.075	4.191	4.210	62.554	-3.456	4.301	14.072	4.449	4.174
$(0.8, 0.2)'$	48.496	-6.859	**0.127**	8.163	0.193	0.299	65.672	-0.275	7.459	17.171	7.110	6.993	65.657	-0.386	7.367	17.157	7.358	6.949
$(0.7, 0.3)'$	46.047	-6.467	**0.312**	7.910	0.412	0.486	67.899	1.927	9.660	19.385	9.200	8.986	67.873	1.806	9.557	19.360	9.438	8.935
$(0.6, 0.4)'$	44.702	-6.193	**0.469**	7.820	0.605	0.648	69.238	3.251	10.983	20.716	10.460	10.189	69.200	3.120	10.869	20.680	10.688	10.131
$(0.5, 0.5)'$	44.460	-6.034	**0.599**	7.893	0.772	0.788	69.689	3.698	11.430	21.165	10.890	10.603	69.640	3.557	11.305	21.117	11.108	10.538
					Panel B: RMSE													
$(0.9, 0.1)'$	16.974	8.556	9.608	**6.591**	18.816	26.264	20.336	8.178	10.520	8.018	19.900	27.713	20.435	8.439	11.038	8.131	20.861	27.852
$(0.8, 0.2)'$	15.809	8.118	9.171	**6.232**	18.286	25.457	21.500	8.013	11.104	9.097	20.138	27.795	21.580	8.254	11.571	9.175	21.110	27.953
$(0.7, 0.3)'$	15.008	7.810	8.882	**5.987**	17.968	24.918	22.385	8.127	11.666	9.987	20.400	27.926	22.446	8.335	12.084	10.032	21.374	28.099
$(0.6, 0.4)'$	14.567	7.631	8.739	**5.852**	17.855	24.644	22.937	8.292	12.059	10.562	20.605	28.049	22.978	8.462	12.432	10.573	21.574	28.234
$(0.5, 0.5)'$	14.487	7.579	8.740	**5.826**	17.943	24.635	23.129	8.375	12.208	10.768	20.706	28.131	23.149	8.505	12.542	10.743	21.666	28.323

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

Table D16: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and additive microstructure noise (noise ratio = 0.0005, $\rho_\nu = 0.5$, 1-min sampling frequency)

$\alpha_\perp$	RV_{GRT}	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias													
$(0.9, 0.1)'$	25.788	-7.384	-0.141	4.137	**-0.050**	0.089	33.292	-3.376	4.324	9.125	4.192	4.208	33.306	-3.476	4.280	9.129	4.456	4.181
$(0.8, 0.2)'$	24.123	-6.875	**0.076**	4.041	0.197	0.300	36.407	-0.296	7.401	12.221	7.111	6.991	36.410	-0.406	7.346	12.214	7.365	6.956
$(0.7, 0.3)'$	22.996	-6.483	**0.264**	4.012	0.416	0.487	38.634	1.906	9.602	14.435	9.201	8.984	38.625	1.786	9.535	14.417	9.445	8.942
$(0.6, 0.4)'$	22.406	-6.208	**0.424**	4.049	0.609	0.650	39.973	3.231	10.926	15.766	10.461	10.187	39.953	3.100	10.848	15.737	10.695	10.138
$(0.5, 0.5)'$	22.353	-6.051	**0.556**	4.153	0.775	0.789	40.424	3.677	11.372	16.215	10.891	10.601	40.393	3.537	11.283	16.175	11.116	10.544
					Panel B: RMSE													
$(0.9, 0.1)'$	8.960	8.548	9.427	**6.064**	18.812	26.258	11.340	8.166	10.314	7.174	19.894	27.705	11.515	8.426	10.844	7.314	20.858	27.847
$(0.8, 0.2)'$	8.361	8.111	9.005	**5.738**	18.283	25.453	12.600	7.999	10.904	8.173	20.132	27.787	12.737	8.240	11.385	8.273	21.108	27.949
$(0.7, 0.3)'$	7.956	7.803	8.726	**5.514**	17.965	24.914	13.573	8.112	11.473	9.030	20.394	27.918	13.678	8.320	11.905	9.092	21.372	28.095
$(0.6, 0.4)'$	7.743	7.624	8.591	**5.388**	17.852	24.640	14.184	8.277	11.870	9.592	20.599	28.041	14.257	8.447	12.258	9.616	21.572	28.229
$(0.5, 0.5)'$	7.718	7.572	8.594	**5.359**	17.940	24.631	14.398	8.359	12.021	9.795	20.700	28.123	14.440	8.489	12.369	9.780	21.665	28.319

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

Table D17: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and additive microstructure noise (MA(1), noise ratio: 0.001, $\rho_\nu = 0.5$, 1-min sampling frequency)

$\alpha_\perp$	RV_{GRT}	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
					Panel A: Bias													
$(0.9, 0.1)'$	38.885	-5.749	-0.162	10.745	**-0.051**	0.066	47.922	-1.528	4.327	16.518	4.222	4.209	47.901	-1.627	4.261	16.514	4.474	4.170
$(0.8, 0.2)'$	36.285	-5.351	**0.062**	10.180	0.199	0.281	51.037	1.551	7.404	19.614	7.142	6.992	51.005	1.442	7.327	19.599	7.383	6.945
$(0.7, 0.3)'$	34.500	-5.037	**0.255**	9.822	0.420	0.471	53.264	3.754	9.605	21.828	9.231	8.985	53.220	3.634	9.517	21.801	9.463	8.930
$(0.6, 0.4)'$	33.531	-4.807	**0.417**	9.670	0.614	0.635	54.603	5.078	10.928	23.160	10.491	10.189	54.548	4.948	10.829	23.121	10.713	10.127
$(0.5, 0.5)'$	33.377	-4.661	**0.548**	9.726	0.780	0.774	55.054	5.525	11.375	23.609	10.922	10.603	54.987	5.385	11.265	23.559	11.134	10.533
					Panel B: RMSE													
$(0.9, 0.1)'$	12.899	8.455	9.644	**6.968**	18.816	26.261	15.793	8.156	10.564	8.545	19.914	27.721	15.909	8.417	11.083	8.646	20.861	27.860
$(0.8, 0.2)'$	12.027	8.026	9.211	**6.587**	18.289	25.458	16.998	8.065	11.149	9.646	20.151	27.804	17.089	8.303	11.614	9.712	21.110	27.962
$(0.7, 0.3)'$	11.429	7.725	8.926	**6.326**	17.972	24.920	17.920	8.231	11.711	10.544	20.413	27.934	17.988	8.433	12.124	10.578	21.374	28.107
$(0.6, 0.4)'$	11.104	7.550	8.785	**6.183**	17.859	24.647	18.497	8.424	12.103	11.120	20.618	28.057	18.541	8.588	12.471	11.123	21.575	28.241
$(0.5, 0.5)'$	11.049	7.500	8.788	**6.155**	17.946	24.638	18.699	8.516	12.253	11.327	20.720	28.139	18.718	8.640	12.582	11.294	21.667	28.331

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

D.7 Two-market system: MSN and jumps

Table D18: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility, jumps and additive microstructure noise (noise ratio: 0.001, $\lambda = 0.014$, $\rho_\nu = 0.5$, 1-min sampling frequency)

$\alpha_\perp$	RV_{GRT}	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_1	$RBPV_1$	PRV_1	$PBPV_1$	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	$RBPV_2$	PRV_2	$PBPV_2$	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
								Panel A: Bias																
$(0.9, 0.1)'$	52.954	56.390	-7.186	-44.234	-0.475	8.656	**-0.034**	0.207	62.436	66.284	-3.550	-42.833	3.600	13.646	3.868	3.912	62.286	66.231	-3.568	-42.841	3.527	13.572	3.698	3.786
$(0.8, 0.2)'$	49.011	52.137	-6.776	-42.244	-0.365	8.086	**0.042**	0.282	65.421	69.27	-0.598	-39.861	6.549	16.613	6.663	6.575	65.261	69.207	-0.626	-39.873	6.466	16.529	6.482	6.436
$(0.7, 0.3)'$	46.304	49.221	-6.437	-40.805	-0.237	7.739	**0.144**	0.383	67.557	71.405	1.514	-37.736	8.660	18.737	8.666	8.484	67.386	71.333	1.477	-37.751	8.565	18.641	8.474	8.333
$(0.6, 0.4)'$	44.834	47.633	-6.169	-39.915	**-0.090**	7.616	0.271	0.509	68.842	72.690	2.786	-36.458	9.931	20.015	9.876	9.641	68.660	72.608	2.739	-36.477	9.826	19.909	9.673	9.478
$(0.5, 0.5)'$	44.600	47.375	-5.972	-39.575	**0.076**	7.715	0.422	0.662	69.278	73.125	3.218	-36.027	10.363	20.449	10.293	10.044	69.085	73.033	3.161	-36.050	10.247	20.332	10.079	9.869
								Panel B: RMSE																
$(0.9, 0.1)'$	17.172	18.356	7.756	23.539	8.711	**5.901**	16.093	21.584	20.156	21.434	7.284	22.844	9.293	6.976	17.128	22.862	20.165	21.434	7.794	22.787	10.131	7.448	17.971	23.975
$(0.8, 0.2)'$	15.888	16.969	7.347	22.465	8.320	**5.539**	15.384	20.689	21.215	22.454	7.030	21.288	9.716	7.926	17.336	22.952	21.194	22.454	7.566	21.229	10.522	8.332	18.124	24.005
$(0.7, 0.3)'$	15.003	16.015	7.057	21.686	8.071	**5.304**	14.908	20.100	22.013	23.225	7.060	20.176	10.159	8.721	17.560	23.072	21.969	23.225	7.589	20.115	10.923	9.072	18.300	24.073
$(0.6, 0.4)'$	14.521	15.492	6.888	21.200	7.966	**5.197**	14.666	19.817	22.511	23.706	7.169	19.507	10.481	9.239	17.724	23.166	22.445	23.706	7.671	19.444	11.203	9.544	18.425	24.126
$(0.5, 0.5)'$	14.441	15.404	6.841	21.009	8.005	**5.219**	14.660	19.836	22.687	23.876	7.231	19.282	10.608	9.430	17.784	23.204	22.602	23.876	7.696	19.214	11.292	9.695	18.455	24.133

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

Table D19: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility, jumps and additive microstructure noise (noise ratio: 0.005, $\lambda = 0.014$, $\rho_\nu = 0.5$, 1-min sampling frequency)

$\alpha_\perp$	RV_{GRT}	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_1	$RBPV_1$	PRV_1	$PBPV_1$	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	$RBPV_2$	PRV_2	$PBPV_2$	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
								Panel A: Bias																
$(0.9, 0.1)'$	26.367	27.484	-7.199	-44.433	-0.432	4.172	**-0.029**	0.211	33.157	34.431	-3.569	-43.053	3.641	8.703	3.869	3.911	33.032	34.371	-3.587	-43.061	3.539	8.634	3.698	3.786
$(0.8, 0.2)'$	24.444	25.443	-6.790	-42.429	-0.328	3.941	**0.046**	0.284	36.143	37.416	-0.617	-40.081	6.591	11.670	6.664	6.574	36.007	37.346	-0.644	-40.093	6.478	11.591	6.482	6.436
$(0.7, 0.3)'$	23.149	24.075	-6.452	-40.979	-0.205	3.831	**0.145**	0.383	38.279	39.552	1.495	-37.956	8.701	13.794	8.666	8.483	38.131	39.471	1.458	-37.971	8.577	13.703	8.473	8.333
$(0.6, 0.4)'$	22.483	23.371	-6.185	-40.084	**-0.063**	3.842	0.271	0.509	39.564	40.837	2.767	-36.679	9.973	15.072	9.876	9.639	39.406	40.746	2.721	-36.697	9.838	14.971	9.672	9.478
$(0.5, 0.5)'$	22.444	23.329	-5.989	-39.743	**0.099**	3.973	0.421	0.660	40.000	41.272	3.199	-36.248	10.405	15.506	10.293	10.043	39.830	41.171	3.143	-36.270	10.259	15.394	10.079	9.869
								Panel B: RMSE																
$(0.9, 0.1)'$	8.983	9.458	7.739	23.574	8.522	**5.330**	16.092	21.586	11.057	11.536	7.264	22.885	9.092	6.081	17.124	22.860	11.119	11.536	7.781	22.827	9.942	6.621	17.967	23.970
$(0.8, 0.2)'$	8.308	8.741	7.332	22.499	8.145	**5.013**	15.383	20.690	12.184	12.620	7.009	21.328	9.526	6.944	17.331	22.950	12.197	12.620	7.552	21.270	10.339	7.409	18.120	24.000
$(0.7, 0.3)'$	7.850	8.253	7.043	21.718	7.906	**4.807**	14.906	20.100	13.053	13.460	7.038	20.216	9.979	7.704	17.555	23.069	13.028	13.460	7.575	20.156	10.747	8.104	18.296	24.068
$(0.6, 0.4)'$	7.608	7.996	6.875	21.232	7.807	**4.715**	14.664	19.816	13.599	13.990	7.147	19.547	10.307	8.209	17.719	23.164	13.542	13.99	7.656	19.484	11.031	8.557	18.421	24.121
$(0.5, 0.5)'$	7.583	7.971	6.829	21.041	7.848	**4.736**	14.657	19.834	13.794	14.180	7.210	19.322	10.437	8.397	17.779	23.201	13.710	14.180	7.681	19.254	11.122	8.702	18.451	24.128

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.7$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

D.8 Simulations with a time-varying α

We follow Dias et al. (2022b) and generate price data with a time-varying α. Since the price discovery measures relate to highly persistent fundamentals, we do not allow α to change suddenly, for example, with daily variation that is unrelated to the previous day. Instead, we generate the values according to a persistent random walk structure but still maintain the conditions necessary to guarantee a cointegration relationship between the prices. We draw data at the 5-minute frequency with $\rho = 0.9$ and at the 1-min frequency with $\rho = 0.7$ according to the two market stochastic volatility model outlined in Section 3 of the main text, expect for the adjustment parameters that evolve according to the following model (Giraitis et al., 2014):

$$\alpha_{1,\delta}^t = \bar{\alpha}\left(\frac{\alpha_1^t}{\max|\alpha_1^t|} - 1\right) - 0.01, \qquad \alpha_{2,\delta}^t = \bar{\alpha}\left(\frac{\alpha_2^t}{\max|\alpha_2^t|} + 1\right) + 0.01 \tag{D.8}$$

where $\bar{\alpha} = 0.24$ and t indicates a 5-minute sampling interval. This configuration guarantees that the discrete time adjustment coefficients are bounded: $\alpha_{1,\delta}^t \in [-0.49, -0.01]$ and $\alpha_{2,\delta}^t \in [0.01, 0.49]$. This implies that cointegration relationship is stable.

The results are reported in Table D20. To compare the results to the constant adjustment case, we take the mean over $(\alpha_{1,\delta}^t, \alpha_{2,\delta}^t)'$. We find that the performance of the two-step estimator is very robust to the time variation of α that is not accounted for by the least squares estimator in the first step.

Table D20: Stochastic volatility simulation with time-varying α (5-min, $\rho = 0.9$ and 1-min, $\rho = 0.7$)

α	RV_{GRT}	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{p}	RV_1	PRV_1	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
5-min:																		
					Panel A: Bias													
const.	44.139	**32.559**	43.631	43.758	43.448	43.728	47.955	33.178	43.917	45.388	43.509	43.765	49.432	33.472	44.099	46.088	43.536	43.721
RW	45.924	**33.192**	44.672	45.397	45.160	45.072	51.499	34.222	45.379	47.889	45.315	45.152	50.582	33.977	45.135	47.414	45.268	45.095
					Panel B: RMSE													
const.	23.040	**18.786**	26.087	23.723	31.089	37.559	25.006	19.111	26.350	24.550	31.154	37.680	25.773	19.266	26.410	24.900	31.116	37.440
RW	27.822	**22.442**	30.621	28.269	38.521	47.767	31.081	23.001	31.374	29.611	38.504	47.638	30.331	23.018	31.055	29.466	38.693	47.963
1-min:																		
					Panel A: Bias													
const.	62.869	**55.751**	63.012	62.882	62.816	62.607	77.520	58.673	65.376	69.999	63.091	62.737	77.070	58.590	65.306	69.773	62.973	62.611
RW	61.521	**55.134**	61.542	61.591	61.573	61.532	72.880	61.608	68.729	70.702	63.522	62.382	72.962	61.598	68.626	70.744	63.592	62.467
					Panel B: RMSE													
const.	32.673	**29.917**	34.097	33.150	36.978	39.414	40.297	31.368	35.296	36.787	37.125	39.558	40.057	31.457	35.495	36.766	37.070	39.378
RW	31.431	**28.643**	32.123	31.659	34.097	36.377	37.257	32.069	35.969	36.395	35.075	36.685	37.249	31.943	35.828	36.317	35.241	37.087

This table reports the average bias ($\times 10^5$) and MSE ($\times 10^9$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.9$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumbs suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and MSE are marked in boldface.

D.9 Co-jumping

In this extension of the jump-diffusion model used in the main paper, the prices are generated according to the model

$$dP(t) = \Pi P(t)dt + \Sigma(t)dW(t) + \iota_N dJ^P(t), \tag{D.9}$$

where the spot variances are also affected by jumps:

$$\sigma_j^2(t) = \exp(\varsigma_0 + \varsigma_1 V_j(t)), \tag{D.10}$$

with

$$dV_j(t) = \gamma_V V_j(t)dt + dB_j(t) + dJ^V(t),\ j = 1, 2. \tag{D.11}$$

The jump components $J^P(t)$ and $J^V(t)$ are determined by a univariate compound Poisson process. More specifically, the jumps are generated by

$$dJ^s(t) = \sum_{i=1}^{N(t)} Y_i^s, \qquad s = P, V, \tag{D.12}$$

where $N(t) \sim \text{Poisson}(\lambda t)$ determines how many jumps occur based on the jump intensity parameter λ. This jump process is identical in both cases and ensures that the jumps occur at the same points in time, i.e., generated co-jumps in the price and volatility process. However, the jump sizes are allowed to differ. $Y_i^P \sim N(0, \sigma_J^2)$ determines the jump size for the price process and $Y_i^V \sim exp(\pi)$ for the volatility process ensures that we obtain positive spot variances. In this simulation experiment, we set $\sigma_J^2 = 7.5 \cdot 10^{-5}$ as for our main results in the paper. Following Li and Linton (2022), we set $\pi = 0.05/252$ but our results do not appear to be sensitive to the magnitude of the volatility jumps.

Table D21: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and co-jumps (5-min, $\lambda = 0.014$, $\sigma_J^2 = 7.5 \cdot 10^{-5}$, $\pi = 0.05/252$)

$\alpha_\perp$	RV_{GRT}	$RBPV_{GRT}$	RV_1	PRV_1	$RBPV_1$	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^{P}	RV_2	PRV_2	$RBPV_2$	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^{P}
Panel A: Bias																
$(0.9, 0.1)'$	1.398	-0.136	1.785	0.251	-8.044	1.219	1.842	0.267	0.143	1.748	0.143	-8.210	0.924	1.587	0.501	**0.006**
$(0.8, 0.2)'$	1.247	**-0.296**	2.588	1.056	-7.265	1.988	2.628	0.956	0.768	2.537	0.936	-7.442	1.679	2.359	1.175	0.616
$(0.7, 0.3)'$	1.099	**-0.450**	3.166	1.635	-6.705	2.539	3.193	1.448	1.214	3.101	1.503	-6.894	2.218	2.911	1.652	1.047
$(0.6, 0.4)'$	0.953	**-0.610**	3.518	1.990	-6.366	2.874	3.537	1.742	1.482	3.439	1.844	-6.565	2.539	3.241	1.932	1.300
$(0.5, 0.5)'$	0.810	**-0.765**	3.646	2.118	-6.247	2.992	3.660	1.838	1.570	3.552	1.960	-6.457	2.643	3.350	2.014	1.373
Panel B: RMSE																
$(0.9, 0.1)'$	**5.558**	5.881	5.602	5.911	9.807	12.637	8.606	18.955	26.489	5.836	5.769	9.992	13.034	8.552	18.945	26.376
$(0.8, 0.2)'$	**5.477**	5.765	5.650	5.891	9.614	12.639	8.64	18.871	26.334	5.861	5.737	9.785	13.02	8.559	18.856	26.225
$(0.7, 0.3)'$	**5.416**	5.686	5.705	5.895	9.485	12.648	8.676	18.816	26.219	5.891	5.729	9.642	13.014	8.572	18.795	26.113
$(0.6, 0.4)'$	**5.376**	5.643	5.747	5.905	9.412	12.655	8.704	18.785	26.143	5.907	5.726	9.555	13.010	8.577	18.758	26.039
$(0.5, 0.5)'$	**5.356**	5.602	5.765	5.910	9.389	12.656	8.715	18.777	26.103	5.898	5.715	9.520	13.002	8.569	18.742	26.002

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.9$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

Table D22: Simulation Results: Bias and relative efficiency of RV_{GRT} estimator for the cointegrated OU type of process with stochastic volatility and co-jumps (1-min, $\lambda = 0.014$, $\sigma_J^2 = 7.5 \cdot 10^{-5}$, $\pi = 0.05/252$)

$\alpha_\perp$	RV_{GRT}	$RBPV_{GRT}$	RV_1	PRV_1	$RBPV_1$	RV_1^{AC}	RV_1^{NW}	RK_1^{MTH}	RK_1^P	RV_2	PRV_2	$RBPV_2$	RV_2^{AC}	RV_2^{NW}	RK_2^{MTH}	RK_2^P
Panel A: Bias																
$(0.9, 0.1)'$	0.972	**0.310**	4.944	4.279	-3.540	4.915	4.948	4.727	4.497	4.893	4.225	-3.468	5.176	5.025	4.908	4.646
$(0.8, 0.2)'$	0.768	**0.107**	8.380	7.714	-0.148	8.305	8.361	7.926	7.532	8.315	7.647	-0.187	8.554	8.425	8.092	7.667
$(0.7, 0.3)'$	0.643	**-0.022**	10.836	10.170	2.278	10.729	10.801	10.213	9.704	10.759	10.091	2.327	10.966	10.853	10.366	9.823
$(0.6, 0.4)'$	0.595	**-0.076**	12.313	11.646	3.737	12.187	12.269	11.590	11.012	12.223	11.555	3.774	12.411	12.308	11.729	11.117
$(0.5, 0.5)'$	0.625	**-0.049**	12.811	12.144	4.228	12.679	12.764	12.057	11.457	12.709	12.040	4.254	12.890	12.79	12.181	11.547
Panel B: RMSE																
$(0.9, 0.1)'$	**3.509**	3.696	4.309	4.437	8.966	11.175	7.030	20.489	27.949	4.631	4.661	9.371	11.458	7.443	21.312	27.991
$(0.8, 0.2)'$	**3.319**	3.469	5.646	5.698	8.621	11.635	7.842	20.64	27.961	5.774	5.737	9.031	11.932	8.183	21.466	28.020
$(0.7, 0.3)'$	**3.180**	3.305	6.776	6.791	8.620	12.133	8.636	20.845	28.042	6.786	6.718	9.011	12.423	8.909	21.657	28.105
$(0.6, 0.4)'$	**3.088**	3.198	7.502	7.501	8.725	12.494	9.183	21.013	28.128	7.431	7.349	9.082	12.764	9.398	21.799	28.183
$(0.5, 0.5)'$	**3.044**	3.149	7.768	7.762	8.785	12.631	9.388	21.09	28.182	7.631	7.546	9.102	12.869	9.551	21.839	28.217

This table reports the average bias and RMSE ($\times 10^5$) for several RV estimators over the full sampling period. The results are based on 1000 draws from the stochastic volatility two-market model with $\rho = 0.9$. The subscripts indicate that the realized measures are computed using prices from market one, two or based on the GRT, respectively. The lag truncation parameter for realized kernel estimators is set according to the rule-of-thumb suggested in Hansen and Lunde (2006) and Barndorff-Nielsen et al. (2008). The estimators with the lowest bias and RMSE are marked in boldface.

D.10 Five-market system

In this section, we consider a more realistic DGP with five markets, jumps, additive microstructure noise, non-synchronously recorded prices and rounding effects. We set

$$\beta = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{D.13}$$

to guarantee that both models are driven by one common stochastic trend. We generate data from two models with different adjustment behavior. The first model imposes equal component shares for all five markets ($\alpha_\perp = (0.2, 0.2, 0.2, 0.2, 0.2)$). Here, the adjustment coefficient matrix is set to

$$\alpha = \begin{bmatrix} -10 & -10 & -10 & -10 \\ 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \tag{D.14}$$

and thereby closely resembles the adjustment coefficients observed in our empirical application.[13]

In the second model, we generate data under the assumption that the first market contributes substantially less adjustment to the long-run equilibrium. This results in a 50% component share of the fifth market ($\alpha_\perp = (0.125, 0.125, 0.125, 0.125, 0.5)$). The corresponding adjustment coefficient matrix is given by

$$\alpha = \begin{bmatrix} -10 & -10 & -10 & -10 \\ 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 2.5 \end{bmatrix}. \tag{D.15}$$

[13] This specific adjustment coefficient matrix was constructed in line with the estimated coefficient matrix for IBM – a stock with a medium trading activity in the DJIA.

As in our two market simulation experiments, we set $\lambda = 0.014$ as the jump intensity and $\sigma_J^2 = 7.5 \cdot 10^{-5}$ as the variance of the jump magnitude. Moreover, the level of microstructure noise is identical in all markets by setting the noise ratio to 0.001. Potential non-synchronicity of the prices are modeled by randomly deleting price observations to reach 2% zero returns for 5-minute data and 4% zero returns for 1-minute data, aligning the experiment with the DJIA data. The prices are then put onto a refresh time grid to synchronize the observations. Finally, the prices are rounded to the third digit.

Table D23: Simulation Results: Bias and relative efficiency of GRT-based estimators in a five market system (mixed sampling frequencies, low noise ratio)

Panel A: $\alpha_{\perp} = (0.2, 0.2, 0.2, 0.2, 0.2)'$

	Bias							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**0.199**	**0.116**	**-0.081**	-3.545	**-0.003**	**0.912**	**0.003**	**0.017**
1	1.895	1.622	1.393	-2.965	1.696	3.953	0.703	0.418
2	2.172	1.899	1.640	-2.869	1.804	4.003	1.398	1.196
3	2.145	1.868	1.613	-2.879	1.794	4.003	1.461	1.277
4	2.210	1.929	1.663	**-2.860**	1.824	4.029	1.444	1.210
5	2.148	1.853	1.634	-2.871	1.785	3.997	1.380	1.180
	RMSE							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**1.060**	**1.159**	**0.833**	5.057	**0.668**	**0.989**	**1.951**	**2.663**
1	2.656	2.499	2.092	4.267	2.543	4.337	2.339	2.855
2	3.092	2.958	2.494	4.119	2.707	4.393	3.077	3.594
3	3.056	2.883	2.525	4.132	2.732	4.414	3.172	3.692
4	3.273	3.183	2.623	**4.081**	2.857	4.480	3.034	3.450
5	3.024	2.774	2.454	4.141	2.671	4.391	3.085	3.618

Panel B: $\alpha_{\perp} = (0.125, 0.125, 0.125, 0.125, 0.5)'$

	Bias							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**0.261**	**0.167**	**-0.048**	-3.688	**0.028**	**1.154**	**0.021**	**0.037**
1	1.657	1.396	1.152	-3.215	1.434	3.697	0.504	0.237
2	1.927	1.663	1.391	-3.122	1.532	3.751	1.185	1.012
3	1.895	1.606	1.364	-3.133	1.524	3.749	1.245	1.090
4	1.957	1.662	1.413	-3.114	1.568	3.771	1.228	1.021
5	1.927	1.636	1.424	**-3.110**	1.553	3.752	1.354	1.272
	RMSE							
	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	**1.123**	**1.221**	**0.864**	5.265	**0.717**	**1.223**	**2.088**	**2.847**
1	2.378	2.264	1.812	4.618	2.206	4.014	2.268	2.851
2	2.788	2.677	2.192	4.474	2.362	4.071	2.931	3.536
3	2.751	2.584	2.232	4.486	2.391	4.090	2.994	3.603
4	2.976	2.876	2.321	**4.436**	2.531	4.147	2.857	3.362
5	2.725	2.509	2.187	4.474	2.357	4.067	3.180	3.910

This table reports the average bias and RMSE (scaled by $\times 10^4$) for several RV estimators over the full sampling period. Prices are simulated from (D.9) and contaminated with additive market microstructure noise simulated from a $VMA(1)$ process, with noise ratio set to 0.0001. Non-synchronous trading and price rounding are then imposed. Finally, prices are aggregated using the refresh time method, resulting in an average of 20,000 observations per day. The columns denote the alternative estimators used in this simulation: the 1-minute realized variance estimator ($RV^{(390)}$), realized bipower variation ($RBPV$), the pre-averaged estimator (PRV), pre-averaged realized bipower variation ($PBRV$), the noise-robust estimators RV^{AC} and RV^{NW} proposed by Hansen and Lunde (2006), and two variants of the realized kernel estimators, RK^{MTH} and RK^{P}. The rows correspond to either the GRT-based estimators (GRT) or the univariate estimators that use prices sampled from a single market, ($n = 1, \ldots, 5$). For each class of estimator, the lowest bias and RMSE are marked in boldface.

Table D24: Simulation Results: Relative efficiency of GRT-based estimators in a five market system (5-min, high noise ratio)

$\alpha_{\perp} = (0.2, 0.2, 0.2, 0.2, 0.2)'$

	RV	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
$\tau = 0.98$								
GRT	**1.678**	**1.837**	3.546	7.259	**4.584**	**2.727**	**6.227**	**8.492**
1	5.246	5.236	3.655	6.929	5.559	3.787	6.432	8.653
2	4.391	4.349	3.366	6.646	5.520	3.941	6.528	8.886
3	4.471	4.329	**3.322**	6.487	5.567	4.214	6.683	8.671
4	4.688	4.587	3.687	**6.452**	5.975	4.691	7.107	9.269
5	4.756	4.702	3.452	6.533	5.493	4.291	6.306	8.558
$\tau = 0.7$								
GRT	**2.424**	**2.399**	4.585	7.976	**5.376**	**3.409**	**6.897**	**9.538**
1	4.565	3.925	4.304	7.756	5.997	3.807	7.105	9.738
2	4.066	3.324	3.866	7.532	6.153	4.134	7.197	9.875
3	4.650	3.757	3.806	7.439	6.332	4.404	7.105	9.661
4	4.798	4.064	**3.671**	**7.397**	6.685	4.658	7.796	10.344
5	5.024	4.221	3.772	7.423	6.100	4.482	6.990	9.577
$\tau = 0.5$								
GRT	**3.174**	**3.263**	5.402	8.558	**6.322**	**4.171**	7.932	**10.435**
1	4.410	3.765	5.245	8.420	6.941	4.426	8.252	10.661
2	4.313	3.279	4.721	8.260	7.107	4.669	8.308	10.746
3	4.737	3.717	4.621	8.231	7.543	4.898	8.077	10.673
4	5.079	3.923	4.658	**8.206**	8.265	5.547	9.001	11.465
5	5.308	4.018	**4.609**	8.224	6.967	4.980	**7.925**	10.488
$\tau = 0.3$								
GRT	**3.971**	3.778	6.178	8.714	7.276	4.795	**8.172**	–
1	4.206	4.034	6.177	**8.700**	**7.040**	**4.740**	8.469	–
2	4.488	**3.777**	5.766	8.560	7.426	5.113	8.890	–
3	4.563	3.895	**5.693**	8.573	7.443	4.940	8.237	–
4	4.925	4.031	5.969	8.499	10.041	6.349	9.544	–
5	5.127	4.173	5.698	8.523	7.638	5.388	8.372	–

This table reports the RMSE (scaled by $\times 10^4$) for several RV estimators over the full sampling period. Prices are simulated from (D.9) and contaminated with additive market microstructure noise simulated from a $VMA(1)$ process, with noise ratio set to 0.0001. Non-synchronous trading and price rounding are then imposed. Finally, prices are aggregated using the refresh time method, resulting in an average of $\tau \times 79$ observations per day. The columns denote the alternative estimators used in this simulation: the 1-minute realized variance estimator ($RV^{(390)}$), realized bipower variation ($RBPV$), the pre-averaged estimator (PRV), pre-averaged realized bipower variation ($PBRV$), the noise-robust estimators RV^{AC} and RV^{NW} proposed by Hansen and Lunde (2006), and two variants of the realized kernel estimators, RK^{MTH} and RK^{P}. The rows correspond to either the GRT-based estimators (GRT) or the univariate estimators that use prices sampled from a single market, ($n = 1, \ldots, 5$). For each class of estimator, the lowest RMSE are marked in boldface.

E Details on the exchanges and the market microstructure

In recent decades, significant regulatory frameworks have been established in the U.S. to enhance the integrity, efficiency, and competitiveness of financial markets. Among these, two key regulations stand out: Regulation ATS (Alternative Trading Systems) and Regulation National Market System (Reg NMS). Each of these regulations played a crucial role in shaping the landscape of trading venues and ensuring that markets are equipped to meet the demands of modern trading practices.

Regulation ATS was enacted in 2000 by the U.S. Securities and Exchange Commission (SEC) and designed to create a regulatory framework for Alternative Trading Systems. Alternative Trading Systems encompass various trading platforms, most notably "dark pools" and Electronic Communication Networks (ECNs). These venues are pivotal for facilitating the matching of large buy and sell orders but do not operate under the same stringent regulations as traditional exchanges. The primary aim of Reg ATS was to account for the emergence and prevalence of non-exchange trading platforms, ensuring that they adhere to standards that promote fair trading practices.

In 2005, the SEC introduced Reg NMS to fortify U.S. securities exchanges in light of evolving technological advancements and changing market conditions. The regulation's primary objective is to enhance market efficiency and fairness by establishing a framework that ensures all investors can access the best available prices for their trades.

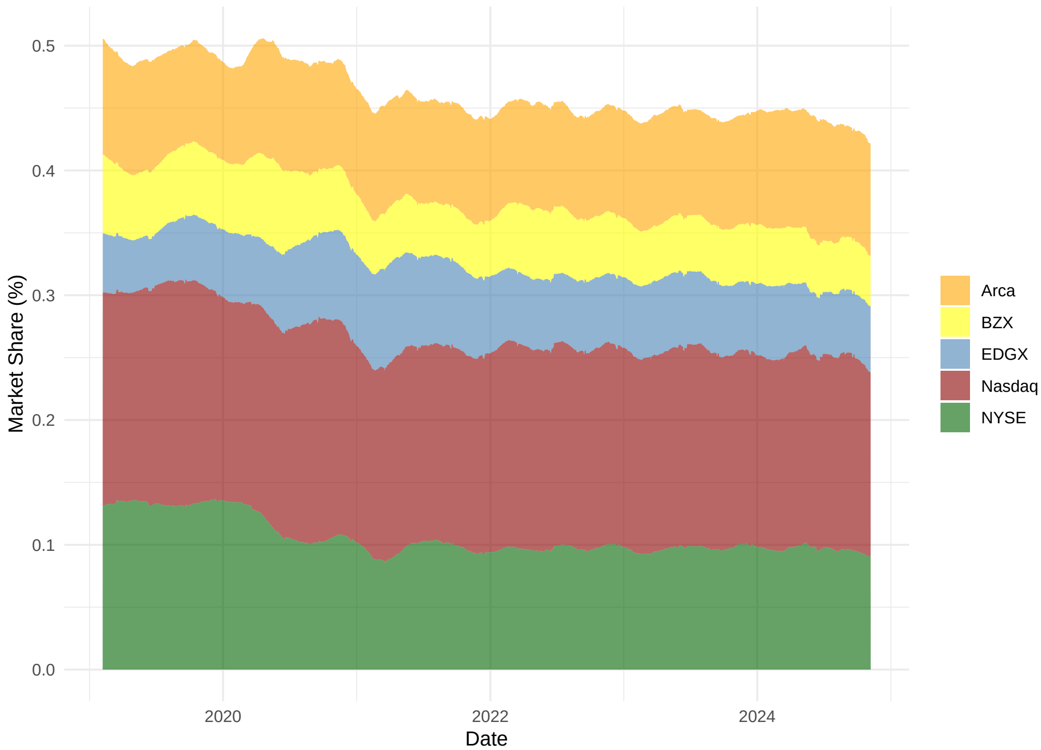
Reg NMS encompasses several critical components, including the implementation of order protection rules designed to prevent trade-throughs, i.e., situations where a trade occurs at a less favorable price than what is available elsewhere in the market. Additionally, the regulation seeks to improve access to market data, ensuring that all market participants have timely and equitable access to essential trading information. Another notable aspect of Reg NMS is the decimalization of price quotes, which allows for finer price distinctions and enhances the price formation process, ultimately leading to narrower bid-ask spreads.

Overall, these regulations have laid the groundwork for the creation and operation of multiple trading venues that are interconnected and compete for liquidity and trades. In our empirical application, we specifically estimate the integrated variance from prices observed on the five main exchanges in the U.S.: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z"). A description of those venues in provided in Table E.1 and the evolution of their overall market share is depicted in Figure E.1.

Table E.1: Description of the trading venues

Trading Venue	Symbol	Type	Description
NYSE	N	Limit order market	Combination of electronic trading and floor trading; brokers and specialists to execute trades.
Nasdaq	T	Dealer market	Entirely electronic; utilizes multiple market makers to facilitate trading.
NYSE Arca	P	ECN	Entirely electronic; matches orders based on price/time priority.
Cboe EDGX	K	National securities exchange	Uses a price/time priority model for executing trades. Features advanced technology and supports both maker-taker and taker-maker models.
Cboe BZX	Z	National securities exchange	Features advanced technology and a trading system with a maker-taker fee model.

Figure E.1: U.S. equity market shares
This figure depicts the overall market share in terms of trading volume of the five main trading venues in the U.S.: NYSE (darkgreen), Nasdaq (darkred), Arca (orange), Cboe EDGX (steelblue), and Cboe BZX (yellow) from November 2018 to November 2024.



Symbol	Company	Primary	Industry	Weight
AAPL	Apple	NASDAQ	Information Technology	3.33%
AMGN	Amgen	NASDAQ	Biopharmaceutical	3.76%
AMZN	Amazon	NASDAQ	Retailing	3.02%
AXP	American Express	NYSE	Financial Services	4.12%
BA	Boeing	NYSE	Aerospace & Defense	2.15%
CAT	Caterpillar	NYSE	Construction & Mining	5.41%
CRM	Salesforce	NYSE	Information Technology	4.95%
CSCO	Cisco	NASDAQ	Information Technology	0.82%
CVX	Chevron	NYSE	Petroleum Industry	2.18%
DIS	Disney	NYSE	Entertainment	1.60%
GS	Goldman Sachs	NYSE	Financial Services	8.18%
HD	Home Depot	NYSE	Home Improvement	5.84%
HON	Honeywell	NASDAQ	Conglomerate	3.10%
IBM	IBM	NYSE	Information Technology	3.22 %
JNJ	Johnson & Johnson	NYSE	Pharmaceutical Industry	2.05%
JPM	JPMorgan Chase	NYSE	Financial Services	3.36%
KO	Coca-Cola	NYSE	Drink industry	0.86%
MCD	McDonald's	NYSE	Restaurant	4.10%
MMM	3M	NYSE	Conglomerate	1.83%
MRK	Merck	NYSE	Pharmaceutical Industry	1.42%
MSFT	Microsoft	NASDAQ	Information Technology	6.06%
NKE	Nike, Inc.	NYSE	Clothing industry	1.08%

Symbol	**Company**	**Primary**	**Industry**	**Weight**
NVDA	Nvidia	NASDAQ	Information technology	1.99%
PG	Procter & Gamble	NYSE	Consumer Goods	2.41%
SHW	Sherwin-Williams	NYSE	Speciality chemicals	5.28%
TRV	Travelers	NYSE	Insurance	3.61%
UNH	UnitedHealth Group	NYSE	Healthcare	7.93%
V	Visa	NYSE	Financial Services	4.23%
VZ	Verizon	NYSE	Telecommunications industry	0.58%
WMT	Walmart	NYSE	Retailing	1.31%

Table E.3: Average proportion of daily trading volume

	N	T	K	P	Z
AAPL	0.047	0.452	0.177	0.164	0.160
AMGN	0.027	0.604	0.144	0.152	0.073
AMZN	0.037	0.460	0.182	0.150	0.170
AXP	0.311	0.350	0.106	0.155	0.079
BA	0.276	0.253	0.122	0.129	0.220
CAT	0.317	0.300	0.116	0.156	0.110
CRM	0.311	0.311	0.130	0.137	0.110
CSCO	0.076	0.513	0.158	0.158	0.095
CVX	0.334	0.288	0.115	0.144	0.119
DIS	0.314	0.286	0.119	0.161	0.120
GS	0.295	0.315	0.116	0.161	0.113
HD	0.315	0.302	0.119	0.158	0.106
HON	0.192	0.450	0.136	0.143	0.079
IBM	0.335	0.288	0.121	0.146	0.110
JNJ	0.360	0.274	0.119	0.157	0.091
JPM	0.311	0.294	0.118	0.165	0.112
KO	0.378	0.257	0.125	0.145	0.095
MCD	0.358	0.278	0.128	0.147	0.088
MMM	0.342	0.310	0.116	0.139	0.092
MRK	0.356	0.280	0.121	0.154	0.090
MSFT	0.052	0.515	0.153	0.159	0.121
NKE	0.345	0.288	0.122	0.154	0.091
NVDA	0.029	0.474	0.164	0.131	0.201
PG	0.376	0.280	0.119	0.148	0.078
SHW	0.369	0.328	0.129	0.104	0.070
TRV	0.368	0.330	0.102	0.135	0.065
UNH	0.342	0.300	0.126	0.145	0.087
V	0.355	0.280	0.130	0.141	0.094
VZ	0.340	0.263	0.128	0.156	0.113
WMT	0.340	0.280	0.135	0.137	0.107

The trading venues are denoted with their ticker symbol: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

Table E.4: Average daily 1-min RV ($\cdot 10^4$)

	N	T	K	P	Z
AAPL	3.513	2.345	2.348	2.350	2.360
AMGN	7.701	2.535	5.155	4.131	3.895
AMZN	52.896	3.213	3.260	3.709	4.432
AXP	2.359	2.384	3.697	2.908	2.662
BA	4.376	4.523	4.538	5.117	4.917
CAT	2.592	2.726	4.439	4.082	2.888
CRM	3.129	3.203	4.953	3.716	3.553
CSCO	2.930	1.977	2.035	2.011	2.008
CVX	2.266	2.366	2.612	2.543	2.388
DIS	2.114	2.165	2.273	2.229	2.212
GS	2.385	2.487	4.266	3.552	2.787
HD	1.928	2.060	3.043	2.699	2.167
HON	2.112	1.842	6.148	4.210	2.155
IBM	1.442	1.509	2.118	1.749	1.607
JNJ	1.232	1.319	1.722	1.424	1.328
JPM	1.950	1.975	2.143	2.027	2.015
KO	1.089	1.121	1.214	1.127	1.136
MCD	1.372	1.548	2.590	1.816	1.625
MMM	2.173	2.358	3.505	3.508	2.692
MRK	1.552	1.629	2.337	1.820	1.675
MSFT	3.477	2.359	2.388	2.377	2.409
NKE	2.675	2.806	3.798	3.075	2.924
NVDA	10.141	6.988	7.058	7.191	7.292
PG	1.272	1.342	2.042	1.552	1.404
SHW	2.754	3.944	34.479	10.078	5.050
TRV	1.848	2.113	8.387	4.591	2.379
UNH	2.242	2.456	4.336	3.168	2.872
V	1.744	1.832	2.342	2.063	2.115
VZ	1.321	1.440	1.528	1.433	1.456
WMT	1.318	1.367	1.683	1.471	1.454

The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.5: Average daily 5-min RV ($\cdot 10^4$)

	N	T	K	P	Z
AAPL	2.797	2.380	2.385	2.381	2.393
AMGN	4.253	2.442	3.087	2.953	3.246
AMZN	16.493	3.165	3.133	3.203	3.637
AXP	2.224	2.273	2.583	2.447	2.430
BA	4.399	4.450	4.416	4.569	4.573
CAT	2.497	2.564	2.949	2.899	2.667
CRM	3.028	3.082	3.443	3.220	3.241
CSCO	2.237	1.994	2.006	1.993	2.008
CVX	2.283	2.362	2.423	2.399	2.389
DIS	2.101	2.152	2.176	2.162	2.173
GS	2.290	2.367	2.609	2.599	2.509
HD	1.881	1.950	2.096	2.112	1.994
HON	1.789	1.660	2.760	2.311	1.839
IBM	1.435	1.482	1.635	1.520	1.536
JNJ	1.180	1.222	1.306	1.244	1.234
JPM	1.918	1.938	1.985	1.947	1.970
KO	1.075	1.099	1.118	1.093	1.122
MCD	1.332	1.405	1.516	1.434	1.449
MMM	2.062	2.203	2.423	2.413	2.335
MRK	1.471	1.522	1.714	1.577	1.555
MSFT	2.644	2.312	2.323	2.318	2.339
NKE	2.557	2.611	2.784	2.704	2.676
NVDA	8.389	7.085	7.115	7.092	7.246
PG	1.185	1.240	1.399	1.285	1.265
SHW	2.311	2.875	12.155	4.656	3.511
TRV	1.718	1.859	3.575	2.503	2.021
UNH	2.061	2.185	2.549	2.332	2.400
V	1.693	1.744	1.807	1.772	1.799
VZ	1.275	1.303	1.332	1.303	1.322
WMT	1.271	1.298	1.334	1.303	1.327

The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.6: Average daily ReMeDI estimates ($\cdot 10^8$), 1-sec

	N	T	K	P	Z
AAPL	12.628	0.075	0.200	0.175	0.129
AMGN	75.519	0.803	34.583	15.699	11.356
AMZN	1243.626	0.322	3.352	6.534	12.318
AXP	1.409	0.691	11.284	4.871	2.480
BA	0.679	1.773	2.877	8.005	5.708
CAT	0.813	1.387	17.768	14.103	2.338
CRM	1.422	1.021	17.971	5.836	3.804
CSCO	8.027	0.029	0.551	0.153	0.078
CVX	0.390	0.439	2.220	2.001	0.598
DIS	0.253	0.193	1.146	0.817	0.309
GS	1.800	1.746	22.550	13.765	4.111
HD	0.992	1.240	10.896	6.926	2.424
HON	4.441	0.950	44.217	21.877	3.716
IBM	0.384	0.613	5.995	2.740	1.112
JNJ	0.189	0.554	3.933	1.336	0.581
JPM	0.070	0.137	1.476	0.747	0.184
KO	0.038	0.057	0.780	0.226	0.099
MCD	0.303	1.314	9.442	4.254	2.350
MMM	1.296	1.659	12.362	13.515	4.254
MRK	0.208	0.239	5.693	1.861	0.491
MSFT	10.088	0.030	0.409	0.238	0.180
NKE	0.128	0.831	9.590	3.405	1.466
NVDA	62.003	0.178	2.028	3.182	2.257
PG	0.345	0.398	6.934	2.252	0.876
SHW	5.772	13.986	436.676	98.046	29.811
TRV	1.563	2.502	70.272	27.586	5.955
UNH	1.395	2.162	19.601	9.016	5.969
V	0.475	0.737	5.194	2.751	2.769
VZ	0.047	0.072	0.878	0.284	0.069
WMT	0.035	0.194	3.328	1.368	0.793

The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.7: Average daily jump proportion, 1-min

	N	T	K	P	Z
AAPL	0.150	0.050	0.054	0.051	0.036
AMGN	0.293	0.016	0.193	0.260	0.280
AMZN	0.454	0.076	0.046	0.038	0.031
AXP	0.071	0.045	0.170	0.176	0.158
BA	0.044	0.081	0.045	0.163	0.154
CAT	0.044	0.038	0.115	0.264	0.150
CRM	0.050	0.032	0.207	0.112	0.191
CSCO	0.142	0.089	0.118	0.098	0.095
CVX	0.039	0.067	0.080	0.090	0.110
DIS	0.044	0.052	0.093	0.079	0.105
GS	0.047	0.028	0.171	0.245	0.183
HD	0.035	0.067	0.102	0.209	0.154
HON	0.167	0.030	0.250	0.423	0.171
IBM	0.050	0.045	0.174	0.127	0.161
JNJ	0.047	0.079	0.133	0.122	0.140
JPM	0.054	0.063	0.077	0.070	0.092
KO	0.111	0.116	0.142	0.123	0.166
MCD	0.028	0.078	0.164	0.168	0.211
MMM	0.037	0.050	0.154	0.242	0.225
MRK	0.058	0.064	0.135	0.108	0.152
MSFT	0.141	0.055	0.055	0.055	0.040
NKE	0.043	0.056	0.117	0.099	0.145
NVDA	0.184	0.056	0.043	0.052	0.041
PG	0.052	0.061	0.144	0.110	0.158
SHW	0.049	0.045	0.364	0.449	0.524
TRV	0.049	0.019	0.272	0.463	0.263
UNH	0.047	0.049	0.182	0.197	0.241
V	0.050	0.059	0.130	0.106	0.198
VZ	0.104	0.106	0.133	0.112	0.155
WMT	0.058	0.065	0.103	0.099	0.164

The daily jump proportion is computed as $\max\{RV_t - RBPV_t, 0\}/RV_t$ to measure the proportion of the quadratic variation attributable to jumps. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.8: Average daily effective spread

	N	T	K	P	Z
AAPL	0.062	0.012	0.017	0.016	0.021
AMGN	0.239	0.083	0.184	0.148	0.269
AMZN	9.707	0.555	1.197	1.568	1.170
AXP	0.037	0.032	0.059	0.043	0.082
BA	0.070	0.074	0.138	0.097	0.103
CAT	0.056	0.059	0.130	0.073	0.136
CRM	0.050	0.045	0.078	0.069	0.122
CSCO	0.008	0.006	0.005	0.007	0.007
CVX	0.017	0.016	0.025	0.020	0.025
DIS	0.017	0.014	0.024	0.018	0.028
GS	0.099	0.100	0.216	0.131	0.292
HD	0.067	0.069	0.134	0.085	0.154
HON	0.079	0.038	0.117	0.056	0.134
IBM	0.023	0.022	0.038	0.030	0.056
JNJ	0.018	0.018	0.029	0.021	0.040
JPM	0.013	0.011	0.017	0.014	0.020
KO	0.006	0.006	0.005	0.005	0.006
MCD	0.042	0.050	0.081	0.059	0.114
MMM	0.038	0.039	0.089	0.050	0.089
MRK	0.010	0.010	0.014	0.012	0.019
MSFT	0.072	0.021	0.035	0.036	0.043
NKE	0.015	0.015	0.025	0.020	0.034
NVDA	0.310	0.088	0.153	0.135	0.156
PG	0.014	0.013	0.023	0.019	0.036
SHW	0.229	0.258	0.752	0.423	1.390
TRV	0.050	0.053	0.132	0.067	0.187
UNH	0.118	0.118	0.226	0.163	0.312
V	0.029	0.031	0.055	0.056	0.076
VZ	0.006	0.006	0.005	0.006	0.006
WMT	0.013	0.013	0.021	0.020	0.027

The effective spread is defined as $ES_t = 2D_t \times [PRICE_t - (BID_t + OFR_t)/2]$. The variable D_t is the sign of the trade, indicating whether the buyer (seller) is the initiator of the trade $D_t = 1$ ($D_t = -1$). The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.9: Average daily depth imbalance difference

	N	T	K	P	Z
AAPL	−0.048	−0.062	−0.068	−0.087	−0.020
AMGN	−0.053	−0.023	−0.028	0.005	−0.025
AMZN	−0.012	−0.016	−0.013	−0.012	0.040
AXP	−0.048	−0.055	−0.039	−0.026	−0.015
BA	0.004	−0.016	−0.003	0.019	0.064
CAT	−0.029	−0.029	−0.028	−0.009	−0.000
CRM	−0.032	−0.023	−0.010	−0.007	0.007
CSCO	−0.211	−0.278	−0.238	−0.251	−0.174
CVX	−0.060	−0.056	−0.040	−0.036	−0.003
DIS	−0.070	−0.059	−0.036	−0.036	0.009
GS	−0.009	−0.014	−0.002	0.006	0.015
HD	−0.024	−0.019	−0.037	−0.008	−0.014
HON	−0.053	−0.029	−0.014	−0.003	−0.025
IBM	−0.040	−0.027	−0.019	−0.008	−0.000
JNJ	−0.059	−0.040	−0.045	−0.025	−0.023
JPM	−0.107	−0.094	−0.056	−0.061	−0.023
KO	−0.292	−0.285	−0.215	−0.214	−0.156
MCD	−0.030	−0.030	−0.027	−0.009	−0.002
MMM	−0.027	−0.029	−0.024	−0.011	0.005
MRK	−0.158	−0.131	−0.082	−0.092	−0.060
MSFT	−0.017	−0.013	−0.004	−0.002	0.038
NKE	−0.104	−0.080	−0.077	−0.062	−0.049
NVDA	−0.012	0.032	0.027	0.021	0.084
PG	−0.116	−0.088	−0.046	−0.063	−0.030
SHW	−0.025	−0.010	−0.004	0.005	−0.027
TRV	−0.034	−0.030	−0.012	−0.010	−0.022
UNH	−0.017	−0.009	0.010	0.000	0.020
V	−0.047	−0.033	−0.026	−0.019	−0.014
VZ	−0.268	−0.268	−0.216	−0.222	−0.159
WMT	−0.090	−0.084	−0.058	−0.060	−0.017

The depth imbalance difference is defined as $DId_t = D_t \times (OFRSIZ_t - BIDSIZ_t)/(OFRSIZ_t + BIDSIZ_t)$. The variable D_t is the sign of the trade, indicating whether the buyer (seller) is the initiator of the trade $D_t = 1$ ($D_t = -1$). The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.10: Average daily price impact

	N	T	K	P	Z
AAPL	0.030	0.006	0.008	0.008	0.010
AMGN	0.148	0.034	0.077	0.066	0.108
AMZN	4.167	0.251	0.532	0.720	0.487
AXP	0.018	0.015	0.030	0.020	0.039
BA	0.032	0.035	0.069	0.048	0.048
CAT	0.027	0.028	0.067	0.036	0.061
CRM	0.024	0.021	0.039	0.034	0.055
CSCO	0.004	0.004	0.003	0.004	0.004
CVX	0.008	0.008	0.013	0.009	0.011
DIS	0.009	0.007	0.013	0.009	0.013
GS	0.047	0.045	0.103	0.061	0.122
HD	0.032	0.032	0.065	0.038	0.069
HON	0.031	0.016	0.056	0.026	0.055
IBM	0.012	0.011	0.019	0.013	0.026
JNJ	0.009	0.009	0.014	0.010	0.017
JPM	0.007	0.006	0.009	0.007	0.011
KO	0.003	0.004	0.003	0.003	0.003
MCD	0.020	0.022	0.038	0.027	0.050
MMM	0.017	0.017	0.044	0.022	0.041
MRK	0.006	0.006	0.007	0.006	0.009
MSFT	0.032	0.008	0.015	0.016	0.018
NKE	0.008	0.008	0.012	0.010	0.016
NVDA	0.151	0.040	0.072	0.064	0.073
PG	0.007	0.007	0.011	0.009	0.015
SHW	0.102	0.120	0.380	0.228	0.675
TRV	0.022	0.023	0.069	0.033	0.108
UNH	0.055	0.056	0.107	0.073	0.135
V	0.014	0.014	0.026	0.026	0.034
VZ	0.003	0.004	0.003	0.004	0.004
WMT	0.007	0.007	0.010	0.009	0.013

The price impact is defined as $PI_t = (ES_t - RS_t)/2$. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table E.11: Average daily durations (in seconds)

	N	T	K	P	Z
AAPL	0.129	0.035	0.051	0.049	0.038
AMGN	2.702	0.630	1.145	1.275	0.729
AMZN	4.736	0.356	0.338	0.408	0.260
AXP	0.382	0.416	0.703	0.759	0.390
BA	0.325	0.606	0.527	0.838	0.404
CAT	0.340	0.636	0.711	0.923	0.479
CRM	0.251	0.378	0.546	0.610	0.337
CSCO	0.174	0.084	0.183	0.112	0.161
CVX	0.160	0.209	0.310	0.367	0.222
DIS	0.187	0.220	0.364	0.412	0.238
GS	0.390	0.905	0.983	1.330	0.513
HD	0.309	0.609	0.650	1.025	0.405
HON	1.188	0.637	1.155	1.030	0.643
IBM	0.338	0.495	0.696	0.799	0.469
JNJ	0.208	0.336	0.451	0.500	0.309
JPM	0.077	0.121	0.143	0.175	0.108
KO	0.103	0.153	0.287	0.189	0.234
MCD	0.361	0.684	0.858	0.983	0.520
MMM	0.419	0.747	1.010	1.170	0.605
MRK	0.133	0.223	0.309	0.330	0.235
MSFT	0.168	0.043	0.074	0.080	0.059
NKE	0.188	0.253	0.418	0.422	0.268
NVDA	0.954	0.156	0.201	0.178	0.143
PG	0.163	0.226	0.389	0.395	0.243
SHW	0.863	2.334	2.806	3.429	1.702
TRV	0.638	1.359	2.312	2.080	0.992
UNH	0.392	0.876	0.886	1.445	0.570
V	0.187	0.248	0.367	0.449	0.239
VZ	0.120	0.144	0.249	0.184	0.205
WMT	0.208	0.237	0.370	0.367	0.265

This table reports the average daily time between ticks in seconds. The trading venues are denoted with their ticker symbol: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

F Supplementary empirical results

F.1 Plots, component shares and residual correlation

Figure F.1: The upper panel displays the pre-averaged log-prices for IBM on 20th February 2020 at a tick-by-tick frequency in the five main trading venues in the U.S.: NYSE ("N") (red), Nasdaq ("T") (light green), Arca ("P") (orange), Cboe EDGX ("K") (blue), Cboe BZX ("Z") (magenta). The middle panel displays the observed log-prices for all five exchanges and the estimated efficient price based on the Granger representation decomposition ($\widehat{P}^*$) drawn in black. The lower panel displays the pre-averaged log-prices for the same exchanges (dashed lines) alongside both $\widehat{P}^*$ (solid black line) and pre-averaged $\widehat{P}^*$ (dashed black line).

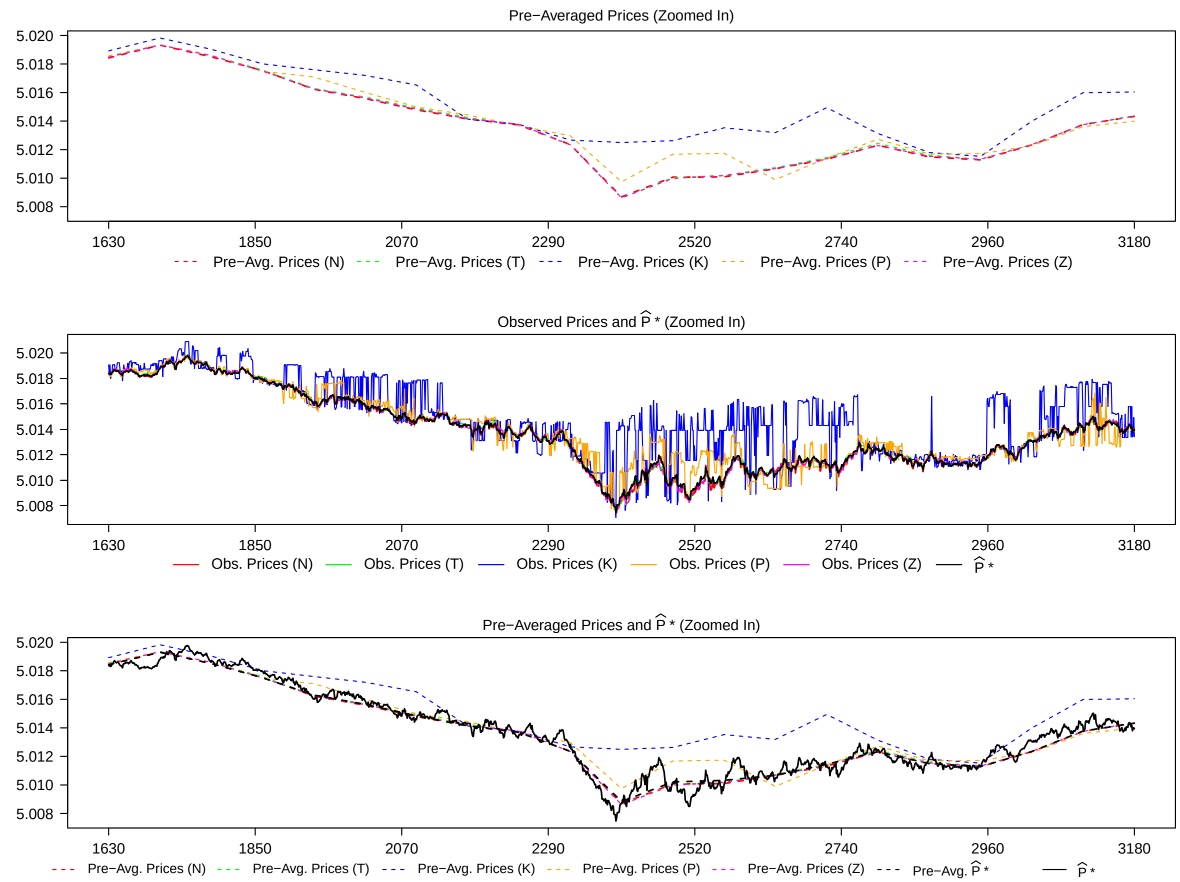
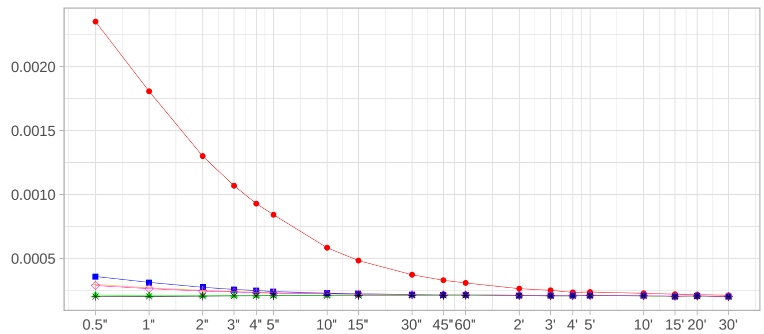
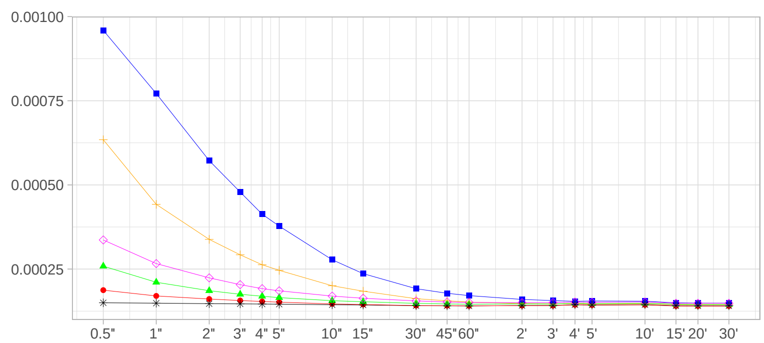


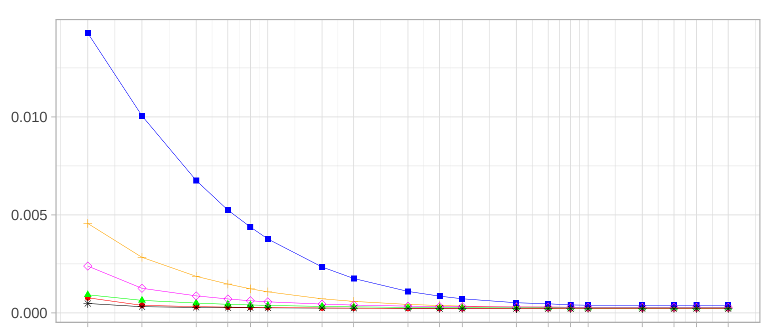
Figure F.2: This figure shows the volatility signature plots for stocks of relatively high (MSFT), medium (IBM) and low (SHW) trading activity computed using the univariate realized variance (RV) and the GRT-based RV estimator. The five trading venues are indicated by different colors and shapes: NYSE ("N") (red, circle), Nasdaq ("T") (light green, triangle), Arca ("P") (orange, plus), Cboe EDGX ("K") (blue, square), Cboe BZX ("Z") (magenta, diamond). The volatility signature plot for the GRT-based RV estimator (RV_{GRT}) is drawn in black and indicated with a star. The tickers MSFT, IBM, and SHW correspond to the companies Microsoft, IBM, and Sherwin-Williams, respectively.



MSFT

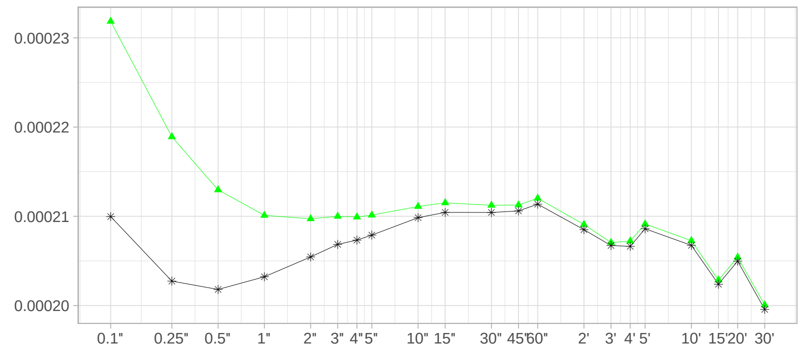


IBM

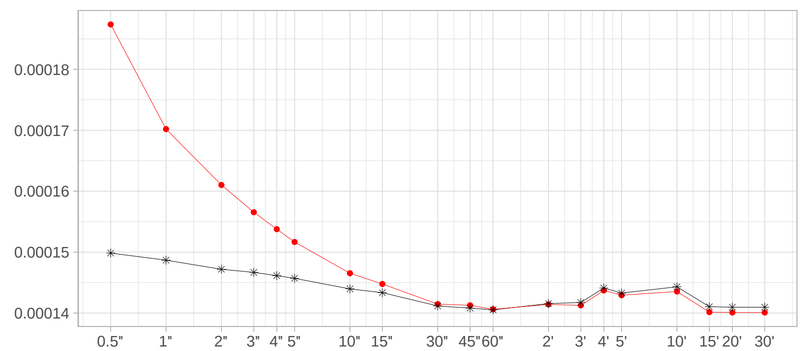


SHW

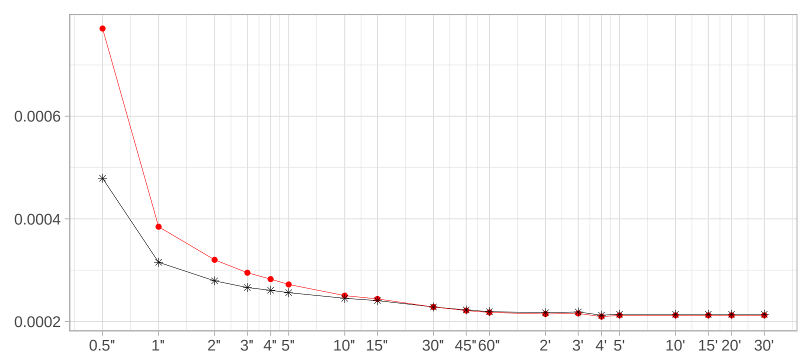
Figure F.3: This figure shows the volatility signature plots for stocks of relatively high (MSFT), medium (IBM) and low (SHW) trading activity. The realized variance estimator is computed using either data from the primary listing exchange (Nasdaq ("T") (light green, triangle) for MSFT or NYSE ("N") (red, circle) for IBM and SHW) or based on the defragemented returns (black, star). We include additional subsecond estimates to accommodate the high trading activity of MSFT. The tickers MSFT, IBM, and SHW correspond to the companies Microsoft, IBM, and Sherwin-Williams, respectively.



MSFT



IBM



SHW

Table F.1: Component shares and residual correlation: 1-min frequency, OLS estimator

	N	T	K	P	Z	$\bar{\rho}$
AAPL	0.034	0.057	0.319	0.204	0.386	0.988
	(0.023)	(0.019)	(0.013)	(0.013)	(0.035)	
AMGN	0.036	0.656	0.071	0.13	0.106	0.665
	(0.005)	(0.002)	(0.003)	(0.001)	(0.004)	
AMZN	0.004	0.610	0.127	0.174	0.085	0.917
	(0.050)	(0.036)	(0.011)	(0.003)	(0.021)	
AXP	0.481	0.272	0.030	0.085	0.132	0.899
	(0.003)	(0.003)	(0.002)	(0.002)	(0.004)	
BA	0.533	0.178	0.089	0.062	0.138	0.961
	(0.005)	(0.005)	(0.003)	(0.003)	(0.005)	
CAT	0.630	0.202	0.012	0.033	0.123	0.873
	(0.004)	(0.004)	(0.002)	(0.002)	(0.005)	
CRM	0.550	0.253	0.017	0.098	0.082	0.917
	(0.004)	(0.004)	(0.002)	(0.003)	(0.005)	
CSCO	0.024	0.309	0.254	0.198	0.214	0.964
	(0.004)	(0.003)	(0.003)	(0.002)	(0.011)	
CVX	0.793	0.103	−0.063	0.056	0.110	0.969
	(0.007)	(0.007)	(0.004)	(0.005)	(0.007)	
DIS	0.650	0.247	−0.028	0.079	0.052	0.980
	(0.006)	(0.006)	(0.005)	(0.005)	(0.008)	
GS	0.527	0.254	0.035	0.067	0.117	0.833
	(0.003)	(0.003)	(0.002)	(0.002)	(0.004)	
HD	0.489	0.259	0.045	0.069	0.138	0.877
	(0.003)	(0.003)	(0.002)	(0.002)	(0.004)	
HON	0.211	0.503	0.037	0.048	0.201	0.73
	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	
IBM	0.581	0.208	0.058	0.040	0.113	0.922
	(0.004)	(0.004)	(0.002)	(0.003)	(0.005)	
JNJ	0.688	0.095	0.054	0.056	0.107	0.926
	(0.005)	(0.004)	(0.002)	(0.003)	(0.005)	
JPM	0.526	0.191	0.055	0.173	0.055	0.982
	(0.006)	(0.005)	(0.004)	(0.004)	(0.008)	
KO	0.615	0.268	0.083	−0.101	0.135	0.978
	(0.007)	(0.006)	(0.005)	(0.006)	(0.007)	
MCD	0.601	0.151	−0.012	0.107	0.153	0.862
	(0.004)	(0.003)	(0.002)	(0.002)	(0.004)	
MMM	0.578	0.214	0.050	0.068	0.090	0.862
	(0.004)	(0.004)	(0.002)	(0.002)	(0.005)	
MRK	0.754	0.030	0.010	0.072	0.134	0.940
	(0.006)	(0.006)	(0.003)	(0.004)	(0.006)	
MSFT	0.063	0.181	0.230	0.417	0.110	0.953
	(0.011)	(0.002)	(0.007)	(0.005)	(0.015)	
NKE	0.747	0.088	0.039	0.068	0.058	0.953
	(0.009)	(0.008)	(0.004)	(0.006)	(0.009)	
NVDA	0.021	0.613	0.235	0.095	0.037	0.990
	(0.031)	(0.020)	(0.015)	(0.009)	(0.025)	
PG	0.621	0.077	0.058	0.023	0.220	0.906
	(0.005)	(0.005)	(0.002)	(0.003)	(0.005)	
SHW	0.674	0.169	0.015	0.044	0.098	0.691
	(0.003)	(0.003)	(0.002)	(0.002)	(0.004)	
TRV	0.596	0.209	0.014	0.045	0.136	0.683
	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	
UNH	0.578	0.206	0.048	0.081	0.087	0.829
	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)	
V	0.546	0.201	0.104	0.096	0.053	0.927
	(0.003)	(0.003)	(0.002)	(0.002)	(0.004)	
VZ	1.046	−0.106	0.070	−0.013	0.003	0.977
	(0.007)	(0.009)	(0.006)	(0.009)	(0.007)	
WMT	0.644	0.012	0.093	0.131	0.120	0.987
	(0.012)	(0.010)	(0.005)	(0.008)	(0.011)	

This tables reports the component share of the cross-listed DJIA stocks at the 1-min sampling frequency. The standard errors of the component shares are given in parentheses (computed with the delta-method). The average residual correlation across all markets is denoted with $\bar{\rho}$. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.2: Component shares and residual correlation: 1-sec frequency, OLS estimator

	N	T	K	P	Z	$\bar{\rho}$
AAPL	0.010	0.445	0.112	0.159	0.273	0.880
	(0.003)	(0.001)	(0.001)	(0.000)	(0.003)	
AMGN	0.015	0.728	0.041	0.106	0.110	0.350
	(0.002)	(0.001)	(0.001)	(0.000)	(0.002)	
AMZN	0.002	0.621	0.180	0.132	0.064	0.718
	(0.004)	(0.002)	(0.001)	(0.001)	(0.003)	
AXP	0.296	0.391	0.040	0.094	0.180	0.566
	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	
BA	0.455	0.206	0.123	0.095	0.121	0.630
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
CAT	0.471	0.247	0.033	0.046	0.203	0.510
	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)	
CRM	0.349	0.353	0.037	0.114	0.147	0.545
	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	
CSCO	0.014	0.334	0.132	0.216	0.303	0.700
	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)	
CVX	0.362	0.250	0.059	0.083	0.246	0.720
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
DIS	0.273	0.311	0.062	0.110	0.244	0.791
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
GS	0.380	0.341	0.036	0.058	0.185	0.447
	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	
HD	0.380	0.302	0.048	0.075	0.195	0.462
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
HON	0.160	0.539	0.027	0.043	0.231	0.359
	(0.000)	(0.001)	(0.001)	(0.000)	(0.001)	
IBM	0.443	0.258	0.043	0.075	0.181	0.598
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
JNJ	0.461	0.198	0.036	0.094	0.211	0.584
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
JPM	0.339	0.315	0.033	0.077	0.235	0.781
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
KO	0.361	0.225	0.062	0.124	0.228	0.796
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
MCD	0.534	0.200	0.036	0.086	0.144	0.471
	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	
MMM	0.445	0.294	0.053	0.058	0.15	0.474
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	
MRK	0.399	0.285	0.016	0.066	0.234	0.639
	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)	
MSFT	0.011	0.650	0.077	0.134	0.127	0.692
	(0.003)	(0.001)	(0.001)	(0.000)	(0.002)	
NKE	0.522	0.213	0.030	0.082	0.153	0.670
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
NVDA	0.012	0.628	0.104	0.129	0.128	0.901
	(0.002)	(0.001)	(0.001)	(0.000)	(0.002)	
PG	0.391	0.278	0.035	0.085	0.211	0.558
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
SHW	0.579	0.265	0.017	0.043	0.095	0.471
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
TRV	0.508	0.272	0.018	0.044	0.159	0.395
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
UNH	0.437	0.291	0.045	0.096	0.131	0.429
	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	
V	0.453	0.295	0.051	0.102	0.100	0.550
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
VZ	0.432	0.184	0.070	0.112	0.203	0.800
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	
WMT	0.481	0.264	0.036	0.093	0.126	0.868
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	

This tables reports the component share of the cross-listed DJIA stocks at the 1-sec sampling frequency. The standard errors of the component shares are given in parentheses (computed with the delta-method). The average residual correlation across all markets is denoted with $\bar{\rho}$. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

F.2 Additional results – mixed frequencies 1-min/5-min RV vs 1-sec noise-robust estimators (QLIKE)

Table F.3: Equal accuracy: ΔL (QLIKE), mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}
AAPL	0.032	0.011	-0.016	-0.024	0.048	0.09	0.035	0.016	0.007	0.009	0.032	0.121	0.268
AMGN	0.057	0.067	0.02	-0.002	0.184	0.372	0.104	0.102	0.08	0.258	0.117	0.561	1.288
AMZN	0.036	0.016	0	-0.008	0.137	0.687	0.428	0.769	0.282	0.381	0.919	0.629	0.805
AXP	0.042	0.034	0.006	-0.008	0.118	0.262	0.042	-0.003	0.032	0.01	-0.008	0.144	0.277
BA	0.032	0.021	0.007	-0.012	0.104	0.21	0.03	-0.005	0.019	0.007	-0.017	0.112	0.232
CAT	0.042	0.026	0.009	-0.006	0.107	0.187	0.042	-0.002	0.026	0.01	-0.007	0.117	0.21
CRM	0.041	0.024	0.003	-0.01	0.111	0.217	0.041	0	0.024	0.005	-0.008	0.136	0.232
CSCO	0.049	0.03	0	-0.015	0.088	0.148	0.057	0.001	0.028	0.004	-0.007	0.217	0.397
CVX	0.031	0.016	-0.002	-0.009	0.07	0.133	0.032	0.001	0.017	-0.001	-0.009	0.075	0.142
DIS	0.046	0.029	0.004	-0.007	0.095	0.176	0.047	0.001	0.029	0.005	-0.007	0.107	0.2
GS	0.031	0.023	0.007	-0.008	0.103	0.201	0.028	-0.003	0.022	0.008	-0.009	0.116	0.253
HD	0.037	0.027	0.005	-0.008	0.111	0.195	0.036	-0.001	0.028	0.007	-0.008	0.128	0.232
HON	0.048	0.061	0.026	-0.007	0.155	0.265	0.057	0.028	0.063	0.061	0.034	0.234	0.475
IBM	0.036	0.03	0.005	-0.01	0.109	0.207	0.035	0	0.031	0.007	-0.008	0.117	0.22
JNJ	0.05	0.037	0.007	-0.01	0.101	0.189	0.05	0	0.037	0.007	-0.008	0.113	0.211
JPM	0.032	0.014	-0.001	-0.011	0.066	0.118	0.036	0.001	0.015	0	-0.011	0.075	0.133
KO	0.035	0.021	0	-0.009	0.07	0.133	0.039	0	0.022	0.002	-0.007	0.081	0.155
MCD	0.04	0.029	0.011	-0.001	0.122	0.236	0.037	0.001	0.03	0.012	0.001	0.126	0.248
MMM	0.042	0.035	0.006	-0.006	0.107	0.176	0.044	0.003	0.035	0.009	-0.002	0.117	0.193
MRK	0.045	0.028	0	-0.009	0.097	0.179	0.049	0.003	0.029	0.004	-0.006	0.113	0.204
MSFT	0.031	0.01	-0.009	-0.013	0.041	0.073	0.026	0	0.005	0.015	0.01	0.095	0.184
NKE	0.036	0.022	0.003	-0.004	0.076	0.139	0.038	0	0.022	0.004	-0.004	0.08	0.146
NVDA	0.031	0.004	-0.011	-0.017	0.047	0.107	0.047	0.036	0.005	0.035	0.063	0.129	0.527
PG	0.048	0.026	0.004	-0.006	0.099	0.186	0.049	0.004	0.027	0.005	-0.003	0.121	0.225
SHW	0.072	0.126	0.063	0.001	0.412	0.837	0.091	0.021	0.139	0.148	0.02	0.497	2.385
TRV	0.05	0.058	0.029	-0.001	0.194	0.358	0.048	0	0.059	0.032	-0.001	0.213	0.399
UNH	0.048	0.045	0.016	-0.005	0.183	0.344	0.046	-0.001	0.043	0.017	-0.005	0.213	0.393
V	0.044	0.027	0	-0.011	0.103	0.19	0.045	0	0.027	0.002	-0.009	0.117	0.21
VZ	0.042	0.017	-0.006	-0.011	0.086	0.163	0.045	0	0.017	-0.003	-0.009	0.099	0.19
WMT	0.039	0.02	0.001	-0.007	0.093	0.174	0.041	0.003	0.021	0.005	-0.004	0.1	0.188

	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}
AAPL	0.032	0.001	0.011	-0.016	-0.023	0.049	0.092	0.031	0	0.012	-0.015	-0.025	0.049	0.092
AMGN	0.057	0	0.067	0.02	-0.002	0.191	0.433	0.042	0.116	0.056	0.114	0.13	0.223	0.56
AMZN	0.032	-0.004	0.011	-0.003	-0.012	0.122	0.264	0.036	-0.004	0.026	0.017	-0.011	0.119	0.246
AXP	0.039	-0.001	0.034	0.007	-0.008	0.121	0.237	0.036	0.046	0.027	0.107	0.061	0.155	0.304
BA	0.026	-0.01	0.017	0.003	-0.02	0.122	0.282	0.032	-0.004	0.025	0.021	-0.014	0.117	0.288
CAT	0.034	-0.006	0.024	0.006	-0.011	0.116	0.214	0.031	0.031	0.028	0.088	0.051	0.155	0.339
CRM	0.037	-0.003	0.023	0.002	-0.012	0.122	0.92	0.045	0.096	0.023	0.148	0.145	0.156	0.308
CSCO	0.056	-0.004	0.032	-0.002	-0.015	0.093	0.155	0.05	-0.003	0.03	0.004	-0.016	0.091	0.152
CVX	0.03	-0.001	0.016	-0.001	-0.011	0.075	0.146	0.03	0.003	0.017	0.012	-0.005	0.075	0.148
DIS	0.043	-0.003	0.029	0.003	-0.011	0.097	0.181	0.041	0.001	0.03	0.167	-0.003	0.094	0.181
GS	0.027	-0.003	0.02	0.008	-0.007	0.116	0.233	0.02	0.095	0.022	0.118	0.141	0.177	0.331
HD	0.028	-0.002	0.024	0.004	-0.006	0.114	0.202	0.034	0.038	0.031	0.099	0.063	0.159	0.286
HON	0.044	0	0.062	0.046	-0.007	0.156	0.281	0.145	0.359	0.083	0.211	0.372	0.241	0.536
IBM	0.032	-0.003	0.029	0.007	-0.013	0.115	0.219	0.024	0.029	0.027	0.363	0.045	0.136	0.275
JNJ	0.043	-0.004	0.036	0.011	-0.016	0.109	0.217	0.044	0.006	0.036	0.022	0.014	0.14	0.291
JPM	0.031	-0.001	0.014	-0.002	-0.012	0.064	0.117	0.033	0	0.015	-0.002	-0.008	0.076	0.138
KO	0.034	0	0.021	0	-0.01	0.072	0.139	0.031	-0.003	0.02	0.001	-0.009	0.075	0.151
MCD	0.04	0.004	0.023	0.029	0.005	0.148	0.412	0.028	0.045	0.027	0.08	0.075	0.192	0.378
MMM	0.036	0	0.032	0.006	-0.003	0.109	0.189	0.028	0.038	0.029	0.075	0.069	0.146	0.251
MRK	0.04	-0.003	0.028	-0.002	-0.012	0.099	0.188	0.041	0.012	0.025	0.028	0.021	0.137	0.261
MSFT	0.031	0	0.01	-0.009	-0.013	0.042	0.074	0.03	-0.001	0.01	-0.009	-0.014	0.041	0.075
NKE	0.032	-0.003	0.021	0.005	-0.006	0.089	0.166	0.028	0.009	0.022	0.015	0.027	0.108	0.199
NVDA	0.032	-0.002	0.004	-0.012	-0.02	0.049	0.112	0.031	-0.002	0.007	-0.009	-0.017	0.046	0.101
PG	0.047	0.001	0.026	0.004	-0.005	0.11	0.204	0.043	0.019	0.025	0.036	0.038	0.152	0.304
SHW	0.049	0.071	0.086	0.462	0.034	0.475	0.93	0.528	1	0.297	0.428	0.886	1.867	1.386
TRV	0.045	0.015	0.054	0.425	0.008	0.217	0.449	0.149	0.482	0.048	0.385	0.462	0.42	1.061
UNH	0.042	0.004	0.041	0.028	-0.001	0.211	0.415	0.043	0.086	0.039	0.177	0.103	0.29	0.635
V	0.036	-0.003	0.025	-0.002	-0.011	0.106	0.217	0.036	0.007	0.028	0.015	0.038	0.142	0.258
VZ	0.04	-0.001	0.017	-0.007	-0.014	0.089	0.171	0.038	-0.003	0.016	-0.005	-0.012	0.091	0.174
WMT	0.03	-0.007	0.02	-0.004	-0.015	0.096	0.189	0.039	-0.009	0.018	0.637	-0.004	0.134	0.265

	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	0.032	0	0.011	-0.016	-0.025	0.05	0.094	0.031	-0.002	0.011	-0.018	-0.026	0.047	0.088
AMGN	0.042	0.087	0.032	0.061	0.083	0.257	0.526	0.08	0.07	0.053	0.087	0.081	0.255	0.702
AMZN	0.028	0.019	0.012	0.014	0.025	0.187	0.384	0.045	0.084	0.008	0.078	0.101	0.244	0.532
AXP	0.031	0.025	0.025	0.029	0.027	0.175	0.416	0.034	0.001	0.029	0.013	0.001	0.139	0.271
BA	0.013	0.011	0.004	0.029	0.021	0.156	1.726	0.019	-0.009	0.011	0.01	-0.006	0.149	1.289
CAT	0.037	0.083	0.014	0.053	0.122	0.194	0.412	0.037	0.002	0.022	0.013	0.001	0.127	0.232
CRM	0.032	0.018	0.016	0.037	0.025	0.198	0.298	0.031	0.009	0.016	0.014	0.018	0.136	0.271
CSCO	0.054	0	0.031	0	-0.015	0.099	0.164	0.047	-0.003	0.031	-0.001	-0.017	0.087	0.147
CVX	0.026	-0.001	0.015	0.011	-0.003	0.089	0.187	0.029	-0.001	0.016	-0.003	-0.008	0.067	0.133
DIS	0.041	-0.007	0.027	0.002	-0.013	0.11	0.215	0.043	-0.005	0.028	0	-0.012	0.1	0.198
GS	0.021	0.077	0.009	0.08	0.102	0.187	0.499	0.033	0.016	0.015	0.033	0.019	0.135	0.321
HD	0.029	0.046	0.016	0.054	0.072	0.18	0.352	0.033	0.001	0.023	0.009	0.002	0.143	0.265
HON	0.059	0.245	0.025	0.407	0.275	0.36	0.892	0.046	0.039	0.047	0.044	0.026	0.209	0.41
IBM	0.03	0	0.024	0.025	0.004	0.17	0.346	0.031	-0.003	0.026	0.006	-0.004	0.122	0.243
JNJ	0.039	0.002	0.033	0.019	0.004	0.137	0.285	0.046	0.001	0.035	0.014	-0.009	0.114	0.221
JPM	0.031	-0.002	0.014	-0.001	-0.011	0.082	0.147	0.029	-0.003	0.014	-0.003	-0.013	0.067	0.121
KO	0.035	0	0.021	0.001	-0.011	0.087	0.168	0.037	-0.001	0.021	0.002	-0.01	0.073	0.143
MCD	0.023	0.026	0.017	0.118	0.039	0.194	0.377	0.033	0.004	0.021	0.038	0.016	0.163	0.312
MMM	0.043	0.126	0.02	0.209	0.156	0.186	0.397	0.041	0.012	0.023	0.036	0.026	0.15	0.259
MRK	0.042	-0.001	0.025	0.017	-0.003	0.143	0.277	0.037	-0.003	0.026	-0.002	-0.009	0.098	0.193
MSFT	0.03	-0.001	0.009	-0.009	-0.015	0.047	0.084	0.03	-0.003	0.009	-0.01	-0.016	0.046	0.084
NKE	0.028	-0.011	0.018	0.009	-0.003	0.111	0.194	0.028	-0.006	0.018	-0.003	-0.012	0.092	0.174
NVDA	0.028	-0.004	0.005	-0.008	-0.019	0.061	0.13	0.027	-0.005	0.003	-0.012	-0.017	0.052	0.115
PG	0.046	0.004	0.024	0.02	0.008	0.152	0.306	0.041	-0.002	0.024	0.005	0	0.114	0.222
SHW	0.143	0.536	0.007	0.272	0.402	1.986	2.629	0.094	0.198	0.073	0.171	0.151	0.663	1.331
TRV	0.085	0.323	0.032	0.782	0.282	1.926	0.998	0.05	0.037	0.044	0.07	0.036	0.244	0.49
UNH	0.026	0.048	0.029	0.068	0.057	0.293	0.543	0.041	0.021	0.032	0.198	0.033	0.239	0.486
V	0.023	-0.005	0.013	-0.003	0.006	0.141	0.263	0.036	0.008	0.02	0.014	0.033	0.149	0.283
VZ	0.043	-0.002	0.016	-0.006	-0.014	0.111	0.208	0.037	-0.002	0.017	-0.009	-0.015	0.079	0.152
WMT	0.029	-0.013	0.012	0.706	-0.011	0.131	0.26	0.032	-0.006	0.017	0.004	-0.004	0.114	0.213

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.4: Tests of equal accuracy for DJIA stocks with mixed frequencies: 5-minute RV and RK estimators

Panel A: Tests of equal accuracy (QLIKE)

	$RV_{GRT}^{(78)}$	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{T}^{(78)}$	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMGN	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AXP	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
BA	△	▲	▲	△	▲	▲	△	▲	▲	△	▲	▲	△	▲	▲	△	▲	▲
CAT	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
CRM	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
CSCO	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
CVX	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
DIS	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
GS	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
HD	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
HON	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
IBM	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
JNJ	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
JPM	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
KO	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
MCD	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
MMM	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
MRK	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
MSFT	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
NKE	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
NVDA	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
PG	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
SHW	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
TRV	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
UNH	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
V	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
VZ	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
WMT	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△

Panel B: In-sample inclusion rates (MCS, QLIKE loss)

	$RV_{GRT}^{(78)}$	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{T}^{(78)}$	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	RK_{Z}^{MTH}	RK_{Z}^{P}
	6	7	6	11	5	3	3	3	2	8	4	4	4	4	2	1	3	1

Panel C: Δ% RMSE with respect to $RV_{GRT}^{(390)}$

$RV_{GRT}^{(78)}$	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	RK_{Z}^{MTH}	RK_{Z}^{P}
6.22%	4.81%	12.26%	8.41%	10.48%	15.72%	7.92%	7.19%	14.79%	22.49%	9.21%	18.83%	13.73%	12.12%	20.51%	12.97%	12.71%	22.32%

Panel A reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. Panel B reports the average inclusion rates in the SSM. Panel C reports the median percentage RMSE difference of all estimators with respect to the $RV_{GRT}^{(390)}$ benchmark. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z")..

Table F.5: Diebold-Mariano Tests: GRT weights vs equal/trading volume; QLIKE (1-min RV benchmark)

	DM_{equal}	p-val	$DM_{tradvol}$	p-val
AAPL	−2.08	0.04	−2.11	0.03
AMGN	−2.81	0.00	−3.12	0.00
AMZN	−4.04	0.00	−4.43	0.00
AXP	−2.85	0.00	−2.86	0.00
BA	−3.33	0.00	−3.22	0.00
CAT	−1.93	0.05	−1.93	0.05
CRM	−3.16	0.00	−3.05	0.00
CSCO	−1.79	0.07	−1.80	0.07
CVX	−2.50	0.01	−2.53	0.01
DIS	−4.36	0.00	−3.77	0.00
GS	−2.45	0.01	−2.45	0.01
HD	−3.03	0.00	−3.15	0.00
HON	−2.57	0.01	−2.83	0.00
IBM	−2.82	0.00	−2.81	0.00
JNJ	−2.99	0.00	−2.95	0.00
JPM	−2.05	0.04	−2.13	0.03
KO	−1.90	0.06	−1.73	0.08
MCD	−2.92	0.00	−2.99	0.00
MMM	−2.64	0.01	−2.80	0.01
MRK	−2.91	0.00	−2.90	0.00
MSFT	−2.57	0.01	−2.81	0.00
NKE	−2.48	0.01	−2.63	0.01
NVDA	−2.03	0.04	−3.14	0.00
PG	−1.55	0.12	−1.53	0.13
SHW	−6.19	0.00	−5.52	0.00
TRV	−3.07	0.00	−3.01	0.00
UNH	−2.70	0.01	−2.77	0.01
V	−2.36	0.02	−2.48	0.01
VZ	−1.84	0.07	−1.66	0.10
WMT	−2.52	0.01	−2.45	0.01

This table reports the Diebold-Mariano tests evaluating the prediction errors of RV_{GRT} versus the equal-weighted RV or the volume-weighted RV. The evaluation is based on the QLIKE loss function. A negative DM statistic implies that RV_{GRT} has smaller errors than the corresponding equal-weighted RV or the volume-weighted RV.

Table F.6: Forecast evaluation: inclusion in MCS (QLIKE), $h = 1, \ldots, 5$ (mixed frequencies)

	$RV_{GRT}^{(390)}$	$RV_{GRT}^{(78)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	$RBPV_{N}$	PRV_{N}	$PBPV_{N}$	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}
AAPL	5	5	5	5	5	5	5	0	0	5	5	0	5	5	5	5	5	5
AMGN	2	0	5	0	0	0	0	1	4	1	4	4	0	1	4	4	4	4
AMZN	0	0	5	0	1	2	2	5	5	2	0	0	5	5	0	0	5	5
AXP	3	4	5	4	3	2	4	1	0	1	2	5	4	3	2	5	0	0
BA	0	0	5	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
CAT	4	3	3	4	4	3	3	3	3	4	3	2	4	4	3	3	3	3
CRM	5	5	5	4	4	1	5	5	4	5	5	4	2	3	1	5	5	4
CSCO	3	3	5	0	4	2	4	0	0	4	5	0	0	5	5	5	0	2
CVX	5	4	5	5	5	5	5	4	4	5	5	5	5	5	5	5	4	4
DIS	4	5	5	5	5	5	5	5	4	5	5	5	5	5	5	5	5	4
GS	2	5	0	5	5	2	2	4	4	3	4	0	5	5	3	4	4	4
HD	5	5	5	3	5	5	5	4	3	5	5	5	4	5	5	5	4	3
HON	3	5	5	4	5	4	5	1	3	0	4	5	0	0	0	4	0	0
IBM	3	2	5	1	1	1	2	2	2	4	2	5	2	2	1	2	2	2
JNJ	2	2	5	1	1	2	2	0	0	2	2	5	1	1	2	2	0	0
JPM	4	5	4	5	5	5	1	4	4	4	5	3	5	5	5	5	4	4
KO	3	5	5	5	5	4	5	1	1	4	5	5	5	5	5	5	3	2
MCD	2	5	5	5	5	5	5	1	0	3	5	4	5	5	5	5	1	0
MMM	2	0	5	0	0	0	0	4	4	3	0	4	0	0	0	0	4	4
MRK	3	5	5	5	5	5	5	2	0	3	5	5	5	5	5	5	1	0
MSFT	5	5	5	5	5	4	5	5	5	5	5	0	5	5	5	5	5	3
NKE	2	0	5	5	5	0	2	0	0	1	0	5	5	5	0	2	0	0
NVDA	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5
PG	0	1	2	4	4	1	1	0	0	0	1	5	4	4	1	2	0	0
SHW	1	2	5	0	0	0	4	0	0	0	0	0	0	0	0	2	0	0
TRV	5	5	5	5	5	5	4	4	4	5	5	4	5	5	5	5	4	4
UNH	5	4	5	4	5	5	5	1	2	5	4	5	0	0	5	5	1	1
V	0	2	5	2	0	2	3	0	0	0	2	5	1	0	3	3	0	0
VZ	5	5	5	5	5	3	3	5	2	5	5	5	5	5	3	1	5	1
WMT	3	2	5	1	1	2	2	1	1	3	2	5	0	1	2	2	1	0
	0.607	0.660	0.927	0.647	0.687	0.567	0.660	0.453	0.427	0.613	0.667	0.700	0.613	0.660	0.600	0.707	0.500	0.427

	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}
AAPL	5	5	5	0	0	5	5	1	1	5	5	5	5	5	5	5	0	0
AMGN	2	0	5	0	0	0	0	1	4	3	3	4	0	0	0	4	4	4
AMZN	5	5	4	5	5	2	5	5	5	1	5	4	0	0	0	5	5	5
AXP	3	2	2	4	2	1	0	1	1	4	5	4	5	5	5	5	0	0
BA	0	0	5	0	0	0	0	0	0	5	5	5	4	3	0	5	0	0
CAT	4	2	2	4	4	2	3	3	2	5	3	2	5	5	3	3	3	2
CRM	5	5	4	4	4	3	5	4	4	5	4	3	5	5	5	4	5	4
CSCO	3	4	5	0	4	3	4	0	0	2	3	5	0	4	0	4	0	0
CVX	5	5	5	5	5	5	5	4	4	5	5	4	5	5	5	5	4	4
DIS	3	5	5	5	5	5	5	4	4	5	5	5	5	5	5	5	5	4
GS	0	3	0	5	5	2	0	1	4	0	4	0	5	5	5	5	4	4
HD	2	5	0	5	5	3	3	3	3	5	5	1	5	5	5	5	2	1
HON	3	5	5	4	5	5	5	2	3	5	4	3	5	5	1	4	4	4
IBM	3	3	5	1	1	1	3	2	2	5	5	3	4	5	5	5	3	3
JNJ	2	5	1	1	1	2	3	0	0	5	5	5	5	5	5	5	0	0
JPM	4	5	0	4	3	5	1	4	4	4	5	2	5	5	5	5	1	4
KO	4	5	5	5	5	1	5	0	0	5	5	1	5	5	5	5	3	1
MCD	0	1	1	5	5	4	3	0	0	5	5	5	5	5	5	5	2	0
MMM	3	0	5	0	0	0	0	4	4	4	5	5	0	0	1	5	4	4
MRK	3	5	4	5	5	4	3	1	0	4	4	2	5	5	5	4	0	0
MSFT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NKE	4	0	5	5	5	2	5	1	2	5	5	5	5	5	5	5	1	5
NVDA	5	5	5	5	5	5	0	5	5	5	5	5	5	5	5	5	5	5
PG	0	2	2	4	4	1	1	0	0	2	5	0	5	5	5	2	0	0
SHW	3	4	3	0	2	0	4	1	0	5	5	5	5	5	4	5	1	0
TRV	1	1	1	5	5	0	1	0	0	4	4	5	5	5	4	5	0	0
UNH	5	4	3	5	5	5	5	4	3	5	5	5	5	5	5	5	4	3
V	0	2	2	1	0	2	3	0	0	2	5	0	5	5	1	2	0	0
VZ	5	5	4	5	5	2	2	5	2	5	5	3	5	5	5	5	5	2
WMT	4	3	2	0	1	2	2	1	0	5	5	5	5	5	5	4	0	0
	0.607	0.673	0.667	0.647	0.673	0.513	0.573	0.413	0.413	0.833	0.927	0.707	0.853	0.880	0.760	0.907	0.467	0.427

	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	5	5	5	5	4	5	5	0	0	5	5	5	5	5	5	5	0	0
AMGN	2	0	4	0	0	0	0	3	4	3	0	4	0	0	3	4	3	4
AMZN	5	4	0	4	5	5	0	5	5	5	4	0	3	5	5	0	5	5
AXP	5	2	3	5	5	3	5	3	3	3	2	0	2	3	2	2	1	0
BA	4	5	1	0	0	0	5	0	1	5	3	2	3	3	0	2	0	0
CAT	2	2	0	4	4	2	2	3	2	2	0	0	2	2	0	0	2	2
CRM	5	5	0	5	5	4	4	4	4	5	4	4	5	5	1	5	4	4
CSCO	4	4	5	0	4	0	4	0	0	3	3	5	0	4	3	4	0	0
CVX	5	5	4	5	5	5	5	4	4	5	4	0	5	5	5	2	4	4
DIS	5	5	5	5	5	5	5	3	3	5	5	5	5	5	3	5	4	3
GS	0	0	0	3	3	4	0	4	4	2	0	0	4	4	0	0	3	4
HD	5	5	3	5	5	5	5	3	1	4	5	3	5	5	5	5	4	3
HON	3	2	3	5	5	3	2	0	0	4	5	5	0	5	4	5	0	1
IBM	3	5	4	2	3	1	5	3	3	1	0	0	2	2	0	0	2	1
JNJ	3	3	0	1	1	3	1	1	0	0	0	0	1	1	0	1	0	0
JPM	4	5	4	5	5	5	5	4	4	4	5	2	5	5	5	3	4	4
KO	4	5	5	5	5	5	5	3	2	1	5	4	5	5	4	5	1	1
MCD	0	2	4	5	5	5	5	0	0	0	3	4	5	5	3	1	0	0
MMM	4	5	5	2	3	4	5	4	4	2	3	5	0	0	1	1	4	4
MRK	3	5	4	5	5	4	5	1	0	1	0	0	5	5	4	1	0	0
MSFT	5	0	3	5	5	4	4	5	5	5	5	5	5	5	5	5	5	5
NKE	5	2	5	5	5	2	5	2	5	1	1	5	5	5	4	5	1	1
NVDA	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5
PG	0	0	0	5	5	0	0	0	0	0	2	0	5	5	1	0	0	0
SHW	3	1	2	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0
TRV	1	1	2	5	5	0	1	0	0	1	0	0	5	5	0	0	0	0
UNH	4	4	5	5	5	4	5	0	0	4	4	5	5	5	4	5	3	3
V	1	0	0	1	2	0	0	0	0	0	1	0	1	3	0	0	0	0
VZ	5	5	0	5	5	4	5	5	1	5	5	5	5	5	3	3	5	0
WMT	4	2	0	4	4	3	3	1	1	0	2	0	1	1	1	1	1	0
	0.693	0.627	0.540	0.740	0.787	0.613	0.673	0.453	0.407	0.540	0.540	0.480	0.660	0.753	0.507	0.500	0.407	0.360

This table reports the number of inclusions in the model confidence set. The HAR model is used to forecast the sparsely sampled (20-minute) RV obtained from the defragmented prices which acts as the unbiased volatility proxy for day $t + h$. The forecast error are evaluated with the QLIKE loss function. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.7: Forecast evaluation: average inclusion rate in MCS (QLIKE), $h = 1, \ldots, 5$ (mixed frequencies)

	Inclusion rates (MCS)								
	$RV^{(78)}$	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
					$h = 1$				
GRT	0.533	0.767	**0.967**	0.667	0.800	0.700	0.900	0.267	0.200
N	0.500	0.767	0.433	0.633	0.733	0.700	0.800	0.333	0.200
T	0.500	0.600	0.500	0.667	0.833	0.567	0.767	0.233	0.167
K	0.633	0.767	0.467	0.867	0.900	0.667	0.733	0.267	0.167
P	0.500	0.567	0.267	0.833	0.867	0.567	0.633	0.200	0.233
Z	0.367	0.433	0.400	0.767	0.900	0.433	0.500	0.200	0.133
					$h = 2$				
GRT	0.567	0.700	**0.967**	0.667	0.633	0.633	0.800	0.433	0.433
N	0.700	0.667	0.700	0.667	0.633	0.633	0.700	0.533	0.467
T	0.700	0.667	0.600	0.633	0.633	0.633	0.600	0.367	0.400
K	0.867	**0.967**	0.667	0.800	0.867	0.733	0.933	0.467	0.400
P	0.667	0.667	0.567	0.733	0.800	0.667	0.667	0.400	0.400
Z	0.567	0.533	0.500	0.633	0.700	0.467	0.500	0.400	0.367
					$h = 3$				
GRT	0.633	0.667	**0.967**	0.667	0.733	0.467	0.633	0.433	0.433
N	0.667	0.667	0.733	0.633	0.667	0.500	0.600	0.567	0.500
T	0.633	0.667	0.600	0.633	0.667	0.367	0.500	0.433	0.367
K	0.800	0.933	0.733	0.867	0.900	0.800	0.900	0.467	0.500
P	0.733	0.633	0.567	0.733	0.800	0.633	0.667	0.500	0.400
Z	0.567	0.533	0.500	0.733	0.733	0.533	0.467	0.400	0.433
					$h = 4$				
GRT	0.600	0.667	**0.967**	0.633	0.633	0.567	0.567	0.567	0.500
N	0.567	0.667	0.733	0.600	0.667	0.633	0.667	0.567	0.500
T	0.567	0.633	0.767	0.633	0.633	0.467	0.500	0.467	0.467
K	0.900	0.933	0.867	0.900	0.900	0.800	**0.967**	0.467	0.533
P	0.667	0.567	0.667	0.667	0.733	0.600	0.633	0.533	0.467
Z	0.600	0.567	0.533	0.567	0.667	0.500	0.433	0.467	0.467
					$h = 5$				
GRT	0.667	0.633	0.967	0.667	0.700	0.433	0.633	0.533	0.467
N	0.667	0.633	0.733	0.633	0.667	0.600	0.667	0.600	0.500
T	0.733	0.667	0.867	0.667	0.667	0.533	0.567	0.567	0.533
K	0.867	**1.000**	0.867	0.867	0.933	0.767	**1.000**	0.600	0.533
P	0.900	0.700	0.667	0.700	0.833	0.567	0.733	0.533	0.500
Z	0.700	0.567	0.533	0.633	0.733	0.467	0.533	0.533	0.467

This table reports the inclusions rates in the model confidence set. $RV^{(78)}$ and $RV^{(390)}$ denote the 5-minute and 1-minute RV estimator, respectively. The pre-averaging, bipower variation, and kernel-based estimators are computed using data sampled at 1-second intervals. The HAR model is used to forecast the sparsely sampled (20-minute) RV obtained from the defragmented prices which acts as the unbiased volatility proxy for day $t+h$. We generate forecasts for $h = 1, \ldots, 5$ and evaluate the forecast errors with the QLIKE loss function. The inclusion into the MCS is counted for h and we report the average inclusion rate over all 30 DJIA constituents. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.8: Forecast evaluation: inclusion in MCS (QLIKE), $h = 1, \ldots, 5$ (mixed frequencies), GRT only

	$RV_{GRT}^{(78)}$	$RV_{GRT}^{(390)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}
AAPL	4	5	5	4	4	5	5	0	0
AMGN	0	0	5	0	0	0	1	0	4
AMZN	5	5	5	5	5	2	2	5	5
AXP	4	4	5	5	5	4	5	4	2
BA	5	5	5	5	5	5	5	5	2
CAT	5	5	5	5	5	5	5	4	4
CRM	4	4	5	0	0	0	5	3	3
CSCO	0	0	5	0	0	0	5	0	0
CVX	5	5	5	5	5	5	5	4	4
DIS	4	5	5	5	5	5	5	4	4
GS	3	5	1	0	5	1	2	3	3
HD	2	5	5	1	5	5	5	5	3
HON	1	3	5	0	4	1	5	0	0
IBM	5	5	5	5	5	3	5	4	3
JNJ	4	5	5	5	5	5	5	3	1
JPM	4	5	2	5	5	5	1	4	4
KO	0	4	4	5	5	3	4	0	0
MCD	2	5	5	5	5	5	5	3	1
MMM	0	0	5	0	0	0	0	2	3
MRK	3	5	5	5	5	5	5	2	0
MSFT	5	5	5	5	5	5	5	5	5
NKE	1	0	5	4	2	1	2	1	1
NVDA	5	5	5	5	5	5	5	5	5
PG	0	1	2	5	5	2	2	0	0
SHW	0	0	5	0	0	0	5	0	0
TRV	5	5	5	4	5	1	2	2	1
UNH	5	3	5	0	5	5	3	0	0
V	0	0	5	4	2	0	0	0	0
VZ	5	5	5	5	5	5	4	5	1
WMT	3	3	5	5	5	3	3	2	2
	0.593	0.713	0.927	0.680	0.780	0.607	0.740	0.500	0.407

This table reports the number of inclusions in the model confidence set. The HAR model is used to forecast the sparsely sampled (20-minute) RV obtained from the defragmented prices which acts as the unbiased volatility proxy for day $t+h$. The forecast error are evaluated with the QLIKE loss function.

F.3 Robustness check – 1-sec frequency

Table F.9: Equal accuracy: ΔL (QLIKE), 1-sec

	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_N	PRV_N	RV_N^{AC}	RV_N^{NW}	RK_N^{MTH}	RK_N^{P}
AAPL	0.035	0.008	0.001	0.072	0.114	0.369	0.031	0.034	0.056	0.145	0.292
AMGN	0.073	0.026	0.004	0.19	0.378	0.525	0.086	0.264	0.123	0.568	1.297
AMZN	0.033	0.017	0.009	0.158	0.724	1.911	0.298	0.398	0.935	0.831	1.134
AXP	0.035	0.008	-0.006	0.12	0.264	0.041	0.034	0.012	-0.006	0.145	0.279
BA	0.036	0.022	0.003	0.119	0.225	0.006	0.034	0.022	-0.002	0.127	0.247
CAT	0.024	0.007	-0.008	0.105	0.185	0.011	0.024	0.008	-0.009	0.115	0.208
CRM	0.026	0.005	-0.007	0.113	0.219	0.034	0.027	0.007	-0.006	0.138	0.235
CSCO	0.055	0.025	0.01	0.113	0.172	0.122	0.052	0.029	0.017	0.242	0.422
CVX	0.026	0.007	0	0.08	0.142	0.011	0.026	0.008	0.001	0.084	0.151
DIS	0.031	0.006	-0.005	0.097	0.178	0.001	0.031	0.007	-0.005	0.109	0.202
GS	0.019	0.003	-0.012	0.099	0.197	0.049	0.018	0.004	-0.013	0.112	0.249
HD	0.019	-0.003	-0.016	0.103	0.187	0.04	0.02	-0.001	-0.016	0.12	0.224
HON	0.059	0.023	-0.009	0.153	0.263	0.252	0.061	0.058	0.032	0.232	0.472
IBM	0.033	0.008	-0.006	0.112	0.211	0.009	0.034	0.01	-0.005	0.12	0.223
JNJ	0.043	0.014	-0.003	0.107	0.196	0	0.044	0.014	-0.002	0.12	0.218
JPM	0.025	0.009	0	0.077	0.128	-0.003	0.025	0.01	0	0.086	0.143
KO	0.029	0.008	-0.001	0.077	0.141	-0.003	0.029	0.01	0	0.089	0.162
MCD	0.015	-0.003	-0.015	0.108	0.222	0.006	0.016	-0.002	-0.013	0.112	0.234
MMM	0.028	-0.001	-0.012	0.1	0.169	0.034	0.028	0.002	-0.008	0.11	0.187
MRK	0.036	0.008	-0.001	0.105	0.186	0.001	0.036	0.011	0.001	0.121	0.211
MSFT	0.017	-0.001	-0.005	0.049	0.08	0.271	0.013	0.023	0.018	0.103	0.192
NKE	0.026	0.007	0	0.08	0.143	-0.003	0.026	0.008	0.001	0.084	0.15
NVDA	0.011	-0.005	-0.011	0.053	0.113	0.504	0.012	0.041	0.069	0.136	0.532
PG	0.021	-0.001	-0.011	0.094	0.181	0.012	0.022	0	-0.008	0.116	0.22
SHW	0.134	0.071	0.009	0.42	0.845	0.038	0.147	0.156	0.028	0.505	2.393
TRV	0.056	0.028	-0.003	0.193	0.357	0.024	0.057	0.03	-0.003	0.212	0.398
UNH	0.039	0.009	-0.011	0.177	0.337	0.036	0.036	0.01	-0.011	0.207	0.386
V	0.034	0.007	-0.004	0.11	0.197	0.021	0.034	0.009	-0.001	0.124	0.218
VZ	0.026	0.003	-0.002	0.095	0.172	-0.003	0.026	0.006	0	0.108	0.199
WMT	0.032	0.013	0.004	0.105	0.185	0.002	0.033	0.016	0.008	0.112	0.2

	RV_T	PRV_T	RV_T^{AC}	RV_T^{NW}	RK_T^{MTH}	RK_T^{P}	RV_K	PRV_K	RV_K^{AC}	RV_K^{NW}	RK_K^{MTH}	RK_K^{P}
AAPL	-0.002	0.036	0.008	0.001	0.073	0.116	0.002	0.036	0.009	-0.001	0.073	0.116
AMGN	0.001	0.073	0.027	0.004	0.197	0.44	0.745	0.062	0.12	0.136	0.229	0.566
AMZN	0.015	0.028	0.014	0.005	0.143	0.304	0.09	0.042	0.034	0.006	0.14	0.287
AXP	0.039	0.036	0.008	-0.007	0.123	0.239	0.428	0.029	0.109	0.063	0.156	0.306
BA	0.07	0.032	0.018	-0.005	0.137	0.297	0.093	0.04	0.036	0.001	0.132	0.303
CAT	0.053	0.023	0.004	-0.013	0.114	0.212	0.425	0.026	0.086	0.049	0.153	0.337
CRM	0.039	0.025	0.004	-0.01	0.124	0.922	0.663	0.025	0.15	0.148	0.158	0.31
CSCO	0	0.056	0.022	0.01	0.117	0.179	0.031	0.055	0.029	0.009	0.116	0.177
CVX	0.017	0.026	0.008	-0.001	0.084	0.156	0.093	0.026	0.021	0.004	0.084	0.157
DIS	-0.004	0.031	0.005	-0.009	0.099	0.183	0.104	0.032	0.169	-0.001	0.096	0.183
GS	0.082	0.016	0.004	-0.011	0.113	0.229	0.748	0.018	0.114	0.137	0.173	0.327
HD	0.097	0.016	-0.004	-0.013	0.106	0.195	0.415	0.023	0.091	0.055	0.151	0.278
HON	0.017	0.06	0.044	-0.01	0.154	0.279	1.053	0.08	0.209	0.369	0.239	0.533
IBM	0.044	0.032	0.01	-0.009	0.118	0.222	0.367	0.03	0.366	0.048	0.14	0.279
JNJ	0.043	0.043	0.018	-0.009	0.115	0.224	0.24	0.042	0.028	0.021	0.147	0.297
JPM	0.003	0.025	0.009	-0.001	0.075	0.128	0.06	0.026	0.009	0.003	0.087	0.149
KO	0.006	0.029	0.008	-0.002	0.08	0.147	0.043	0.028	0.008	-0.001	0.083	0.159
MCD	0.141	0.01	0.015	-0.009	0.134	0.398	0.473	0.013	0.066	0.061	0.178	0.364
MMM	0.122	0.025	0	-0.01	0.103	0.182	0.535	0.022	0.069	0.062	0.139	0.244
MRK	0.004	0.036	0.006	-0.004	0.107	0.195	0.196	0.033	0.035	0.029	0.145	0.269
MSFT	-0.007	0.018	-0.001	-0.005	0.049	0.082	0.01	0.018	-0.001	-0.006	0.049	0.083
NKE	0.044	0.026	0.009	-0.002	0.093	0.17	0.293	0.027	0.02	0.031	0.113	0.203
NVDA	-0.015	0.01	-0.005	-0.014	0.056	0.118	0.016	0.013	-0.002	-0.011	0.053	0.108
PG	0.02	0.021	-0.001	-0.01	0.104	0.199	0.28	0.02	0.031	0.033	0.147	0.299
SHW	0.284	0.094	0.47	0.042	0.483	0.937	1.737	0.304	0.436	0.894	1.875	1.399
TRV	0.129	0.053	0.423	0.006	0.216	0.448	1.343	0.047	0.383	0.461	0.419	1.06
UNH	0.097	0.034	0.021	-0.008	0.205	0.409	0.582	0.032	0.17	0.097	0.283	0.628
V	0.077	0.032	0.005	-0.004	0.113	0.224	0.359	0.035	0.023	0.045	0.15	0.265
VZ	-0.003	0.026	0.002	-0.005	0.098	0.18	0.043	0.025	0.004	-0.004	0.1	0.183
WMT	0.01	0.032	0.007	-0.003	0.107	0.2	0.155	0.029	0.649	0.007	0.145	0.277

	RV_P	PRV_P	RV_P^{AC}	RV_P^{NW}	RK_P^{MTH}	RK_P^{P}	RV_Z	PRV_Z	RV_Z^{AC}	RV_Z^{NW}	RK_Z^{MTH}	RK_Z^{P}
AAPL	-0.004	0.035	0.008	0	0.074	0.118	-0.006	0.035	0.006	-0.002	0.071	0.112
AMGN	0.481	0.038	0.067	0.089	0.263	0.532	0.416	0.059	0.093	0.088	0.261	0.708
AMZN	0.301	0.028	0.031	0.042	0.223	0.45	0.572	0.025	0.095	0.118	0.288	0.607
AXP	0.283	0.026	0.03	0.028	0.177	0.418	0.134	0.031	0.014	0.003	0.141	0.273
BA	0.37	0.019	0.044	0.036	0.171	1.741	0.244	0.026	0.025	0.009	0.164	1.304
CAT	0.583	0.012	0.051	0.12	0.193	0.41	0.108	0.02	0.011	-0.001	0.125	0.23
CRM	0.276	0.018	0.039	0.027	0.2	0.3	0.287	0.018	0.016	0.02	0.139	0.274
CSCO	0.002	0.055	0.024	0.009	0.123	0.188	0.005	0.055	0.024	0.007	0.112	0.171
CVX	0.121	0.024	0.02	0.007	0.098	0.196	0.057	0.025	0.007	0.002	0.076	0.143
DIS	0.056	0.029	0.003	-0.011	0.112	0.217	0.005	0.03	0.002	-0.01	0.102	0.2
GS	0.656	0.005	0.076	0.099	0.183	0.495	0.25	0.011	0.029	0.015	0.131	0.317
HD	0.445	0.009	0.046	0.064	0.172	0.344	0.151	0.015	0.002	-0.006	0.135	0.257
HON	0.929	0.022	0.404	0.273	0.357	0.89	0.213	0.044	0.042	0.024	0.207	0.408
IBM	0.216	0.027	0.028	0.007	0.173	0.349	0.139	0.029	0.01	-0.001	0.125	0.246
JNJ	0.157	0.039	0.026	0.011	0.143	0.292	0.067	0.042	0.02	-0.002	0.121	0.227
JPM	0.044	0.024	0.01	-0.001	0.093	0.158	0.015	0.025	0.008	-0.002	0.078	0.132
KO	0.019	0.028	0.009	-0.003	0.094	0.176	0.022	0.029	0.009	-0.002	0.081	0.151
MCD	0.326	0.003	0.104	0.025	0.18	0.363	0.198	0.007	0.024	0.002	0.149	0.298
MMM	0.633	0.014	0.202	0.149	0.179	0.39	0.291	0.016	0.029	0.019	0.143	0.253
MRK	0.107	0.032	0.024	0.004	0.151	0.285	0.05	0.034	0.006	-0.002	0.106	0.2
MSFT	0.003	0.016	-0.002	-0.008	0.054	0.092	0.009	0.017	-0.002	-0.008	0.054	0.091
NKE	0.191	0.022	0.013	0.001	0.115	0.199	0.103	0.022	0.002	-0.008	0.097	0.178
NVDA	0.066	0.011	-0.002	-0.013	0.067	0.136	0.088	0.009	-0.006	-0.011	0.059	0.121
PG	0.151	0.019	0.015	0.003	0.147	0.301	0.096	0.019	0	-0.005	0.109	0.217
SHW	1.242	0.015	0.28	0.41	1.995	2.642	0.659	0.081	0.179	0.159	0.67	1.34
TRV	1.03	0.031	0.78	0.281	1.924	0.997	0.266	0.043	0.068	0.035	0.242	0.488
UNH	0.439	0.023	0.062	0.05	0.287	0.536	0.297	0.025	0.192	0.026	0.232	0.479
V	0.247	0.02	0.004	0.013	0.148	0.27	0.303	0.027	0.021	0.04	0.156	0.29
VZ	0.005	0.025	0.002	-0.005	0.12	0.217	0	0.025	-0.001	-0.006	0.088	0.161
WMT	0.133	0.024	0.718	0.001	0.142	0.271	0.145	0.029	0.016	0.008	0.126	0.225

This table reports the QLIKE distances from the benchmark estimator (RV_{GRT}). The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The RV_{GRT}, RV, pre-averaging and the kernel-based estimators are all computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.10: Tests of equal accuracy for DJIA stocks (QLIKE): 1-sec

	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_N	PRV_N	RV_N^{AC}	RV_N^{NW}	RK_N^{MTH}	RK_N^{P}	RV_T	PRV_T	RV_T^{AC}	RV_T^{NW}	RK_T^{MTH}	RK_T^{P}	RV_K	PRV_K	RV_K^{AC}	RV_K^{NW}	RK_K^{MTH}	RK_K^{P}	RV_P	PRV_P	RV_P^{AC}	RV_P^{NW}	RK_P^{MTH}	RK_P^{P}	RV_Z	PRV_Z	RV_Z^{AC}	RV_Z^{NW}	RK_Z^{MTH}	RK_Z^{P}
AAPL	▲	△	△	▲	▲	▲	▲	▲	▲	▲	▲	▽	▲	△	△	▲	▲	△	▲	▲	▽	▲	▲	▽	▲	△	▽	▲	▲	▽	▲	△	▽	▲	▲
AMGN	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AXP	▲	△	▽	▲	▲	▲	△	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	△	▲	▲	▲	▲	▲	△	△	△	▲	▲	▲	△	△	△	▲	▲
BA	△	△	△	▲	▲	△	△	△	▽	▲	▲	▲	△	△	▽	▲	▲	▲	△	△	△	▲	▲	▲	△	▲	△	▲	▲	▲	△	△	△	▲	▲
CAT	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	△	▽	▲	▲
CRM	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	△	△	▲	▲
CSCO	▲	△	△	▲	▲	▲	▲	▲	△	▲	▲	△	▲	△	△	▲	▲	▲	▲	▲	△	▲	▲	△	▲	△	△	▲	▲	△	▲	△	△	▲	▲
CVX	△	△	△	▲	▲	△	△	△	△	▲	▲	△	△	△	▽	▲	▲	▲	△	△	△	▲	▲	▲	△	△	△	▲	▲	▲	△	△	△	▲	▲
DIS	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	▽	▲	△	▽	▲	▲	▲	▲	▲	▽	▲	▲	▲	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲
GS	△	△	▽	▲	▲	▲	△	△	▽	▲	▲	▲	△	△	▽	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	△	▲	▲
HD	△	▽	▽	▲	▲	▲	△	▽	▽	▲	▲	▲	△	▽	▽	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	△	▽	▲	▲
HON	△	△	▽	▲	▲	▲	▲	△	△	▲	▲	△	△	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	△	△	▲	▲
IBM	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	△	▽	▲	▲
JNJ	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	▲	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▽	▲	▲
JPM	▲	△	△	▲	▲	▽	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	▲	▲	△	△	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲
KO	△	△	▽	▲	▲	▽	△	△	△	▲	▲	△	△	△	▽	▲	▲	▲	△	△	▽	▲	▲	△	△	△	▽	▲	▲	△	△	△	▽	▲	▲
MCD	△	▽	▽	▲	▲	△	△	▽	▽	▲	▲	▲	△	△	▽	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	△	▲	▲	▲	△	△	△	▲	▲
MMM	△	▽	▽	▲	▲	▲	△	△	▽	▲	▲	▲	△	▽	▽	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	△	△	▲	▲
MRK	▲	△	▽	▲	▲	△	▲	△	△	▲	▲	△	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	△	▽	▲	▲
MSFT	▲	▽	▽	▲	▲	▲	▲	▲	▲	▲	▲	▽	▲	▽	▽	▲	▲	▲	▲	▽	▽	▲	▲	△	▲	▽	▽	▲	▲	▲	▲	▽	▽	▲	▲
NKE	▲	△	△	▲	▲	▽	▲	△	△	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	△	▲	▲	▲	▲	△	▽	▲	▲
NVDA	△	▽	▽	▲	▲	▲	△	▲	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	△	▽	▽	▲	▲	▲	△	▽	▽	▲	▲	▲	△	▽	▽	▲	▲
PG	△	▽	▽	▲	▲	△	△	△	▽	▲	▲	△	△	▽	▽	▲	▲	▲	△	△	△	▲	▲	▲	△	△	△	▲	▲	▲	△	△	▽	▲	▲
SHW	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
TRV	▲	▲	▽	▲	▲	▲	▲	▲	▽	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
UNH	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	△	▲	▲
V	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	△	▲	▲	▲	▲	△	△	△	▲	▲	▲	△	△	▲	▲	▲
VZ	▲	△	▽	▲	▲	▽	▲	△	▽	▲	▲	▽	▲	△	▽	▲	▲	▲	▲	△	▽	▲	▲	△	▲	△	▽	▲	▲	△	▲	▽	▽	▲	▲
WMT	△	△	△	△	△	△	△	△	△	△	△	△	△	△	▽	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△
24	30	23	29	6	7	20	28	23	25	6	3	12	28	16	15	3	2	24	28	22	26	4	4	11	26	12	17	5	3	6	29	8	7	3	1

This table reports the QLIKE distances from the benchmark estimator (RV_{GRT}). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The RV_{GRT}, RV, pre-averaging, and the kernel-based estimators are all computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.11: Forecast evaluation: inclusion in MCS (QLIKE), $h = 1, \ldots, 5$

	RV_{GRT}	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	RV_N	$RBPV_N$	PRV_N	$PBPV_N$	RV_N^{AC}	RV_N^{NW}	RK_N^{MTH}	RK_N^{P}	RV_T	$RBPV_T$	PRV_T	$PBPV_T$	RV_T^{AC}	RV_T^{NW}	RK_T^{MTH}	RK_T^{P}
AAPL	5	5	5	5	5	5	0	0	0	0	5	5	5	5	5	5	5	5	0	0	4	5	1	0
AMGN	5	5	0	0	0	0	1	4	5	4	0	0	4	5	4	4	5	5	0	0	0	0	1	4
AMZN	5	4	1	2	1	1	5	5	0	0	5	5	0	0	5	5	3	3	5	5	2	2	5	5
AXP	5	5	3	3	2	4	1	0	5	5	3	3	2	5	0	0	2	1	3	2	1	1	1	0
BA	5	5	0	0	0	0	0	0	3	0	0	0	0	0	0	0	4	5	0	0	0	0	0	0
CAT	3	3	4	4	3	3	3	3	0	2	4	4	3	3	3	3	2	2	4	4	2	3	3	2
CRM	5	5	1	2	0	5	4	4	4	4	2	2	1	5	4	4	3	3	2	2	3	5	4	4
CSCO	5	5	0	0	0	0	0	0	0	0	0	1	1	3	0	0	5	5	0	0	0	0	0	0
CVX	5	5	5	5	5	5	4	4	5	5	5	5	5	5	4	4	3	5	5	5	5	5	4	4
DIS	5	5	5	5	4	5	4	4	5	5	5	5	4	5	4	4	5	5	5	5	4	5	4	4
GS	1	0	4	5	2	2	2	4	0	0	4	4	3	4	4	4	0	0	4	5	2	0	1	4
HD	5	5	4	5	5	5	4	3	5	4	4	5	5	5	4	3	0	0	5	5	3	2	3	2
HON	5	5	4	5	4	5	1	1	5	5	0	0	0	3	0	0	5	5	4	5	5	5	1	1
IBM	5	5	1	1	1	2	2	2	5	5	1	1	1	2	2	2	4	5	1	1	1	3	2	2
JNJ	3	4	1	1	2	2	1	0	3	3	1	1	2	2	0	0	2	1	1	1	2	3	0	0
JPM	3	3	5	5	5	1	4	4	4	3	5	5	5	3	4	4	0	0	4	3	3	1	4	4
KO	5	4	5	5	3	5	0	0	5	4	5	5	5	5	3	1	5	4	5	5	1	4	0	0
MCD	5	5	5	5	5	5	1	0	4	4	5	5	5	5	1	0	0	1	5	5	4	3	0	0
MMM	3	5	0	0	0	0	4	4	4	4	0	0	0	0	4	4	5	5	0	0	0	0	4	4
MRK	5	5	5	5	5	5	2	0	5	5	5	5	5	5	2	0	4	4	5	5	4	3	1	0
MSFT	5	5	5	5	5	5	5	5	0	0	5	5	5	5	5	2	5	5	5	5	5	5	5	5
NKE	5	5	5	5	0	1	3	1	5	5	5	5	0	0	4	2	5	5	5	5	2	5	4	5
NVDA	5	5	5	5	5	5	5	5	4	4	5	5	5	5	5	5	5	5	5	5	4	0	5	5
PG	5	3	5	5	1	1	0	0	5	5	4	5	2	3	0	0	3	3	5	5	1	2	0	0
SHW	5	5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	2	2	0	1	0	1	0	0
TRV	5	5	5	5	5	5	4	4	4	4	5	5	5	5	4	4	1	1	5	5	0	0	0	0
UNH	5	5	4	5	5	5	1	1	4	4	0	0	5	5	1	1	2	2	5	5	5	5	3	3
V	4	5	1	0	2	2	0	0	2	4	1	0	2	2	0	0	1	2	1	0	2	2	0	0
VZ	5	5	5	5	3	2	5	1	3	5	5	5	3	1	5	1	3	3	5	5	1	2	5	2
WMT	2	5	0	1	2	2	1	1	2	5	0	0	2	2	1	0	0	1	0	0	2	1	1	0
	0.893	0.907	0.620	0.660	0.533	0.593	0.447	0.400	0.640	0.653	0.593	0.607	0.567	0.653	0.520	0.413	0.593	0.620	0.627	0.627	0.453	0.487	0.413	0.400

	RV_K	$RBPV_K$	PRV_K	$PBPV_K$	RV_K^{AC}	RV_K^{NW}	RK_K^{MTH}	RK_K^{P}	RV_P	$RBPV_P$	PRV_P	$PBPV_P$	RV_P^{AC}	RV_P^{NW}	RK_P^{MTH}	RK_P^{P}	RV_Z	$RBPV_Z$	PRV_Z	$PBPV_Z$	RV_Z^{AC}	RV_Z^{NW}	RK_Z^{MTH}	RK_Z^{P}
AAPL	5	5	5	5	4	5	0	0	5	5	5	4	5	5	0	0	5	5	5	5	5	5	0	0
AMGN	4	4	0	0	0	4	4	4	4	4	0	0	0	0	2	4	5	5	0	0	3	4	2	3
AMZN	0	4	1	1	1	4	5	5	0	0	3	5	2	0	5	5	0	0	3	5	2	0	5	5
AXP	3	4	5	5	5	5	0	0	3	3	4	4	3	5	3	3	0	0	2	3	2	1	1	0
BA	4	5	2	2	0	5	0	0	1	1	1	1	0	5	0	0	1	1	2	2	0	2	0	0
CAT	1	2	5	5	3	3	3	2	0	0	4	4	2	2	3	2	0	0	1	1	0	0	2	2
CRM	0	3	5	5	5	3	4	4	0	0	5	5	4	4	4	4	3	4	4	5	0	4	4	4
CSCO	5	4	0	0	0	0	0	0	5	5	0	0	0	0	0	0	3	5	0	0	0	0	0	0
CVX	1	4	5	5	5	5	4	4	4	4	5	5	5	5	4	4	0	0	5	5	5	3	4	4
DIS	5	5	5	5	5	5	5	4	5	5	5	5	5	5	3	3	4	5	5	5	3	5	4	4
GS	0	0	5	5	5	4	4	4	0	0	3	2	4	0	4	4	0	0	4	4	0	0	3	4
HD	0	1	5	5	5	5	2	1	4	3	5	5	4	5	3	1	1	3	5	5	5	5	4	3
HON	3	3	5	5	1	4	4	4	3	3	5	5	1	2	0	0	5	5	0	5	4	5	0	1
IBM	3	3	4	5	5	5	3	3	3	4	2	2	1	5	3	3	0	0	1	1	0	0	1	1
JNJ	4	4	4	5	4	5	0	0	0	0	1	1	3	2	0	0	0	0	1	1	0	1	0	0
JPM	1	1	5	5	5	5	0	3	4	4	5	5	5	5	4	4	2	2	5	5	5	2	4	4
KO	0	0	5	5	5	5	1	0	5	4	5	5	4	5	1	0	4	2	5	5	4	5	0	0
MCD	4	5	5	5	5	5	2	0	4	4	5	5	5	5	0	0	2	5	5	5	3	1	0	0
MMM	5	5	0	1	1	5	4	4	4	5	1	4	4	5	4	4	5	5	0	0	0	0	3	4
MRK	2	2	5	5	5	4	1	0	4	4	5	5	3	5	1	0	0	0	5	5	4	1	0	0
MSFT	5	5	5	5	5	5	5	5	3	3	5	5	4	4	5	5	5	5	5	5	5	5	5	5
NKE	5	5	5	5	5	5	5	5	5	5	5	5	2	5	5	5	5	5	5	5	4	5	5	5
NVDA	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	4	5	5	5	5	5	5
PG	0	1	5	5	5	3	0	0	0	0	5	5	0	0	0	0	0	0	5	5	1	0	0	0
SHW	5	5	5	5	4	5	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TRV	4	4	5	5	4	5	0	0	1	1	5	5	0	1	0	0	0	0	5	5	0	0	0	0
UNH	2	5	5	5	5	5	4	2	5	5	5	5	4	5	0	0	4	5	5	5	4	5	3	3
V	1	2	5	5	0	3	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0
VZ	4	3	5	5	5	5	5	2	0	0	5	5	4	5	5	1	5	5	5	5	3	3	5	0
WMT	3	5	5	5	5	5	2	0	0	0	3	2	2	2	1	1	0	0	0	1	1	0	1	0
	0.560	0.693	0.840	0.860	0.747	0.880	0.480	0.407	0.540	0.513	0.720	0.733	0.540	0.647	0.433	0.387	0.420	0.473	0.627	0.693	0.453	0.447	0.407	0.380

This table reports the number of inclusions in the model confidence set. The HAR model is used to forecast the sparsely sampled (20-minute) RV obtained from the defragmented prices which acts as the unbiased volatility proxy for day $t + h$. The forecast error are evaluated with the QLIKE loss function. The RV_{GRT}, RV, pre-averaging, and the kernel-based estimators are all computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

F.4 Robustness check – mixed frequencies 1-min/5-min RV vs 1-sec noise-robust estimators (MSE)

Table F.12: Equal accuracy losses (MSE) $\times 10^8$, mixed frequencies

	$RV_{GRT}^{(390)}$	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}
AAPL	7.772	9.304	7.23	7.398	7.178	8.044	8.415	22.855	150.075	10.845	19.854	603.001	9.596	11.129
AMGN	17.79	19.343	13.243	14.01	11.843	13.482	13.762	206.609	2275.033	44.652	172.709	2230.934	19.621	21.418
AMZN	4.275	5.421	4.405	4.551	3.972	5.921	6.827	1428.187	21037.059	544.128	2284.186	35751.455	15.811	18.82
AXP	7.345	7.836	5.954	5.642	5.443	9.221	11.223	8.129	7.414	5.843	5.978	6.231	10.087	12.045
BA	71.416	84.211	63.785	70.508	66.041	76.575	91.612	85.201	71.959	63.851	71.089	68.718	78.614	93.583
CAT	5.581	6.401	5.071	5.339	5.26	6.251	7.144	6.514	6.152	5.119	5.519	5.934	6.436	7.559
CRM	5.852	7.079	5.189	4.998	4.833	7.374	8.843	7.282	6.603	5.278	5.26	6.559	7.704	9.532
CSCO	5.646	6.954	5.176	5.567	5.759	5.782	6.147	28.25	289.448	16.467	26.251	2679.805	7.934	10.997
CVX	6.488	7.877	6.431	5.832	6.127	7.35	8.696	7.657	6.498	6.518	5.809	6.478	7.38	8.658
DIS	4.441	5.366	4.336	4.37	4.48	5.518	6.096	5.341	4.41	4.313	4.31	4.628	5.563	6.093
GS	2.686	3.176	2.6	2.433	2.575	4.05	5.107	3.335	3.391	2.603	2.45	3.862	4.228	5.364
HD	4.473	6.267	4.576	4.339	4.258	6.335	7.183	6.087	4.593	4.71	4.484	4.598	6.253	7.476
HON	7.388	6.733	6.254	5.898	5.527	8.733	10.765	7.7	11.455	6.464	10.288	11.423	10.172	14.15
IBM	2.467	2.823	2.214	2.328	2.287	2.904	3.119	2.812	2.622	2.237	2.308	2.428	2.89	3.062
JNJ	3.101	3.385	3.159	3.001	3.079	3.265	3.263	3.526	3.116	3.176	2.982	3.132	3.356	3.367
JPM	5.947	7.568	5.555	5.908	5.383	7.071	7.155	7.661	6.013	5.558	5.917	5.393	7.217	7.317
KO	3.085	3.646	2.695	2.978	2.635	3.304	3.621	3.73	2.97	2.694	2.684	2.472	3.31	3.679
MCD	6.106	5.752	4.096	5.499	5.824	6.082	7.006	6.029	5.971	4.158	5.739	6.088	6.497	7.445
MMM	4.117	5.082	4.075	4.423	4.652	5.524	6.649	5.062	4.661	4.098	4.55	5.446	5.699	7.073
MRK	4.933	5.21	4.662	5.118	4.879	5.136	5.383	5.145	4.58	4.667	4.828	4.583	5.183	5.517
MSFT	7.425	7.552	6.55	6.579	6.081	6.987	7.354	19.473	202.879	12.703	18.826	891.644	8.97	10.087
NKE	13.788	13.559	10.153	13.345	12.975	13.715	15.054	13.709	13.621	10.181	13.323	13.148	13.894	15.3
NVDA	26.252	29.466	23.12	25.439	22.971	29.863	33.735	73.214	557.216	39.67	65.179	1221.343	42.329	53.243
PG	3.619	4.413	3.23	3.189	3.772	4.578	4.998	4.537	3.765	3.267	3.321	3.988	4.674	5.094
SHW	9.562	11.02	9.787	8.641	8.204	23.122	32.925	12.184	14.329	10.181	9.151	11.482	25.701	37.112
TRV	3.984	4.094	3.721	3.79	3.571	3.61	4.067	4.302	5.07	3.8	4.212	5.279	3.608	4.089
UNH	4.973	6.584	5.019	4.391	4.345	6.936	7.952	6.681	5.219	5.033	4.399	4.785	7.432	8.487
V	5.063	4.572	3.589	4.854	4.991	4.295	4.176	5.003	5.064	3.597	4.927	5.148	4.647	4.496
VZ	8.636	3.875	3.136	3.259	3.088	3.28	3.438	3.893	3.446	3.127	3.236	3.091	3.251	3.43
WMT	5.288	4.621	4.025	5.307	5.022	4.283	4.369	4.892	5.541	4.055	5.507	5.125	4.497	4.608

	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}
AAPL	9.341	7.901	7.265	7.407	7.161	8.107	8.492	9.49	7.922	7.262	7.395	7.381	8.144	8.42
AMGN	19.711	18.008	13.168	14.045	11.999	13.447	14.008	25.967	115.382	14.402	23.798	97.615	13.111	14.916
AMZN	5.654	4.621	4.526	4.755	4.25	6.094	7.063	5.352	4.845	4.233	4.539	4.725	5.941	6.965
AXP	7.828	7.734	6.007	5.788	5.72	9.678	11.818	13.362	117.957	8.3	29.696	355.325	9.417	10.776
BA	85.171	74.834	63.831	72.747	74.416	80.891	94.962	84.511	76.052	63.594	69.11	69.588	78.452	93.718
CAT	6.445	5.876	5.049	5.381	5.761	6.352	7.482	21.181	256.522	10.602	43.923	618.961	7.688	9.977
CRM	7.225	6.189	5.203	5.064	5.155	7.729	9.443	11.452	83.505	6.646	17.801	297.648	8.356	10.7
CSCO	7.092	5.49	5.166	5.558	5.575	6.062	6.428	6.892	5.81	5.101	5.176	10.437	5.877	6.171
CVX	8.504	7.249	6.403	6.4	7.062	7.9	9.081	8.29	14.592	6.272	8.87	35.758	8.633	10.101
DIS	5.402	4.482	4.355	4.444	4.658	5.648	6.245	5.484	4.824	4.373	4.544	5.802	5.622	6.34
GS	3.358	2.897	2.606	2.549	2.994	4.304	5.367	5.495	85.201	3.457	21.055	314.6	5.202	5.935
HD	6.631	4.995	4.645	4.456	4.905	6.878	7.981	9.512	67.455	5.125	21.216	341.348	6.721	7.921
HON	6.834	8.426	6.246	6.262	6.418	9.371	11.807	32.931	400.613	13.133	106.99	971.013	10.332	15.191
IBM	2.935	2.66	2.241	2.401	2.602	3.098	3.273	4.42	20.208	2.349	4.533	76.016	3.048	3.307
JNJ	3.373	3.322	3.175	3.113	3.551	3.201	3.311	3.583	20.373	3.404	7.688	83.456	3.387	3.755
JPM	7.704	6.142	5.549	5.918	5.47	7.386	7.521	8.472	10.77	5.688	7.397	33.602	7.04	7.292
KO	3.707	3.226	2.687	3.123	2.742	3.381	3.642	4.168	5.921	2.813	4.005	19.738	3.569	3.729
MCD	5.971	6.759	4.227	5.601	6.784	6.547	8.166	8.374	156.964	6.37	35.513	249.19	5.588	7.933
MMM	5.292	4.721	4.119	4.516	5.737	5.82	7.133	9.141	64.141	4.407	8.222	146.786	6.668	8.348
MRK	5.243	5.368	4.672	5.399	5.508	5.154	5.511	7.526	59.302	5.389	19.691	335.616	5.371	6.003
MSFT	7.547	7.451	6.563	6.656	6.177	6.907	7.312	7.517	8.008	6.412	6.204	7.325	7.015	7.646
NKE	13.743	14.991	10.165	13.829	13.942	14.265	15.585	14.168	85.397	10.268	37.897	421.25	13.544	15.518
NVDA	29.668	26.964	23.223	25.861	23.521	30.46	34.44	29.185	26.726	22.898	28.014	27.128	30.634	34.895
PG	4.591	4.105	3.203	3.285	4.379	4.546	4.826	6.825	59.214	3.321	5.175	217.328	4.959	5.436
SHW	11.749	14.63	9.287	9.087	10.137	29.188	40.371	416.167	4900.538	103.803	248.987	2979.164	36.11	54.979
TRV	4.499	4.728	3.694	4.055	4.017	4.136	5.033	45.138	605.78	13.559	54.248	903.366	6.362	8.439
UNH	6.646	5.88	5.039	4.842	6.069	7.107	8.059	13.066	197.274	8.48	38.975	530.132	7.92	8.928
V	5.084	5.148	3.587	5.055	5.329	4.588	4.411	4.469	26.571	3.517	10.407	83.529	4.099	4.197
VZ	4.28	21.509	3.157	3.481	3.38	3.264	3.395	4.202	17.145	3.212	3.787	16.898	3.239	3.502
WMT	4.776	5.658	4.059	5.537	5.447	4.369	4.554	4.408	18.281	4.254	11.16	65.969	3.969	4.292

	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	9.318	7.99	7.248	7.472	7.358	8.173	8.546	9.35	7.713	7.248	7.45	7.264	8.008	8.433
AMGN	22.977	41.849	13.512	15.682	28.699	13.735	14.779	20.291	26.439	13.634	18.13	22.571	17.072	19.464
AMZN	5.737	6.544	4.474	4.852	9.681	6.704	8.012	7.184	10.695	4.63	6.682	17.207	7.922	9.88
AXP	8.973	10.793	6.056	8.112	17.771	11.302	13.319	8.856	10.403	6.05	6.917	9.369	10.596	12.637
BA	81.965	69.77	64.763	72.905	73.198	86.017	98.735	84.092	79.072	63.8	79.19	92.882	86.805	98.292
CAT	7.274	18.324	5.098	7.179	34.005	7.301	10.631	7.048	6.728	5.084	5.814	7.018	7.094	8.617
CRM	7.788	11.619	5.101	5.129	25.776	8.306	10.244	8.16	7.125	5.228	5.619	7.527	8.743	10.734
CSCO	7.194	6.092	5.196	5.923	6.139	5.961	6.299	6.916	5.489	5.187	5.489	5.675	5.699	6.108
CVX	8.009	10.266	6.226	6.077	19.422	8.222	9.146	8.219	6.568	6.483	5.666	6.218	7.801	9.189
DIS	5.429	4.614	4.319	4.405	5.315	5.713	6.28	5.448	4.663	4.336	4.529	4.825	5.661	6.324
GS	3.678	9.689	2.6	3.309	21.083	5.523	5.754	3.865	3.862	2.608	3.193	4.539	5.47	6.788
HD	7.774	10.382	4.665	5.504	26.53	7.425	8.7	8.213	5.863	4.656	4.95	7.414	7.955	8.752
HON	8.159	36.087	6.198	12.005	68.866	13.1	19.144	7.753	9.711	6.289	5.996	9.677	10.159	13.856
IBM	2.946	5.66	2.166	2.277	12.159	2.914	3.218	3.275	2.959	2.244	2.783	3	3.512	3.849
JNJ	3.393	4.469	3.159	3.394	8.233	3.187	3.433	3.529	3.424	3.137	3.359	3.768	3.23	3.327
JPM	7.572	6.254	5.6	6.217	8.873	7.204	7.493	7.485	5.972	5.546	6.006	5.582	6.876	6.902
KO	3.96	3.253	2.674	3.233	3.559	3.505	3.749	3.618	2.967	2.702	3	2.676	3.339	3.783
MCD	6.278	10.743	3.962	6.12	21.156	6.287	8.294	7.144	7.043	4.138	5.546	9.667	7.837	10.209
MMM	5.89	15.804	4.086	6.191	33.517	7.228	9.374	6.195	6.812	4.132	5.924	11.013	6.834	8.366
MRK	5.458	8.993	4.539	6.647	26.904	5.205	5.947	5.231	5.205	4.674	5.201	5.074	5.108	5.51
MSFT	7.56	7.211	6.479	6.455	6.497	6.911	7.323	7.719	7.693	6.559	6.634	6.373	7.167	7.909
NKE	14.387	18.364	10.176	15.026	27.285	15.113	16.121	12.888	15.083	10.213	13.831	14.187	13.9	14.932
NVDA	29.429	28.392	23.286	25.356	25.762	30.261	34.68	29.617	28.111	23.144	25.949	26.605	29.944	35.294
PG	6.114	10.793	3.3	4.046	24.52	4.975	5.609	4.003	3.681	3.242	3.101	4.684	4.835	5.51
SHW	26.177	162.104	10.569	18.552	99.213	41.55	60.237	14.663	24.476	9.145	14.186	24.064	34.053	48.2
TRV	7.248	31.075	3.734	6.15	31.546	4.989	6.065	5.185	6.004	3.704	4.343	7.998	4.944	5.613
UNH	7.353	14.608	5.234	6.367	30.011	8.582	9.602	7.815	9.608	5.094	6.771	14.474	8.661	9.166
V	4.431	9.392	3.589	6.138	17.618	4.17	4.191	4.505	8.847	3.607	6.636	9.739	4.245	4.332
VZ	3.981	15.026	3.131	3.477	5.105	3.277	3.553	3.938	20.077	3.148	3.356	3.359	3.442	3.737
WMT	4.47	5.553	3.974	5.438	8.16	4.119	4.403	4.477	5.771	4.055	4.875	5.493	3.962	4.124

This table reports the MSE distances from the proxy. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

Table F.13: Equal accuracy: ΔL (MSE) $\times 10^8$, mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}
AAPL	1.531	-0.542	-0.375	-0.595	0.272	0.643	15.083	142.303	3.072	12.082	595.229	1.824	3.357	1.569	0.128	-0.508	-0.365	-0.611	0.335	0.72
AMGN	1.553	-4.547	-3.78	-5.946	-4.308	-4.028	188.82	2257.243	26.862	154.919	2213.144	1.832	3.628	1.921	0.219	-4.622	-3.745	-5.79	-4.343	-3.782
AMZN	1.145	0.13	0.276	-0.304	1.646	2.551	1423.911	21032.783	539.852	2279.91	35747.18	11.536	14.545	1.378	0.345	0.251	0.479	-0.025	1.818	2.787
AXP	0.491	-1.391	-1.703	-1.902	1.876	3.878	0.784	0.069	-1.502	-1.367	-1.114	2.742	4.7	0.483	0.389	-1.338	-1.557	-1.625	2.333	4.473
BA	12.795	-7.631	-0.908	-5.375	5.159	20.196	13.784	0.542	-7.565	-0.327	-2.698	7.198	22.167	13.755	3.417	-7.586	1.33	3	9.475	23.546
CAT	0.82	-0.51	-0.241	-0.32	0.67	1.563	0.933	0.571	-0.462	-0.062	0.353	0.855	1.978	0.864	0.295	-0.532	-0.2	0.18	0.771	1.901
CRM	1.227	-0.663	-0.854	-1.019	1.522	2.991	1.43	0.751	-0.574	-0.591	0.707	1.852	3.68	1.373	0.338	-0.649	-0.788	-0.697	1.878	3.591
CSCO	1.308	-0.47	-0.079	0.113	0.136	0.501	22.605	283.802	10.821	20.605	2674.159	2.288	5.351	1.446	-0.156	-0.48	-0.088	-0.071	0.416	0.782
CVX	1.389	-0.057	-0.657	-0.361	0.862	2.208	1.168	0.009	0.029	-0.679	-0.01	0.892	2.169	2.016	0.76	-0.086	-0.089	0.574	1.412	2.592
DIS	0.925	-0.104	-0.071	0.039	1.077	1.655	0.9	-0.031	-0.128	-0.131	0.187	1.122	1.653	0.962	0.041	-0.086	0.004	0.217	1.207	1.804
GS	0.49	-0.086	-0.253	-0.111	1.364	2.421	0.649	0.705	-0.083	-0.236	1.176	1.542	2.678	0.672	0.211	-0.08	-0.137	0.307	1.617	2.681
HD	1.793	0.103	-0.135	-0.215	1.862	2.71	1.614	0.12	0.237	0.011	0.125	1.78	3.003	2.157	0.522	0.172	-0.017	0.432	2.405	3.508
HON	-0.655	-1.133	-1.49	-1.861	1.346	3.378	0.313	4.067	-0.924	2.9	4.036	2.784	6.762	-0.554	1.039	-1.142	-1.125	-0.969	1.984	4.419
IBM	0.357	-0.253	-0.139	-0.18	0.437	0.653	0.345	0.155	-0.23	-0.159	-0.039	0.423	0.595	0.468	0.193	-0.226	-0.066	0.135	0.632	0.806
JNJ	0.284	0.059	-0.1	-0.022	0.164	0.162	0.426	0.015	0.075	-0.119	0.031	0.256	0.266	0.272	0.222	0.074	0.013	0.45	0.1	0.211
JPM	1.62	-0.393	-0.039	-0.564	1.123	1.207	1.713	0.065	-0.39	-0.03	-0.554	1.269	1.369	1.757	0.194	-0.399	-0.03	-0.477	1.439	1.574
KO	0.561	-0.39	-0.107	-0.45	0.219	0.536	0.645	-0.115	-0.392	-0.401	-0.613	0.225	0.594	0.621	0.141	-0.398	0.038	-0.343	0.296	0.557
MCD	-0.354	-2.01	-0.607	-0.282	-0.024	0.9	-0.077	-0.135	-1.948	-0.367	-0.018	0.391	1.34	-0.135	0.653	-1.879	-0.505	0.678	0.441	2.06
MMM	0.965	-0.041	0.306	0.536	1.407	2.533	0.945	0.544	-0.019	0.433	1.329	1.582	2.956	1.176	0.605	0.002	0.399	1.62	1.703	3.017
MRK	0.276	-0.272	0.185	-0.054	0.203	0.449	0.212	-0.354	-0.266	-0.105	-0.35	0.25	0.584	0.309	0.435	-0.261	0.466	0.574	0.22	0.577
MSFT	0.127	-0.876	-0.846	-1.344	-0.439	-0.071	12.048	195.454	5.278	11.401	884.218	1.545	2.662	0.122	0.026	-0.863	-0.77	-1.248	-0.518	-0.113
NKE	-0.229	-3.634	-0.443	-0.813	-0.073	1.266	-0.079	-0.166	-3.606	-0.464	-0.64	0.107	1.513	-0.045	1.203	-3.622	0.042	0.155	0.477	1.798
NVDA	3.214	-3.132	-0.813	-3.281	3.611	7.483	46.962	530.964	13.418	38.927	1195.091	16.077	26.991	3.416	0.712	-3.03	-0.391	-2.731	4.208	8.188
PG	0.794	-0.389	-0.43	0.153	0.959	1.379	0.918	0.146	-0.352	-0.298	0.369	1.055	1.475	0.972	0.486	-0.416	-0.334	0.76	0.927	1.207
SHW	1.458	0.225	-0.921	-1.358	13.56	23.363	2.622	4.767	0.619	-0.411	1.92	16.139	27.55	2.187	5.068	-0.275	-0.475	0.575	19.626	30.809
TRV	0.11	-0.264	-0.194	-0.413	-0.374	0.082	0.317	1.085	-0.185	0.228	1.295	-0.376	0.105	0.515	0.744	-0.29	0.071	0.033	0.151	1.049
UNH	1.611	0.047	-0.581	-0.627	1.963	2.979	1.708	0.246	0.06	-0.574	-0.188	2.459	3.514	1.673	0.908	0.067	-0.131	1.096	2.135	3.086
V	-0.491	-1.474	-0.209	-0.072	-0.768	-0.887	-0.059	0.001	-1.466	-0.136	0.085	-0.416	-0.567	0.021	0.085	-1.476	-0.008	0.267	-0.475	-0.652
VZ	-4.761	-5.5	-5.377	-5.548	-5.355	-5.198	-4.742	-5.19	-5.508	-5.4	-5.545	-5.384	-5.205	-4.356	12.873	-5.479	-5.155	-5.256	-5.372	-5.241
WMT	-0.666	-1.262	0.019	-0.266	-1.004	-0.918	-0.395	0.254	-1.233	0.219	-0.162	-0.791	-0.679	-0.511	0.37	-1.229	0.25	0.16	-0.919	-0.733

	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	1.717	0.15	-0.51	-0.377	-0.392	0.372	0.648	1.545	0.218	-0.524	-0.3	-0.414	0.4	0.774	1.578	-0.059	-0.524	-0.322	-0.508	0.236	0.66
AMGN	8.177	97.592	-3.388	6.008	79.825	-4.679	-2.873	5.187	24.059	-4.278	-2.108	10.909	-4.055	-3.011	2.502	8.65	-4.156	0.341	4.781	-0.718	1.675
AMZN	1.077	0.569	-0.043	0.263	0.45	1.665	2.689	1.461	2.269	0.198	0.576	5.405	2.428	3.737	2.909	6.42	0.355	2.406	12.932	3.647	5.604
AXP	6.017	110.612	0.955	22.351	347.98	2.072	3.431	1.628	3.448	-1.289	0.767	10.426	3.957	5.974	1.511	3.058	-1.295	-0.428	2.024	3.251	5.292
BA	13.094	4.636	-7.822	-2.306	-1.828	7.036	22.302	10.549	-1.646	-6.653	1.489	1.782	14.601	27.319	12.676	7.655	-7.616	7.774	21.466	15.389	26.875
CAT	15.6	250.941	5.021	38.342	613.38	2.107	4.396	1.693	12.743	-0.483	1.598	28.424	1.72	5.05	1.467	1.147	-0.496	0.233	1.437	1.513	3.036
CRM	5.601	77.653	0.794	11.95	291.796	2.505	4.848	1.936	5.767	-0.751	-0.723	19.924	2.455	4.392	2.308	1.273	-0.624	-0.233	1.676	2.891	4.883
CSCO	1.246	0.164	-0.545	-0.47	4.791	0.231	0.525	1.548	0.446	-0.45	0.277	0.494	0.315	0.653	1.27	-0.157	-0.459	-0.157	0.029	0.053	0.463
CVX	1.802	8.104	-0.216	2.381	29.27	2.144	3.612	1.52	3.777	-0.263	-0.412	12.933	1.734	2.658	1.731	0.079	-0.006	-0.822	-0.27	1.312	2.701
DIS	1.044	0.383	-0.067	0.103	1.362	1.182	1.899	0.989	0.174	-0.121	-0.036	0.874	1.272	1.84	1.008	0.223	-0.105	0.088	0.385	1.22	1.884
GS	2.809	82.515	0.771	18.369	311.914	2.515	3.249	0.992	7.003	-0.086	0.623	18.397	2.837	3.068	1.179	1.176	-0.078	0.507	1.853	2.784	4.101
HD	5.038	62.982	0.652	16.743	336.875	2.248	3.448	3.301	5.909	0.192	1.031	22.057	2.952	4.226	3.74	1.39	0.183	0.477	2.941	3.482	4.279
HON	25.543	393.225	5.745	99.602	963.625	2.944	7.804	0.771	28.699	-1.189	4.617	61.478	5.713	11.756	0.365	2.324	-1.098	-1.392	2.289	2.771	6.469
IBM	1.953	17.741	-0.118	2.066	73.549	0.581	0.84	0.479	3.193	-0.301	-0.19	9.692	0.447	0.752	0.809	0.492	-0.222	0.316	0.533	1.045	1.382
JNJ	0.482	17.273	0.303	4.587	80.355	0.287	0.654	0.292	1.369	0.058	0.294	5.132	0.086	0.332	0.429	0.324	0.036	0.258	0.667	0.129	0.227
JPM	2.525	4.823	-0.259	1.45	27.655	1.093	1.345	1.625	0.306	-0.348	0.27	2.926	1.257	1.545	1.538	0.025	-0.402	0.059	-0.366	0.928	0.954
KO	1.083	2.836	-0.273	0.92	16.653	0.484	0.644	0.875	0.167	-0.411	0.148	0.474	0.42	0.664	0.533	-0.118	-0.383	-0.085	-0.409	0.254	0.698
MCD	2.268	150.858	0.264	29.408	243.085	-0.518	1.827	0.172	4.637	-2.144	0.014	15.05	0.181	2.188	1.038	0.937	-1.968	-0.559	3.561	1.731	4.103
MMM	5.024	60.025	0.29	4.105	142.669	2.551	4.232	1.773	11.688	-0.031	2.075	29.4	3.111	5.258	2.078	2.695	0.015	1.807	6.896	2.717	4.249
MRK	2.593	54.368	0.456	14.758	330.683	0.437	1.069	0.524	4.06	-0.394	1.713	21.97	0.272	1.013	0.298	0.271	-0.259	0.268	0.141	0.175	0.577
MSFT	0.092	0.583	-1.013	-1.222	-0.1	-0.411	0.221	0.135	-0.214	-0.946	-0.97	-0.928	-0.515	-0.102	0.294	0.268	-0.866	-0.791	-1.052	-0.259	0.483
NKE	0.38	71.609	-3.52	24.109	407.462	-0.244	1.73	0.599	4.577	-3.611	1.239	13.497	1.325	2.334	-0.9	1.296	-3.575	0.043	0.399	0.113	1.144
NVDA	2.933	0.474	-3.354	1.762	0.876	4.382	8.643	3.177	2.14	-2.966	-0.896	-0.49	4.009	8.428	3.365	1.859	-3.108	-0.303	0.353	3.692	9.042
PG	3.206	55.595	-0.298	1.556	213.709	1.34	1.817	2.495	7.174	-0.319	0.427	20.901	1.356	1.99	0.384	0.062	-0.377	-0.518	1.065	1.216	1.891
SHW	406.605	4890.976	94.241	239.425	2969.602	26.548	45.417	16.615	152.542	1.008	8.99	89.651	31.988	50.675	5.102	14.914	-0.417	4.624	14.502	24.491	38.638
TRV	41.153	601.796	9.574	50.263	899.382	2.377	4.455	3.264	27.091	-0.25	2.166	27.561	1.004	2.081	1.201	2.019	-0.28	0.359	4.014	0.96	1.629
UNH	8.093	192.302	3.507	34.003	525.16	2.947	3.955	2.38	9.636	0.261	1.394	25.038	3.61	4.629	2.842	4.635	0.122	1.799	9.501	3.688	4.194
V	-0.594	21.508	-1.545	5.344	78.466	-0.964	-0.866	-0.631	4.33	-1.474	1.075	12.556	-0.892	-0.872	-0.557	3.785	-1.456	1.573	4.676	-0.818	-0.731
VZ	-4.433	8.509	-5.424	-4.849	8.262	-5.397	-5.134	-4.655	6.39	-5.505	-5.159	-3.531	-5.359	-5.082	-4.698	11.442	-5.487	-5.28	-5.277	-5.193	-4.898
WMT	-0.88	12.993	-1.033	5.873	60.682	-1.319	-0.995	-0.818	0.266	-1.313	0.15	2.873	-1.169	-0.884	-0.81	0.484	-1.233	-0.412	0.205	-1.326	-1.164

This table reports the MSE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.14: Tests of equal accuracy for DJIA stocks (MSE): mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}
AAPL	△	▽	▽	▽	△	△	△	△	△	△	▲	△	△	△	△	▽	▽	▽	△	△
AMGN	△	▽	▽	▽	▽	▽	△	△	△	△	△	△	△	△	△	▽	▽	▽	▽	▽
AMZN	▲	△	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	△	△	▽	▲	▲
AXP	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△
BA	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△	△	△	▽	△	△	△	△
CAT	△	▽	▽	▽	△	△	△	△	▽	▽	△	△	△	△	△	▽	▽	△	△	△
CRM	△	▽	▽	▽	△	△	△	△	▽	▽	△	△	△	△	△	▽	▽	▽	△	△
CSCO	△	▽	▽	△	△	△	△	△	△	△	▲	△	△	△	▽	▽	▽	▽	△	△
CVX	△	▽	▽	▽	△	△	△	△	△	▽	▽	△	△	△	△	▽	▽	△	△	△
DIS	△	▽	▽	△	△	△	△	▽	▽	▽	△	△	△	△	△	▽	△	△	△	△
GS	△	▽	▽	▽	△	△	△	△	▽	▽	△	△	△	△	△	▽	▽	△	△	△
HD	△	△	▽	▽	△	△	△	△	△	△	△	△	△	△	△	△	▽	△	△	△
HON	▽	▽	▽	▽	△	△	△	△	▽	△	△	△	△	▽	△	▽	▽	▽	△	△
IBM	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△	△	△	▽	▽	△	△	△
JNJ	△	△	▽	▽	△	△	△	△	△	▽	△	△	△	△	△	△	△	△	△	△
JPM	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△
KO	△	▽	▽	▽	△	△	△	▽	▽	▽	▽	△	△	△	△	▽	△	▽	△	△
MCD	▽	▽	▽	▽	▽	△	▽	▽	▽	▽	▽	△	△	▽	△	▽	▽	△	△	△
MMM	△	▽	△	△	△	△	△	△	▽	△	△	△	△	△	△	△	△	△	△	△
MRK	△	▽	△	▽	△	△	△	▽	▽	▽	▽	△	△	△	△	▽	△	△	△	△
MSFT	△	▽	▽	▽	▽	▽	△	△	△	△	▲	△	△	△	△	▽	▽	▽	▽	▽
NKE	▽	▽	▽	▽	▽	△	▽	▽	▽	▽	▽	△	△	▽	△	▽	△	△	△	△
NVDA	△	▽	▽	▽	△	△	△	△	△	△	▲	△	△	△	△	▽	▽	▽	△	△
PG	△	▽	▽	△	△	△	△	△	▽	▽	△	△	△	△	△	▽	▽	△	△	△
SHW	△	△	▽	▽	△	△	△	△	△	▽	△	△	△	△	△	▽	▽	△	△	△
TRV	△	▽	▽	▽	▽	△	△	△	▽	△	△	▽	△	△	△	▽	△	△	△	△
UNH	△	△	▽	▽	△	△	△	△	△	▽	▽	△	△	△	△	△	▽	△	△	△
V	▽	▽	▽	▽	▽	▽	▽	△	▽	▽	△	▽	▽	△	△	▽	▽	△	▽	▽
VZ	▽	▽	▽	▽	▽	▽	▽	▽	▽	▽	▽	▽	▽	▽	△	▽	▽	▽	▽	▽
WMT	▽	▽	△	▽	▽	▽	▽	△	▽	△	▽	▽	▽	▽	△	▽	△	△	▽	▽
30	29	30	30	30	30	30	30	30	30	30	30	28	28	28	30	30	29	30	30	29

	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	△	△	▽	▽	▽	△	△	△	△	▽	▽	▽	△	△	△	▽	▽	▽	▽	△	△
AMGN	△	△	▽	△	△	▽	▽	△	△	▽	▽	△	▽	▽	△	△	▽	△	△	▽	△
AMZN	▲	△	▽	△	△	▲	▲	▲	▲	△	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲
AXP	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	▽	△	△	△
BA	△	△	▽	▽	▽	△	△	△	▽	▽	△	△	△	△	△	△	▽	△	△	△	△
CAT	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	△	△	△	△
CRM	△	△	△	△	▲	△	△	△	△	▽	▽	△	△	△	△	△	▽	▽	△	△	△
CSCO	△	△	▽	▽	△	△	△	△	△	▽	△	△	△	△	△	▽	▽	▽	△	△	△
CVX	△	△	▽	△	▲	△	△	△	△	▽	▽	△	△	△	△	△	▽	▽	▽	△	△
DIS	△	△	▽	△	△	△	△	△	△	▽	▽	△	△	△	△	△	▽	△	△	△	△
GS	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	△	△	△	△
HD	△	△	△	△	▲	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△
HON	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	▽	△	△	△
IBM	△	△	▽	△	▲	△	△	△	△	▽	▽	△	△	△	△	△	▽	△	△	△	△
JNJ	△	△	△	△	▲	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△
JPM	△	△	▽	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	△	▽	△	△
KO	△	△	▽	△	▲	△	△	△	△	▽	△	△	△	△	△	▽	▽	▽	▽	△	△
MCD	△	△	△	△	▲	▽	△	△	△	▽	△	△	△	△	△	△	▽	▽	△	△	△
MMM	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	△	△	△	△	△
MRK	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	△	△	△	△
MSFT	△	△	▽	▽	▽	▽	△	△	▽	▽	▽	▽	▽	▽	△	△	▽	▽	▽	▽	△
NKE	△	△	▽	△	▲	▽	△	△	△	▽	△	△	△	△	▽	△	▽	△	△	△	△
NVDA	△	△	▽	△	△	△	△	△	△	▽	▽	▽	△	△	△	△	▽	▽	△	△	△
PG	△	△	▽	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	▽	△	△	△
SHW	▲	▲	▲	▲	▲	△	△	△	▲	△	△	▲	△	△	△	△	▽	△	△	△	△
TRV	△	△	△	△	▲	△	△	△	△	▽	△	△	△	△	△	△	▽	△	△	△	△
UNH	△	△	△	△	▲	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△	△
V	▽	△	▽	△	▲	▽	▽	▽	△	▽	△	△	▽	▽	▽	△	▽	△	△	▽	▽
VZ	▽	△	▽	▽	△	▽	▽	▽	△	▽	▽	▽	▽	▽	▽	△	▽	▽	▽	▽	▽
WMT	▽	△	▽	△	▲	▽	▽	▽	△	▽	△	△	▽	▽	▽	△	▽	▽	△	▽	▽
30	29	29	29	29	29	29	29	28	29	30	29	29	28	29	26	27	30	28	27	28	27

This table reports the MSE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.15: One-step ahead forecast error loss (MSE) $\times 10^8$, mixed frequencies

	$RV^{(390)}_{GRT}$	$RV^{(78)}_{GRT}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV^{AC}_{GRT}	RV^{NW}_{GRT}	RK^{MTH}_{GRT}	RK^{P}_{GRT}	$RV^{(78)}_{N}$	$RV^{(390)}_{N}$	$RBPV_{N}$	PRV_{N}	$PBPV_{N}$	RV^{AC}_{N}	RV^{NW}_{N}	RK^{MTH}_{N}	RK^{P}_{N}
AAPL	1.757	1.636	1.543	1.633	1.633	1.574	1.553	1.719	1.76	1.886	2.11	2.665	1.638	1.649	1.653	2.28	1.794	1.881
AMGN	1.229	1.208	1.152	1.175	1.173	1.189	1.153	1.239	1.258	1.373	2.003	1.686	1.439	1.461	1.654	2.311	1.302	1.331
AMZN	3.108	2.953	2.987	2.964	2.961	2.898	2.835	3.114	3.158	5.915	6.659	8.038	5.541	5.609	6.225	7.113	3.428	3.063
AXP	8.188	9.341	7.431	8.925	8.995	8.268	8.319	9.466	9.72	8.776	9.203	7.924	8.961	9.01	8.727	7.908	9.854	10.144
BA	203.414	211.985	159.397	187.393	187.955	232.206	225.784	177.882	190.937	209.488	219.179	160.025	188.447	188.991	237.776	232.199	180.76	193.481
CAT	4.557	4.445	4.992	4.326	4.306	4.468	4.58	3.683	3.676	4.58	4.898	6.777	4.374	4.357	4.601	5.133	3.676	3.724
CRM	6.167	6.084	5.642	5.97	5.965	5.662	5.205	6.235	6.497	6.216	6.31	7.473	6.068	6.063	5.963	5.532	6.237	6.658
CSCO	0.959	0.954	0.918	0.961	0.96	0.944	0.926	0.988	1.001	0.958	0.948	1.198	0.932	0.931	0.929	0.948	1.005	1.024
CVX	8.809	9.193	11.316	9.876	10.002	9.288	10.403	9.5	10.068	9.136	9.54	11.454	10.003	10.128	9.605	10.92	9.763	10.11
DIS	6.202	6.164	6.717	6.205	6.204	6.14	6.718	6.177	6.475	6.191	6.131	7.457	6.162	6.162	6.29	6.734	6.293	6.546
GS	3.483	3.19	3.89	3.308	3.306	3.08	3.282	3.927	4.033	3.58	3.535	6.97	3.311	3.308	3.031	3.588	3.933	4.066
HD	5.964	5.54	5.057	6.326	6.337	5.621	5.409	6.234	6.992	5.997	5.602	5.16	6.475	6.486	5.851	5.59	6.144	7.732
HON	10.003	10.599	8.135	11.085	11.086	9.496	9.051	15.188	22.049	10.345	11.878	29.632	11.372	11.379	15.098	10.881	18.538	31.68
IBM	3.207	3.185	3.019	3.012	3.011	3.163	3.155	3.013	2.923	3.257	3.402	3.618	3.049	3.048	3.13	3.344	3.036	2.93
JNJ	3.117	2.82	3.214	2.732	2.731	2.898	3.044	2.891	3.133	3.142	2.85	3.253	2.737	2.735	2.943	3.045	2.938	3.179
JPM	6.751	6.539	9.166	6.632	6.662	7.128	7.252	7.1	7.019	6.835	6.563	8.836	6.637	6.679	7.228	7.278	7.21	7.238
KO	4.456	4.174	4.05	3.887	3.914	4.133	4.027	4.679	5.088	4.445	4.039	3.805	3.882	3.909	3.927	3.819	4.592	4.934
MCD	7.354	6.502	5.877	6.2	6.216	6.397	6.331	7.509	8.097	7.19	6.521	7.908	6.27	6.286	6.436	6.546	7.421	8.006
MMM	2.8	2.741	2.731	2.73	2.73	2.747	2.723	2.846	2.892	2.808	2.764	2.82	2.731	2.731	2.76	2.741	2.855	2.892
MRK	6.798	7.114	7.042	6.489	6.486	7.6	7.612	7.056	7.107	6.888	6.649	7.804	6.499	6.495	7.169	6.847	7.181	7.302
MSFT	1.174	1.073	1.053	1.076	1.077	1.106	1.086	1.153	1.14	1.111	1.072	2.23	1	0.999	1.064	1.188	1.161	1.237
NKE	7.792	8.326	6.919	6.938	6.938	8.199	8.393	7.758	7.637	7.818	8.295	6.948	6.939	6.938	8.256	8.407	7.764	7.631
NVDA	18.08	17.489	16.935	17.085	17.094	17.47	17.198	17.56	17.641	19.66	24.326	34.678	17.271	17.356	18.968	29.295	18.615	19.064
PG	7.186	7.401	8.194	6.499	6.488	7.092	7.729	6.851	7.177	7.59	7.65	8.363	6.582	6.571	7.386	7.54	7.144	7.451
SHW	19.109	13.026	12.951	17.619	17.609	12.627	12.09	39.866	49.446	19.745	15.468	17.502	18.756	18.77	12.779	14.075	42.593	53.586
TRV	6.249	8.861	9.55	6.578	6.623	7.951	7.922	5.209	5.254	6.826	11.654	20.503	6.896	6.977	9.236	13.224	4.983	5.047
UNH	6.265	6.042	5.179	6.306	6.303	5.451	5.565	7.403	8.013	6.299	5.977	9.402	6.377	6.37	5.504	5.494	7.798	8.42
V	4.753	4.972	3.989	4.528	4.534	4.8	4.746	4.592	4.603	4.802	4.915	4.396	4.534	4.559	4.823	4.686	4.642	4.651
VZ	4.629	5.606	5.156	3.987	3.983	4.066	4.037	4.007	4.254	4.655	4.146	4.213	4.005	4	4.059	4.07	3.991	4.274
WMT	5.213	5.427	4.74	4.97	4.978	5.374	5.368	5.12	5.084	5.324	5.518	4.6	4.984	4.998	5.485	5.382	5.267	5.241

	$RV^{(78)}_{T}$	$RV^{(390)}_{T}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV^{AC}_{T}	RV^{NW}_{T}	RK^{MTH}_{T}	RK^{P}_{T}	$RV^{(78)}_{K}$	$RV^{(390)}_{K}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV^{AC}_{K}	RV^{NW}_{K}	RK^{MTH}_{K}	RK^{P}_{K}
AAPL	1.746	1.635	1.551	1.631	1.631	1.571	1.558	1.71	1.751	1.754	1.631	1.556	1.633	1.633	1.576	1.55	1.716	1.756
AMGN	1.234	1.212	1.146	1.175	1.173	1.19	1.155	1.243	1.262	1.217	1.216	1.319	1.173	1.172	1.187	1.221	1.263	1.295
AMZN	3.092	2.93	3.341	2.947	2.944	2.882	2.825	3.108	3.163	3.147	2.893	3.853	3.013	3.01	2.96	2.867	3.11	3.136
AXP	8.124	9.414	8.831	9.036	9.086	8.264	8.36	9.476	9.708	8.363	9.595	10.671	7.818	7.841	8.139	11.449	9.434	9.908
BA	201.823	210.023	193.298	188.348	188.873	232.558	229.109	181.383	190.492	189.689	169.989	110.8	176.462	176.82	208.146	176.151	175.735	190.428
CAT	4.642	4.531	5.013	4.326	4.305	4.502	4.579	3.706	3.781	4.589	6.961	10.429	5.197	5.24	10.379	6.932	3.784	4.074
CRM	6.23	6.185	6.217	5.966	5.96	5.601	5.287	6.253	6.663	6.788	8.78	9.238	6.343	6.379	6.799	8.987	6.348	6.787
CSCO	0.957	0.957	0.927	0.962	0.961	0.939	0.923	0.988	1.001	0.966	0.962	0.95	0.961	0.961	0.949	0.914	0.994	1.008
CVX	8.757	9.213	12.004	9.852	9.979	9.614	10.374	9.604	10.122	8.009	8.253	12.916	9.464	9.551	9.659	9.249	8.609	9.232
DIS	6.289	6.171	7.15	6.239	6.238	6.119	6.775	6.254	6.554	5.907	6.096	8.896	6.279	6.276	5.863	6.721	6.201	6.521
GS	3.496	3.287	6.416	3.311	3.31	3.231	3.688	3.892	3.994	3.413	4.991	10.073	3.726	3.748	6.33	5.923	4.782	4.605
HD	6.075	5.597	7.539	6.358	6.37	5.612	5.515	6.248	7.631	5.732	7.14	8.776	5.278	5.304	7.042	7.635	6.481	8.461
HON	10.031	11.031	8.29	11.105	11.106	10.142	9.858	16.011	24.135	9.898	9.91	10.439	9.154	9.214	11.331	9.633	18.323	30.094
IBM	3.191	3.228	3.048	3.052	3.051	3.224	3.137	3.007	2.887	3.347	3.818	4.416	2.946	2.958	2.708	3.662	3.008	2.938
JNJ	3.1	2.777	3.725	2.749	2.749	2.924	3.081	2.936	3.16	3.007	3.638	5.285	2.742	2.751	3.053	4.365	3.159	3.627
JPM	6.81	6.675	8.708	6.647	6.689	7.157	7.324	7.214	7.122	6.795	7.565	22.014	6.701	6.758	7.02	12.331	7.031	6.938
KO	4.535	4.258	3.875	3.886	3.913	4.239	4.082	4.738	5.145	4.446	4.935	11.326	3.957	3.992	3.985	5.98	4.722	4.931
MCD	7.452	6.844	6.384	6.309	6.326	6.221	6.392	7.675	8.591	6.567	15.397	6.779	7.088	7.083	9.501	7.393	6.974	7.691
MMM	2.812	2.735	2.783	2.732	2.731	2.735	2.709	2.866	2.894	2.761	2.692	2.866	2.715	2.714	2.721	2.681	2.877	2.874
MRK	6.751	7.481	8.522	6.514	6.513	8.035	8.231	6.995	7.158	8.52	10.559	7.634	7.31	7.382	8.647	9.212	6.832	7.319
MSFT	1.16	1.07	1.104	1.068	1.069	1.092	1.086	1.143	1.131	1.185	1.051	1.164	1.08	1.08	1.131	1.046	1.159	1.146
NKE	7.741	8.205	6.948	6.933	6.932	8.08	8.099	7.714	7.585	7.437	8.429	7.284	6.94	6.938	7.909	7.743	7.49	7.416
NVDA	18.054	17.495	17.116	17.077	17.085	17.467	17.204	17.551	17.668	18.054	17.513	17.991	17.068	17.072	17.296	17.12	17.499	17.622
PG	6.912	7.496	8.403	6.476	6.469	7.084	7.965	6.17	6.165	8.032	9.006	9.719	6.455	6.521	5.923	9.794	5.921	6.828
SHW	15.718	11.385	11.989	15.979	15.908	11.146	10.416	36.327	47.263	18.094	26.706	52.033	17.128	17.528	18.788	22.631	29.267	41.548
TRV	6.193	8.088	6.562	6.572	6.601	7.491	6.461	5.968	5.762	5.398	6.814	8.218	5.476	5.465	8.557	5.26	5.542	5.718
UNH	6.094	6.295	7.092	6.377	6.375	5.613	6.225	7.247	7.693	6.323	7.197	7.103	6.473	6.46	7.712	7.632	7.504	7.765
V	4.78	4.851	4.476	4.494	4.521	4.813	4.598	4.667	4.67	4.783	6.189	8.156	4.514	4.549	5.575	7.427	4.654	4.873
VZ	5.029	9.618	5.265	4.022	4.019	4.309	4.353	4.005	4.214	4.84	7.988	8.496	4.008	4.013	3.941	4.988	3.971	4.325
WMT	5.244	5.494	5.582	4.98	4.996	5.388	5.487	5.089	5.102	4.927	6.014	7.25	4.75	4.765	5.884	6.847	4.994	5.242

	$RV^{(78)}_{P}$	$RV^{(390)}_{P}$	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$	RV^{AC}_{P}	RV^{NW}_{P}	RK^{MTH}_{P}	RK^{P}_{P}	$RV^{(78)}_{Z}$	$RV^{(390)}_{Z}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV^{AC}_{Z}	RV^{NW}_{Z}	RK^{MTH}_{Z}	RK^{P}_{Z}
AAPL	1.752	1.636	1.564	1.633	1.633	1.575	1.558	1.716	1.758	1.76	1.635	1.596	1.632	1.632	1.573	1.552	1.726	1.767
AMGN	1.256	1.215	1.326	1.174	1.173	1.205	1.208	1.303	1.341	1.249	1.195	1.191	1.169	1.166	1.226	1.145	1.295	1.325
AMZN	3.153	3.66	6.612	2.987	2.985	2.869	4.612	3.16	3.183	3.414	3.885	6.773	2.953	2.954	3.074	4.696	3.314	3.423
AXP	8.109	8.362	7.808	8.75	8.818	8.16	7.876	9.567	10.014	8.312	10.24	16.407	9.021	9.101	8.648	9.575	9.411	9.746
BA	185.401	157.711	94.221	190.045	190.151	208.581	142.378	188.827	191.179	181.626	190.245	155.953	182.685	183.086	240.892	204.215	173.446	176.673
CAT	4.718	5.196	7.276	4.305	4.293	4.398	4.842	3.718	4.233	4.614	4.444	11.634	4.299	4.28	4.457	4.868	3.636	3.705
CRM	5.807	6.079	8.094	5.768	5.76	5.322	5.985	6.352	6.764	6.424	6.148	8.221	5.911	5.905	5.787	5.707	6.611	7.137
CSCO	0.958	0.952	0.921	0.96	0.96	0.94	0.923	0.987	1	0.958	0.954	0.941	0.961	0.961	0.946	0.927	0.989	1.002
CVX	8.141	7.943	8.267	9.22	9.336	7.927	8.334	9.705	10.124	8.584	9.04	9.623	9.794	9.92	8.614	9.317	9.55	9.911
DIS	6.273	6.145	11.03	6.18	6.177	6.211	6.872	6.282	6.527	6.106	6.215	6	6.195	6.194	6.126	6.644	6.198	6.533
GS	3.382	3.648	7.341	3.224	3.22	3.025	3.631	4.272	3.977	3.618	3.341	6.332	3.304	3.303	3.707	3.95	4.277	4.399
HD	5.545	5.335	5.246	6.08	6.096	5.861	5.266	6.751	8.444	6.086	5.739	7.37	6.327	6.349	5.455	5.462	6.415	7.423
HON	8.599	9.333	23.614	9.754	9.699	10.578	8.546	20.326	31.638	10.079	9.741	10.411	11.054	11.04	8.746	8.657	17.149	26.771
IBM	3.048	3.583	4.7	2.865	2.869	2.863	3.632	2.842	2.769	3.324	3.366	4.571	3.025	3.025	3.564	3.476	3.069	2.988
JNJ	2.995	2.927	4.76	2.719	2.721	2.901	3.707	3.017	3.274	3.256	2.904	5.651	2.736	2.736	2.859	3.226	3.022	3.227
JPM	6.646	6.627	13.386	6.537	6.599	6.885	7.725	7.106	7.138	6.836	6.626	8.516	6.655	6.697	7.227	7.299	7.011	6.785
KO	4.644	4.226	5.853	3.856	3.884	4.288	4.435	4.704	5.062	4.448	4.102	5.331	3.886	3.914	4.114	4.067	4.745	5.193
MCD	7.457	5.916	4.691	5.87	5.877	6.145	5.376	7.777	8.548	7.797	6.134	10.361	6.153	6.171	6.007	6.2	8.551	9.588
MMM	2.846	2.853	3.107	2.741	2.743	2.762	2.862	2.87	2.851	2.769	2.762	2.866	2.722	2.722	2.751	2.712	2.868	2.874
MRK	6.912	8.093	12.8	6.521	6.54	8.211	9.697	6.854	7.419	6.945	7.365	7.87	6.443	6.441	7.488	7.699	7.156	7.356
MSFT	1.173	1.072	1.245	1.073	1.074	1.095	1.071	1.156	1.148	1.187	1.074	1.174	1.068	1.069	1.096	1.079	1.165	1.153
NKE	7.838	8.108	7.375	6.931	6.93	8.284	7.868	7.735	7.617	7.551	8.281	7.115	6.929	6.928	7.863	7.973	7.544	7.461
NVDA	18.064	17.515	22.344	17.04	17.047	17.414	17.186	17.517	17.781	18.172	17.659	25.88	17.099	17.105	17.576	17.748	17.615	17.878
PG	8.058	9.895	9.718	6.752	6.773	8.051	9.15	6.591	7.642	6.739	6.932	15.261	6.574	6.569	6.549	8.303	7.497	8.639
SHW	14.948	18.273	31.262	17.502	17.356	13.957	15.917	36.337	45.334	14.169	13.026	17.021	15.726	15.524	14.159	11.101	29.178	35.954
TRV	5.42	6.243	9.055	5.875	5.865	5.059	5.007	5.549	5.643	6.455	8.28	9.047	6.193	6.226	6.351	6.445	6.158	5.557
UNH	6.677	6.27	6.568	6.343	6.333	5.59	5.937	8.712	8.867	6.719	6.083	6.322	6.348	6.345	6.833	5.927	8.142	8.004
V	4.74	5.935	8.092	4.502	4.518	5.418	5.947	4.688	4.736	4.797	5.567	6.681	4.519	4.547	5.129	5.076	4.778	4.872
VZ	4.697	7.522	11.738	3.951	3.957	4.165	4.926	3.971	4.332	4.647	8.766	8.509	3.995	3.991	4.092	4.204	4.151	4.561
WMT	5.166	5.603	7.267	4.967	4.974	5.382	5.862	4.93	5.036	4.975	5.418	5.966	4.979	4.994	5.194	5.331	4.792	4.787

This table reports the MSE losses of the one-step ahead volatility forecasts. The HAR model is used to forecast the sparsely sampled (20-minute) RV obtained from the defragmented prices which acts as the unbiased volatility proxy for day $t+1$. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE (“N”), Nasdaq (“T”), Arca (“P”), Cboe EDGX (“K”), and Cboe BZX (“Z”).

Table F.16: Forecast evaluation: inclusion in MCS (MSE), $h = 1, \ldots, 5$ (mixed frequencies)

	$RV_{GRT}^{(390)}$	$RV_{GRT}^{(78)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	$RBPV_{N}$	PRV_{N}	$PBPV_{N}$	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}
AAPL	5	5	5	5	5	5	5	2	1	5	5	1	5	5	5	5	5	5	5	5	5	5	5	5	5	2	2
AMGN	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
AMZN	5	5	5	5	5	5	5	5	5	5	5	0	5	5	5	2	5	5	5	5	5	5	5	5	5	5	5
AXP	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
BA	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CAT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CRM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CSCO	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CVX	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
DIS	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
GS	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
HD	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
HON	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
IBM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
JNJ	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
JPM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
KO	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MCD	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MMM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MRK	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MSFT	5	5	5	5	5	5	5	5	5	5	5	0	5	5	5	5	5	4	5	5	5	5	5	5	5	5	5
NKE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NVDA	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
PG	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
SHW	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
TRV	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
UNH	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
V	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
VZ	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
WMT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.980	0.973	1.000	1.000	0.900	1.000	1.000	1.000	0.980	1.000	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.980	0.980

	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	5	5	5	5	5	5	5	2	1	5	5	5	5	5	5	5	2	2	5	5	5	5	5	5	4	2	1
AMGN	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
AMZN	5	5	5	5	5	5	5	5	5	5	5	0	5	5	5	3	5	5	5	5	0	5	5	5	3	5	5
AXP	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
BA	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CAT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CRM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CSCO	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CVX	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
DIS	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
GS	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
HD	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
HON	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
IBM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
JNJ	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
JPM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
KO	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MCD	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MMM	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MRK	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
MSFT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5
NKE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NVDA	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
PG	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
SHW	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
TRV	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
UNH	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
V	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
VZ	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
WMT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.980	0.973	1.000	1.000	0.967	1.000	1.000	1.000	0.987	0.980	0.980	1.000	1.000	0.960	1.000	1.000	1.000	0.980	0.980	0.973

This table reports the number of inclusions in the model confidence set. The HAR model is used to forecast the sparsely sampled (20-minute) RV obtained from the defragmented prices which acts as the unbiased volatility proxy for day $t + h$. The forecast error are evaluated with the QLIKE loss function. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.17: Tests of equal accuracy for DJIA stocks (MSE): mixed frequencies, GRT only

	$RV^{(78)}_{GRT}$	PRV_{GRT}	RV^{AC}_{GRT}	RV^{NW}_{GRT}	RK^{MTH}_{GRT}	RK^{P}_{GRT}
AAPL	▲	▽	▽	▽	△	△
AMGN	△	▽	▽	▽	▽	▽
AMZN	▲	△	△	▽	▲	▲
AXP	△	▽	▽	▽	△	△
BA	△	▽	▽	▽	△	△
CAT	△	▽	▽	▽	△	△
CRM	▲	▽	▽	▽	▲	▲
CSCO	△	▽	▽	△	△	△
CVX	△	▽	▽	▽	△	△
DIS	△	▽	▽	△	△	△
GS	△	▽	▽	▽	△	△
HD	△	△	▽	▽	△	△
HON	▽	▽	▽	▽	△	△
IBM	△	▽	▽	▽	△	△
JNJ	△	△	▽	▽	△	△
JPM	△	▽	▽	▽	△	△
KO	△	▽	▽	▽	△	△
MCD	▽	▽	▽	▽	▽	△
MMM	△	▽	△	△	▲	▲
MRK	△	▽	△	▽	△	△
MSFT	△	▽	▽	▽	▽	▽
NKE	▽	▽	▽	▽	▽	△
NVDA	△	▽	▽	▽	△	▲
PG	△	▽	▽	△	△	△
SHW	△	△	▽	▽	△	△
TRV	△	▽	▽	▽	▽	△
UNH	△	△	▽	▽	△	△
V	▽	▽	▽	▽	▽	▽
VZ	▽	▽	▽	▽	▽	▽
WMT	▽	▽	△	▽	▽	▽

This table reports the MSE distances from the benchmark estimator ($RV^{(390)}_{GRT}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days.

F.5 Robustness check – mixed frequencies 1-min/5-min RV vs 1-sec noise-robust estimators (Avg 20-min RV proxy)

Table F.18: Equal accuracy: ΔL (QLIKE), mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_N^{(78)}$	$RV_N^{(390)}$	PRV_N	RV_N^{AC}	RV_N^{NW}	RK_N^{MTH}	RK_N^{P}
AAPL	0.025	0.01	-0.019	-0.027	0.037	0.078	0.029	0.016	0.005	-0.006	0.031	0.113	0.296
AMGN	0.086	0.128	0.05	0.006	0.273	0.507	0.089	0.046	0.125	0.259	0.051	0.763	1.591
AMZN	0.068	0.053	0.014	0.007	0.34	2.13	0.038	0.351	-0.08	0.056	0.494	0.792	0.647
AXP	0.041	0.034	0.01	-0.004	0.1	0.225	0.042	-0.005	0.032	0.016	-0.008	0.124	0.245
BA	0.044	0.03	0.013	-0.011	0.141	0.267	0.04	-0.009	0.027	0.013	-0.022	0.153	0.296
CAT	0.028	0.019	0.007	-0.008	0.097	0.184	0.029	0	0.019	0.01	-0.007	0.107	0.202
CRM	0.048	0.024	0.001	-0.009	0.121	0.227	0.043	-0.003	0.025	0.004	-0.01	0.143	0.273
CSCO	0.045	0.025	0.004	-0.011	0.079	0.131	0.048	0	0.023	0.008	-0.002	0.187	0.334
CVX	0.027	0.012	-0.002	-0.009	0.056	0.116	0.028	0	0.012	-0.002	-0.01	0.063	0.129
DIS	0.046	0.024	0.004	-0.004	0.092	0.17	0.047	0.001	0.024	0.005	-0.004	0.103	0.196
GS	0.032	0.027	0.008	-0.005	0.105	0.209	0.029	-0.005	0.026	0.008	-0.009	0.117	0.255
HD	0.032	0.019	0	-0.012	0.111	0.199	0.033	0.001	0.02	0.007	-0.01	0.136	0.244
HON	0.055	0.072	0.028	-0.005	0.166	0.274	0.059	0.022	0.073	0.064	0.03	0.235	0.47
IBM	0.034	0.035	0.008	-0.01	0.126	0.253	0.035	0.002	0.036	0.011	-0.006	0.135	0.268
JNJ	0.046	0.024	0.007	-0.008	0.078	0.152	0.045	-0.002	0.024	0.005	-0.008	0.085	0.166
JPM	0.033	0.012	-0.002	-0.014	0.062	0.106	0.038	0.002	0.012	-0.001	-0.014	0.073	0.126
KO	0.028	0.01	0.001	-0.005	0.047	0.098	0.027	-0.002	0.011	-0.001	-0.005	0.053	0.114
MCD	0.038	0.028	0.015	0.003	0.117	0.227	0.033	0.001	0.029	0.016	0.005	0.12	0.236
MMM	0.045	0.042	0.005	-0.009	0.121	0.201	0.051	0.007	0.041	0.01	-0.004	0.141	0.226
MRK	0.041	0.022	0.002	-0.007	0.094	0.172	0.047	0.004	0.023	0.008	-0.004	0.111	0.199
MSFT	0.026	0.008	-0.006	-0.008	0.026	0.054	0.023	0.003	0.004	0.025	0.02	0.079	0.166
NKE	0.044	0.02	0.001	-0.006	0.084	0.158	0.046	0.001	0.02	0.002	-0.006	0.088	0.166
NVDA	0.029	0.003	-0.018	-0.027	0.046	0.113	0.039	0.014	-0.003	0.024	0.027	0.156	0.588
PG	0.035	0.014	0.002	-0.006	0.081	0.162	0.034	0.003	0.014	0.002	-0.004	0.097	0.191
SHW	0.134	0.24	0.129	0.017	0.622	1.12	0.18	0.051	0.269	0.265	0.067	0.762	2.801
TRV	0.06	0.074	0.041	-0.002	0.212	0.381	0.058	-0.003	0.073	0.043	-0.004	0.228	0.407
UNH	0.05	0.048	0.017	-0.005	0.203	0.374	0.049	0.001	0.047	0.02	-0.003	0.245	0.429
V	0.046	0.019	-0.003	-0.016	0.111	0.194	0.045	-0.001	0.02	-0.001	-0.013	0.121	0.209
VZ	0.042	0.009	-0.008	-0.012	0.086	0.153	0.045	-0.001	0.01	-0.007	-0.011	0.098	0.176
WMT	0.038	0.011	0.002	-0.007	0.086	0.164	0.039	0.001	0.011	0.003	-0.006	0.09	0.174

	$RV_T^{(78)}$	$RV_T^{(390)}$	PRV_T	RV_T^{AC}	RV_T^{NW}	RK_T^{MTH}	RK_T^{P}	$RV_K^{(78)}$	$RV_K^{(390)}$	PRV_K	RV_K^{AC}	RV_K^{NW}	RK_K^{MTH}	RK_K^{P}
AAPL	0.027	0.001	0.01	-0.018	-0.027	0.04	0.082	0.025	0	0.011	-0.017	-0.028	0.038	0.081
AMGN	0.083	-0.002	0.129	0.05	0.005	0.283	0.6	0.042	0.055	0.107	0.165	0.071	0.272	0.583
AMZN	0.051	-0.026	0.034	0.001	-0.016	0.308	0.581	0.051	-0.058	0.085	0.047	-0.06	0.3	0.555
AXP	0.038	-0.002	0.034	0.009	-0.006	0.105	0.213	0.036	0.049	0.024	0.148	0.069	0.135	0.28
BA	0.035	-0.017	0.025	0.006	-0.029	0.151	0.34	0.047	-0.005	0.039	0.027	-0.016	0.167	0.42
CAT	0.024	-0.006	0.017	0.005	-0.013	0.109	0.21	0.018	0.024	0.02	0.104	0.046	0.143	0.275
CRM	0.049	-0.003	0.023	0	-0.012	0.134	0.361	0.053	0.102	0.027	0.166	0.154	0.174	0.345
CSCO	0.043	-0.006	0.026	-0.003	-0.014	0.076	0.13	0.048	0.002	0.025	0.013	-0.009	0.086	0.14
CVX	0.027	0.002	0.011	-0.001	-0.009	0.061	0.128	0.029	0.007	0.012	0.015	0	0.063	0.128
DIS	0.044	-0.003	0.024	0.003	-0.007	0.093	0.18	0.041	0.004	0.025	0.203	0.003	0.089	0.178
GS	0.027	-0.008	0.023	0.008	-0.008	0.118	0.24	0.021	0.082	0.028	0.134	0.135	0.188	0.354
HD	0.018	-0.006	0.015	-0.008	-0.012	0.102	0.189	0.032	0.042	0.021	0.08	0.066	0.162	0.288
HON	0.048	0.004	0.074	0.054	-0.001	0.164	0.289	0.136	0.342	0.075	0.165	0.355	0.241	0.541
IBM	0.027	-0.004	0.034	0.006	-0.014	0.127	0.249	0.019	0.022	0.03	0.171	0.037	0.158	0.316
JNJ	0.042	0	0.023	0.011	-0.01	0.086	0.178	0.045	0.013	0.025	0.024	0.026	0.115	0.249
JPM	0.031	-0.001	0.011	-0.003	-0.017	0.057	0.101	0.031	0	0.012	-0.001	-0.01	0.068	0.12
KO	0.028	0.001	0.01	0.001	-0.004	0.05	0.106	0.027	0.001	0.01	0.005	0.001	0.054	0.119
MCD	0.038	0.005	0.022	0.029	0.012	0.137	0.273	0.027	0.06	0.025	0.08	0.091	0.189	0.382
MMM	0.032	-0.008	0.04	0	-0.013	0.117	0.208	0.018	0.017	0.034	0.078	0.045	0.156	0.255
MRK	0.037	-0.002	0.021	-0.001	-0.009	0.097	0.184	0.033	0.012	0.019	0.027	0.025	0.123	0.247
MSFT	0.026	0	0.008	-0.006	-0.009	0.027	0.056	0.026	0	0.008	-0.005	-0.008	0.026	0.057
NKE	0.039	-0.004	0.02	0.002	-0.008	0.102	0.193	0.034	0.011	0.024	0.011	0.032	0.123	0.231
NVDA	0.03	-0.002	0.002	-0.019	-0.03	0.05	0.12	0.027	-0.006	0.006	-0.018	-0.029	0.046	0.111
PG	0.033	-0.001	0.014	0	-0.006	0.087	0.168	0.03	0.023	0.013	0.03	0.047	0.134	0.266
SHW	0.014	-0.05	0.171	0.69	-0.064	0.647	1.226	0.34	0.783	0.165	0.262	0.672	1.646	1.642
TRV	0.045	-0.002	0.07	0.275	-0.008	0.237	0.483	0.124	0.44	0.048	0.352	0.42	0.439	0.97
UNH	0.044	0.006	0.043	0.026	-0.002	0.233	0.453	0.043	0.092	0.046	0.187	0.111	0.306	0.721
V	0.042	-0.002	0.018	-0.002	-0.014	0.123	0.217	0.038	0.009	0.018	0.008	0.038	0.147	0.263
VZ	0.038	0.001	0.01	-0.007	-0.012	0.088	0.16	0.04	0	0.008	-0.006	-0.009	0.09	0.168
WMT	0.032	-0.003	0.011	-0.002	-0.01	0.089	0.181	0.042	0.001	0.008	0.577	0.011	0.128	0.255

	$RV_P^{(78)}$	$RV_P^{(390)}$	PRV_P	RV_P^{AC}	RV_P^{NW}	RK_P^{MTH}	RK_P^{P}	$RV_Z^{(78)}$	$RV_Z^{(390)}$	PRV_Z	RV_Z^{AC}	RV_Z^{NW}	RK_Z^{MTH}	RK_Z^{P}
AAPL	0.026	0	0.01	-0.019	-0.028	0.039	0.083	0.023	-0.003	0.01	-0.021	-0.029	0.035	0.076
AMGN	0.023	0.031	0.071	0.072	0.032	0.333	0.645	0.055	0.006	0.106	0.071	0.019	0.325	0.79
AMZN	0.037	-0.06	0.04	0.037	-0.039	0.453	0.682	-0.057	-0.109	0.016	0.015	-0.082	0.448	0.503
AXP	0.026	0.026	0.025	0.032	0.029	0.149	0.416	0.032	-0.002	0.031	0.016	0.001	0.112	0.238
BA	0.017	-0.001	0.01	0.026	0.006	0.194	1.725	0.019	-0.026	0.018	0.006	-0.025	0.173	0.462
CAT	0.035	0.087	0.011	0.056	0.124	0.199	0.397	0.02	-0.004	0.015	0.01	-0.005	0.122	0.231
CRM	0.038	0.02	0.016	0.02	0.026	0.171	0.349	0.037	0.01	0.019	0.014	0.024	0.15	0.303
CSCO	0.049	0.001	0.025	0.003	-0.011	0.088	0.145	0.041	-0.004	0.025	0.002	-0.014	0.077	0.128
CVX	0.027	0.002	0.011	0.019	0.002	0.082	0.173	0.026	-0.001	0.011	-0.002	-0.005	0.054	0.117
DIS	0.041	-0.006	0.023	0.001	-0.009	0.107	0.219	0.043	-0.003	0.024	0.001	-0.008	0.097	0.187
GS	0.016	0.062	0.009	0.06	0.094	0.197	0.589	0.027	0.005	0.018	0.028	0.014	0.141	0.33
HD	0.022	0.041	0.007	0.038	0.065	0.184	0.348	0.026	0.002	0.016	0.007	-0.001	0.139	0.26
HON	0.045	0.236	0.029	0.451	0.269	0.364	0.997	0.046	0.03	0.053	0.042	0.019	0.218	0.424
IBM	0.026	-0.009	0.027	0.025	-0.009	0.196	0.389	0.027	-0.008	0.03	0.007	-0.012	0.143	0.299
JNJ	0.044	0.012	0.023	0.035	0.015	0.115	0.245	0.043	0.006	0.023	0.021	-0.003	0.091	0.182
JPM	0.03	-0.002	0.011	-0.002	-0.013	0.077	0.132	0.029	-0.003	0.011	-0.005	-0.017	0.06	0.105
KO	0.03	0.002	0.01	0	-0.003	0.064	0.133	0.03	0.001	0.01	0.002	-0.003	0.05	0.11
MCD	0.026	0.033	0.018	0.063	0.052	0.202	0.395	0.034	0.015	0.021	0.037	0.028	0.158	0.295
MMM	0.04	0.118	0.025	0.258	0.142	0.201	0.404	0.045	0.003	0.024	0.02	0.012	0.172	0.282
MRK	0.037	-0.003	0.018	0.021	-0.002	0.135	0.271	0.031	-0.006	0.019	0.002	-0.01	0.093	0.181
MSFT	0.026	-0.001	0.007	-0.006	-0.01	0.031	0.064	0.025	-0.002	0.007	-0.006	-0.01	0.029	0.062
NKE	0.035	-0.013	0.017	0.005	-0.003	0.122	0.223	0.035	-0.006	0.016	0.001	-0.011	0.105	0.198
NVDA	0.024	-0.013	0.004	-0.016	-0.037	0.062	0.141	0.019	-0.013	0	-0.027	-0.039	0.05	0.123
PG	0.033	0.007	0.012	0.015	0.015	0.123	0.252	0.034	0.004	0.013	0.009	0.008	0.103	0.203
SHW	0.025	0.349	-0.018	0.198	0.228	1.462	2.817	0.038	0.073	0.131	0.162	0.047	0.89	1.784
TRV	0.06	0.28	0.03	0.95	0.24	3.109	0.989	0.042	0.008	0.056	0.078	0.012	0.257	0.496
UNH	0.022	0.052	0.036	0.076	0.059	0.329	0.588	0.044	0.018	0.035	0.386	0.033	0.255	0.494
V	0.027	-0.002	0.008	-0.006	0.005	0.153	0.273	0.033	0.013	0.012	0.004	0.035	0.153	0.273
VZ	0.042	-0.001	0.009	-0.008	-0.014	0.11	0.198	0.039	0	0.009	-0.007	-0.012	0.08	0.144
WMT	0.029	-0.004	0.006	0.813	0.003	0.12	0.249	0.032	0.001	0.009	0.008	0.009	0.111	0.211

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). The average of the sparsely sampled (20-minute) RV obtained from all five exchanges serves as the proxy for the daily integrated variance. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.19: Tests of equal accuracy for DJIA stocks (QLIKE): mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	▲	△	▽	▽	▲	▲	▲	▲	△	▽	▲	▲	▲	▲	△	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
AMGN	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▽	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	△	▲	▲
AMZN	△	△	△	△	▲	▲	△	▲	▽	△	▲	▲	▲	△	▽	△	△	▽	▲	▲	△	▽	▲	△	▽	▲	▲	△	▽	△	△	▽	▲	▲	▽	▽	△	△	▽	▲	▲
AXP	▲	▲	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▲	△	▲	▲	▲	▲	△	△	△	△	△	▲	▲	△	▽	△	△	△	▲	▲
BA	△	△	△	▽	▲	▲	△	▽	△	△	▽	▲	▲	△	▽	△	△	▽	▲	▲	△	▽	△	△	▽	▲	▲	△	▽	△	△	△	▲	▲	△	▽	△	△	▽	▲	▲
CAT	▲	△	△	▽	▲	▲	▲	△	△	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▽	△	△	▽	▲	▲
CRM	▲	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▽	▲	▽	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	△	△	▲	▲	▲	▲	△	△	△	▲	▲	▲
CSCO	▲	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	▽	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲
CVX	△	△	▽	▽	▲	▲	△	▽	△	▽	▽	▲	▲	△	△	△	▽	▽	▲	▲	△	△	△	△	▽	▲	▲	△	△	△	△	△	▲	▲	△	▽	△	▽	▽	▲	▲
DIS	▲	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	△	▲	▲	△	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲
GS	▲	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲
HD	▲	△	▽	▽	▲	▲	▲	△	△	△	▽	▲	▲	△	▽	△	▽	▽	▲	▲	▲	▲	△	▲	▲	▲	▲	△	▲	△	▲	▲	▲	▲	▲	△	△	△	▽	▲	▲
HON	▲	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
IBM	▲	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▽	▲	▲	▽	▲	▲	▲	▽	▲	△	▽	▲	▲
JNJ	▲	△	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲	▲	△	△	△	△	▲	▲	▲	△	△	▲	△	▲	▲	▲	△	△	△	▽	▲	▲
JPM	▲	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
KO	△	△	△	▽	▲	▲	△	▽	△	▽	▽	▲	▲	△	△	△	△	▽	▲	▲	△	△	△	△	△	▲	▲	△	△	△	△	▽	▲	▲	△	△	△	△	▽	▲	▲
MCD	▲	△	△	△	▲	▲	▲	△	△	△	△	▲	▲	▲	△	△	△	△	▲	▲	△	▲	△	▲	▲	▲	▲	△	▲	△	▲	▲	▲	▲	▲	△	△	▲	△	▲	▲
MMM	▲	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	△	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	△	△	▲	▲
MRK	▲	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	▽	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▽	▲	▲	▽	▲	▲	▲	▽	▲	△	▽	▲	▲
MSFT	▲	△	▽	▽	▲	▲	▲	△	△	▲	▲	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
NKE	▲	▲	△	▽	▲	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	△	▲	△	▲	▲	▲	▲	▽	▲	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲
NVDA	▲	△	▽	▽	▲	▲	▲	▲	▽	▲	▲	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
PG	▲	△	△	▽	▲	▲	▲	△	△	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	△	△	△	▲	▲	▲	△	△	△	△	▲	▲
SHW	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▽	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
TRV	▲	▲	▲	▽	▲	▲	▲	▽	▲	▲	▽	▲	▲	▲	▽	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	△	▲	▲
UNH	▲	▲	▲	▽	▲	▲	▲	△	▲	▲	▽	▲	▲	▲	△	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
V	▲	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	△	△	△	▲	▲	▲	▲	▽	△	▽	△	▲	▲	▲	△	△	△	▲	▲	▲
VZ	▲	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲
WMT	△	△	△	▽	△	△	△	△	△	△	▽	△	△	△	▽	△	▽	▽	△	△	△	△	△	△	△	△	△	△	▽	△	△	△	△	△	△	△	△	△	△	△	△
22	6	30	22	27	5	7	10	21	27	22	24	5	3	3	15	28	14	17	3	3	10	23	28	23	27	3	4	4	15	26	12	19	3	3	1	8	27	8	7	3	3

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The average of the sparsely sampled (20-minute) RV obtained from all five exchanges serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.20: Forecast evaluation: inclusion in MCS (QLIKE), $h = 1, \ldots, 5$ (mixed frequencies)

	$RV_{GRT}^{(390)}$	$RV_{GRT}^{(78)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	$RBPV_{N}$	PRV_{N}	$PBPV_{N}$	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}
AAPL	5	5	5	5	5	5	5	2	1	5	5	0	5	5	5	4	5	4
AMGN	2	0	5	0	0	0	0	1	4	1	4	4	0	1	4	4	4	4
AMZN	0	0	5	2	2	2	2	5	5	2	0	0	5	5	0	0	5	5
AXP	3	3	5	4	4	1	4	1	0	1	2	5	4	4	1	5	0	0
BA	0	0	5	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
CAT	4	3	3	4	4	3	3	3	3	4	3	2	4	4	3	3	3	3
CRM	5	5	5	3	3	0	5	5	4	5	5	5	2	2	1	5	5	4
CSCO	2	2	5	0	3	1	4	0	0	4	5	0	0	5	5	5	0	1
CVX	5	4	5	5	5	5	5	4	4	5	5	5	5	5	5	5	4	4
DIS	4	5	5	5	5	4	5	4	4	5	5	5	5	5	4	5	5	4
GS	1	5	0	5	5	1	2	3	4	2	4	0	5	5	3	4	4	4
HD	5	5	5	4	5	5	5	5	3	5	5	4	4	5	5	5	4	4
HON	3	5	5	4	5	5	5	2	3	0	4	5	0	0	0	4	0	0
IBM	4	2	5	2	2	1	2	2	2	4	2	5	2	2	1	2	4	2
JNJ	2	2	5	1	1	2	2	0	0	3	2	4	1	1	2	2	0	0
JPM	4	5	4	5	5	5	1	4	4	4	5	4	5	5	5	5	4	4
KO	2	4	5	5	5	4	5	1	0	4	5	5	5	5	5	5	3	2
MCD	2	5	5	5	5	5	5	1	0	4	5	4	5	5	5	5	1	0
MMM	2	0	5	0	0	0	0	4	4	3	0	4	0	0	0	0	4	4
MRK	3	5	5	5	5	5	5	2	0	3	5	5	5	5	5	5	1	0
MSFT	5	5	5	5	5	5	5	5	5	5	5	0	5	5	5	5	5	3
NKE	1	0	5	5	5	0	0	0	0	1	0	5	5	5	0	0	0	0
NVDA	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5
PG	0	2	2	4	4	1	2	0	0	0	1	5	4	4	1	2	0	0
SHW	1	3	5	0	0	0	4	0	0	0	0	0	0	0	0	2	0	0
TRV	5	5	5	5	5	5	4	4	4	5	5	5	5	5	5	5	4	4
UNH	5	5	5	4	5	5	5	1	1	5	4	4	0	0	5	5	1	1
V	0	1	5	2	0	2	3	0	0	0	1	5	1	0	2	3	0	0
VZ	5	5	5	5	5	3	4	5	1	5	5	5	5	5	4	1	5	1
WMT	4	3	5	0	0	3	3	4	3	3	2	5	0	0	1	3	1	1
	0.593	0.660	0.927	0.660	0.687	0.553	0.667	0.487	0.427	0.620	0.660	0.700	0.613	0.653	0.580	0.693	0.513	0.427

	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}
AAPL	5	5	5	0	0	5	5	2	1	5	5	5	5	5	5	5	2	1
AMGN	2	0	5	0	0	0	0	1	4	3	3	4	0	0	0	4	4	4
AMZN	5	5	4	5	5	2	5	5	5	2	5	4	0	0	1	5	5	5
AXP	3	2	2	4	2	1	0	1	0	4	5	4	5	5	5	5	0	0
BA	0	0	5	0	0	0	0	0	0	5	5	5	4	4	0	5	0	0
CAT	4	2	2	4	4	2	3	3	2	5	3	2	5	5	3	3	3	2
CRM	5	5	5	5	5	3	5	4	4	5	4	3	5	5	5	4	5	4
CSCO	2	4	5	0	3	2	4	0	0	2	2	5	0	3	0	4	0	0
CVX	5	4	5	5	5	5	5	4	4	5	5	4	5	5	5	5	4	4
DIS	3	5	5	5	5	5	5	4	4	5	5	5	5	5	5	5	5	4
GS	0	3	0	5	5	3	0	1	4	0	4	0	5	5	5	5	4	4
HD	2	5	0	5	5	3	3	4	3	5	5	3	5	5	5	5	3	1
HON	3	5	5	4	5	5	5	2	3	4	4	4	5	5	1	4	4	4
IBM	3	4	5	1	1	1	3	2	2	5	5	3	4	5	5	5	4	3
JNJ	2	4	1	1	1	2	3	0	0	5	5	5	5	5	5	4	0	0
JPM	4	5	0	4	3	4	1	4	4	4	5	2	5	5	5	5	1	4
KO	4	3	5	5	5	1	4	0	0	5	5	0	5	5	5	5	3	1
MCD	0	0	2	5	5	5	3	0	0	5	5	5	5	5	5	5	2	0
MMM	3	0	5	0	0	0	0	4	4	4	5	5	0	0	1	5	4	4
MRK	3	5	4	5	5	4	3	0	0	4	4	2	5	5	5	4	0	0
MSFT	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NKE	4	0	5	5	5	1	5	1	2	5	5	5	5	5	5	5	1	5
NVDA	5	5	5	5	5	5	0	5	5	5	5	5	5	5	5	5	5	5
PG	0	2	2	4	4	1	1	0	0	1	5	0	5	5	5	2	0	0
SHW	4	3	3	0	2	0	3	1	1	4	5	5	5	5	4	5	1	0
TRV	1	1	1	5	5	0	1	0	0	4	4	5	5	5	4	5	0	0
UNH	5	4	3	5	5	5	5	3	3	5	5	5	5	5	5	5	4	3
V	0	1	2	1	0	2	2	0	0	1	5	0	5	5	1	2	0	0
VZ	5	5	4	5	5	2	3	5	2	5	5	3	5	5	5	5	5	2
WMT	4	3	2	0	0	4	3	4	3	5	5	4	5	5	4	4	4	2
	0.607	0.633	0.680	0.653	0.667	0.520	0.567	0.433	0.433	0.813	0.920	0.713	0.853	0.880	0.760	0.900	0.520	0.447

	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	5	5	4	5	3	5	5	2	0	4	5	5	5	5	5	5	2	0
AMGN	2	0	4	0	0	0	0	3	4	3	0	4	0	0	3	4	4	4
AMZN	5	4	0	4	5	5	0	5	5	5	4	0	3	5	5	0	5	5
AXP	5	2	3	4	5	3	5	3	3	3	2	0	3	3	2	1	1	0
BA	4	5	2	0	0	0	5	0	0	5	2	2	3	3	0	1	0	0
CAT	2	2	0	4	4	2	2	3	2	2	0	0	2	2	0	0	2	2
CRM	5	5	0	5	5	4	5	4	4	5	5	4	5	5	1	5	4	4
CSCO	2	4	5	0	3	0	4	0	0	2	3	5	0	3	2	4	0	0
CVX	5	5	4	5	5	5	5	4	4	5	4	0	5	5	5	3	4	4
DIS	5	5	5	5	5	4	5	3	3	5	5	5	5	5	3	5	4	4
GS	0	0	0	2	2	4	0	4	4	2	0	0	4	4	0	0	3	4
HD	5	5	3	5	5	5	5	3	1	4	5	3	5	5	5	5	4	3
HON	3	3	3	5	5	2	2	0	0	4	5	5	0	5	4	5	0	1
IBM	3	5	4	2	2	1	5	3	3	1	0	0	2	2	0	0	2	1
JNJ	3	3	0	1	1	3	1	0	0	0	0	0	1	1	0	1	0	0
JPM	4	5	4	5	5	5	5	4	4	4	5	2	5	5	5	2	4	4
KO	4	5	5	5	5	5	5	3	1	1	4	4	5	5	4	5	1	0
MCD	1	2	4	5	5	5	5	0	0	0	1	5	5	5	3	2	0	0
MMM	4	5	5	2	4	4	5	4	4	2	2	5	0	0	0	0	4	4
MRK	3	5	4	5	5	3	5	1	0	1	0	0	5	5	4	1	0	0
MSFT	5	0	3	5	5	4	4	5	5	5	5	5	5	5	5	5	5	5
NKE	5	2	5	5	5	2	5	1	5	2	1	5	5	5	4	5	0	1
NVDA	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5
PG	0	0	0	5	5	0	0	0	0	0	2	0	5	5	1	0	0	0
SHW	3	2	1	0	1	1	0	3	0	0	0	0	0	0	0	0	0	0
TRV	1	1	2	5	5	0	1	0	0	1	0	0	5	5	0	0	0	0
UNH	4	4	5	5	5	4	5	0	0	4	4	5	5	5	4	5	3	3
V	1	0	0	1	2	0	0	0	0	0	0	0	1	1	0	0	0	0
VZ	5	5	1	5	5	4	5	5	1	5	5	5	5	5	3	4	5	0
WMT	4	3	2	4	4	3	3	4	2	4	2	1	0	0	1	0	4	3
	0.687	0.647	0.553	0.727	0.773	0.587	0.680	0.480	0.400	0.560	0.507	0.493	0.660	0.727	0.493	0.487	0.440	0.380

This table reports the number of inclusions in the model confidence set. The HAR model is used to forecast the average sparsely sampled (20-minute) RV obtained from all exchanges which acts as the unbiased volatility proxy for day $t + h$. The forecast error are evaluated with the QLIKE loss function. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.21: Tests of equal accuracy for DJIA stocks (QLIKE): mixed frequencies, GRT only

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}
AAPL	▲	△	▼	▼	▲	▲
AMGN	▲	▲	▲	△	▲	▲
AMZN	△	△	△	△	△	△
AXP	▲	▲	△	▽	▲	▲
BA	▲	△	△	▽	▲	▲
CAT	▲	▲	△	▽	▲	▲
CRM	▲	▲	△	▽	▲	▲
CSCO	▲	▲	△	▽	▲	▲
CVX	△	△	▽	▽	▲	▲
DIS	▲	▲	△	▽	▲	▲
GS	▲	▲	△	▽	▲	▲
HD	▲	△	▽	▽	▲	▲
HON	▲	▲	△	▽	▲	▲
IBM	▲	▲	△	▽	▲	▲
JNJ	▲	▲	△	▽	▲	▲
JPM	▲	△	▽	▽	▲	▲
KO	△	△	△	▽	▲	▲
MCD	▲	△	△	△	▲	▲
MMM	▲	▲	△	▽	▲	▲
MRK	▲	▲	△	▽	▲	▲
MSFT	▲	△	▽	▽	▲	▲
NKE	▲	▲	△	▽	▲	▲
NVDA	▲	△	▽	▽	▲	▲
PG	▲	△	△	▽	▲	▲
SHW	△	△	△	△	△	△
TRV	▲	▲	▲	▽	▲	▲
UNH	▲	▲	▲	▽	▲	▲
V	▲	△	▽	▽	▲	▲
VZ	▲	△	▽	▽	▲	▲
WMT	▲	△	△	▽	▲	▲

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The average of the sparsely sampled (20-minute) RV obtained from all five exchanges serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

F.6 Robustness check – mixed frequencies 1-min/5-min RV vs 1-sec noise-robust estimators (squared open-to-close returns)

Table F.22: Equal accuracy: ΔL (QLIKE), mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}
AAPL	0.018	0	-0.03	-0.044	0.047	0.099	0.018	0.004	-0.009	-0.003	-0.003	0.147	0.458
AMGN	0.057	0.067	0.02	-0.002	0.184	0.372	0.104	0.102	0.08	0.258	0.117	0.561	1.288
AMZN	0.036	0.016	0	-0.008	0.137	0.687	0.428	0.769	0.282	0.381	0.919	0.629	0.805
AXP	0.042	0.034	0.006	-0.008	0.118	0.262	0.042	-0.003	0.032	0.01	-0.008	0.144	0.277
BA	0.08	0.059	0.046	0	0.256	0.341	0.074	-0.012	0.055	0.054	-0.013	0.276	0.383
CAT	0.042	0.026	0.009	-0.006	0.107	0.187	0.042	-0.002	0.026	0.01	-0.007	0.117	0.21
CRM	0.041	0.024	0.003	-0.01	0.111	0.217	0.041	0	0.024	0.005	-0.008	0.136	0.232
CSCO	0.049	0.03	0	-0.015	0.088	0.148	0.057	0.001	0.028	0.004	-0.007	0.217	0.397
CVX	0.031	0.016	-0.002	-0.009	0.07	0.133	0.032	0.001	0.017	-0.001	-0.009	0.075	0.142
DIS	0.046	0.029	0.004	-0.007	0.095	0.176	0.047	0.001	0.029	0.005	-0.007	0.107	0.2
GS	0.031	0.023	0.007	-0.008	0.103	0.201	0.028	-0.003	0.022	0.008	-0.009	0.116	0.253
HD	0.037	0.027	0.005	-0.008	0.111	0.195	0.036	-0.001	0.028	0.007	-0.008	0.128	0.232
HON	0.048	0.061	0.026	-0.007	0.155	0.265	0.057	0.028	0.063	0.061	0.034	0.234	0.475
IBM	0.036	0.03	0.005	-0.01	0.109	0.207	0.035	0	0.031	0.007	-0.008	0.117	0.22
JNJ	0.05	0.037	0.007	-0.01	0.101	0.189	0.05	0	0.037	0.007	-0.008	0.113	0.211
JPM	0.032	0.014	-0.001	-0.011	0.066	0.118	0.036	0.001	0.015	0	-0.011	0.075	0.133
KO	0.035	0.021	0	-0.009	0.07	0.133	0.039	0	0.022	0.002	-0.007	0.081	0.155
MCD	0.04	0.029	0.011	-0.001	0.122	0.236	0.037	0.001	0.03	0.012	0.001	0.126	0.248
MMM	0.042	0.035	0.006	-0.006	0.107	0.176	0.044	0.003	0.035	0.009	-0.002	0.117	0.193
MRK	0.045	0.028	0	-0.009	0.097	0.179	0.049	0.003	0.029	0.004	-0.006	0.113	0.204
MSFT	0.031	0.01	-0.009	-0.013	0.041	0.073	0.026	0	0.005	0.015	0.01	0.095	0.184
NKE	0.036	0.022	0.003	-0.004	0.076	0.139	0.038	0	0.022	0.004	-0.004	0.08	0.146
NVDA	0.031	0.004	-0.011	-0.017	0.047	0.107	0.047	0.036	0.005	0.035	0.063	0.129	0.527
PG	0.048	0.026	0.004	-0.006	0.099	0.186	0.049	0.004	0.027	0.005	-0.003	0.121	0.225
SHW	0.072	0.126	0.063	0.001	0.412	0.837	0.091	0.021	0.139	0.148	0.02	0.497	2.385
TRV	0.05	0.058	0.029	-0.001	0.194	0.358	0.048	0	0.059	0.032	-0.001	0.213	0.399
UNH	0.048	0.045	0.016	-0.005	0.183	0.344	0.046	-0.001	0.043	0.017	-0.005	0.213	0.393
V	0.044	0.027	0	-0.011	0.103	0.19	0.045	0	0.027	0.002	-0.009	0.117	0.21
VZ	0.042	0.017	-0.006	-0.011	0.086	0.163	0.045	0	0.017	-0.003	-0.009	0.099	0.19
WMT	0.039	0.02	0.001	-0.007	0.093	0.174	0.041	0.003	0.021	0.005	-0.004	0.1	0.188

	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}
AAPL	0.02	0.001	0.001	-0.028	-0.044	0.049	0.101	0.019	0	0.001	-0.028	-0.045	0.049	0.105
AMGN	0.057	0	0.067	0.02	-0.002	0.191	0.433	0.042	0.116	0.056	0.114	0.13	0.223	0.56
AMZN	0.032	-0.004	0.011	-0.003	-0.012	0.122	0.264	0.036	-0.004	0.026	0.017	-0.011	0.119	0.246
AXP	0.039	-0.001	0.034	0.007	-0.008	0.121	0.237	0.036	0.046	0.027	0.107	0.061	0.155	0.304
BA	0.073	-0.021	0.053	0.04	-0.025	0.23	0.375	0.075	-0.007	0.069	0.049	-0.014	0.285	0.699
CAT	0.034	-0.006	0.024	0.006	-0.011	0.116	0.214	0.031	0.031	0.028	0.088	0.051	0.155	0.339
CRM	0.037	-0.003	0.023	0.002	-0.012	0.122	0.92	0.045	0.096	0.023	0.148	0.145	0.156	0.308
CSCO	0.056	-0.004	0.032	-0.002	-0.015	0.093	0.155	0.05	-0.003	0.03	0.004	-0.016	0.091	0.152
CVX	0.03	-0.001	0.016	-0.001	-0.011	0.075	0.146	0.03	0.003	0.017	0.012	-0.005	0.075	0.148
DIS	0.043	-0.003	0.029	0.003	-0.011	0.097	0.181	0.041	0.001	0.03	0.167	-0.003	0.094	0.181
GS	0.027	-0.003	0.02	0.008	-0.007	0.116	0.233	0.02	0.095	0.022	0.118	0.141	0.177	0.331
HD	0.028	-0.002	0.024	0.004	-0.006	0.114	0.202	0.034	0.038	0.031	0.099	0.063	0.159	0.286
HON	0.044	0	0.062	0.046	-0.007	0.156	0.281	0.145	0.359	0.083	0.211	0.372	0.241	0.536
IBM	0.032	-0.003	0.029	0.007	-0.013	0.115	0.219	0.024	0.029	0.027	0.363	0.045	0.136	0.275
JNJ	0.043	-0.004	0.036	0.011	-0.016	0.109	0.217	0.044	0.006	0.036	0.022	0.014	0.14	0.291
JPM	0.031	-0.001	0.014	-0.002	-0.012	0.064	0.117	0.033	0	0.015	-0.002	-0.008	0.076	0.138
KO	0.034	0	0.021	0	-0.01	0.072	0.139	0.031	-0.003	0.02	0.001	-0.009	0.075	0.151
MCD	0.04	0.004	0.023	0.029	0.005	0.148	0.412	0.028	0.045	0.027	0.08	0.075	0.192	0.378
MMM	0.036	0	0.032	0.006	-0.003	0.109	0.189	0.028	0.038	0.029	0.075	0.069	0.146	0.251
MRK	0.04	-0.003	0.028	-0.002	-0.012	0.099	0.188	0.041	0.012	0.025	0.028	0.021	0.137	0.261
MSFT	0.031	0	0.01	-0.009	-0.013	0.042	0.074	0.03	-0.001	0.01	-0.009	-0.014	0.041	0.075
NKE	0.032	-0.003	0.021	0.005	-0.006	0.089	0.166	0.028	0.009	0.022	0.015	0.027	0.108	0.199
NVDA	0.032	-0.002	0.004	-0.012	-0.02	0.049	0.112	0.031	-0.002	0.007	-0.009	-0.017	0.046	0.101
PG	0.047	0.001	0.026	0.004	-0.005	0.11	0.204	0.043	0.019	0.025	0.036	0.038	0.152	0.304
SHW	0.049	0.071	0.086	0.462	0.034	0.475	0.93	0.528	1	0.297	0.428	0.886	1.867	1.386
TRV	0.045	0.015	0.054	0.425	0.008	0.217	0.449	0.149	0.482	0.048	0.385	0.462	0.42	1.061
UNH	0.042	0.004	0.041	0.028	-0.001	0.211	0.415	0.043	0.086	0.039	0.177	0.103	0.29	0.635
V	0.036	-0.003	0.025	-0.002	-0.011	0.106	0.217	0.036	0.007	0.028	0.015	0.038	0.142	0.258
VZ	0.04	-0.001	0.017	-0.007	-0.014	0.089	0.171	0.038	-0.003	0.016	-0.005	-0.012	0.091	0.174
WMT	0.03	-0.007	0.02	-0.004	-0.015	0.096	0.189	0.039	-0.009	0.018	0.637	-0.004	0.134	0.265

	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	0.019	-0.001	0.001	-0.031	-0.046	0.049	0.104	0.017	-0.001	0	-0.028	-0.044	0.045	0.096
AMGN	0.042	0.087	0.032	0.061	0.083	0.257	0.526	0.08	0.07	0.053	0.087	0.081	0.255	0.702
AMZN	0.028	0.019	0.012	0.014	0.025	0.187	0.384	0.045	0.084	0.008	0.078	0.101	0.244	0.532
AXP	0.031	0.025	0.025	0.029	0.027	0.175	0.416	0.034	0.001	0.029	0.013	0.001	0.139	0.271
BA	0.018	-0.045	0.022	0.031	-0.028	0.274	1.709	0.031	-0.052	0.039	-0.004	-0.042	0.206	0.417
CAT	0.037	0.083	0.014	0.053	0.122	0.194	0.412	0.037	0.002	0.022	0.013	0.001	0.127	0.232
CRM	0.032	0.018	0.016	0.037	0.025	0.198	0.298	0.031	0.009	0.016	0.014	0.018	0.136	0.271
CSCO	0.054	0	0.031	0	-0.015	0.099	0.164	0.047	-0.003	0.031	-0.001	-0.017	0.087	0.147
CVX	0.026	-0.001	0.015	0.011	-0.003	0.089	0.187	0.029	-0.001	0.016	-0.003	-0.008	0.067	0.133
DIS	0.041	-0.007	0.027	0.002	-0.013	0.11	0.215	0.043	-0.005	0.028	0	-0.012	0.1	0.198
GS	0.021	0.077	0.009	0.08	0.102	0.187	0.499	0.033	0.016	0.015	0.033	0.019	0.135	0.321
HD	0.029	0.046	0.016	0.054	0.072	0.18	0.352	0.033	0.001	0.023	0.009	0.002	0.143	0.265
HON	0.059	0.245	0.025	0.407	0.275	0.36	0.892	0.046	0.039	0.047	0.044	0.026	0.209	0.41
IBM	0.03	0	0.024	0.025	0.004	0.17	0.346	0.031	-0.003	0.026	0.006	-0.004	0.122	0.243
JNJ	0.039	0.002	0.033	0.019	0.004	0.137	0.285	0.046	0.001	0.035	0.014	-0.009	0.114	0.221
JPM	0.031	-0.002	0.014	-0.001	-0.011	0.082	0.147	0.029	-0.003	0.014	-0.003	-0.013	0.067	0.121
KO	0.035	0	0.021	0.001	-0.011	0.087	0.168	0.037	-0.001	0.021	0.002	-0.01	0.073	0.143
MCD	0.023	0.026	0.017	0.118	0.039	0.194	0.377	0.033	0.004	0.021	0.038	0.016	0.163	0.312
MMM	0.043	0.126	0.02	0.209	0.156	0.186	0.397	0.041	0.012	0.023	0.036	0.026	0.15	0.259
MRK	0.042	-0.001	0.025	0.017	-0.003	0.143	0.277	0.037	-0.003	0.026	-0.002	-0.009	0.098	0.193
MSFT	0.03	-0.001	0.009	-0.009	-0.015	0.047	0.084	0.03	-0.003	0.009	-0.01	-0.016	0.046	0.084
NKE	0.028	-0.011	0.018	0.009	-0.003	0.111	0.194	0.028	-0.006	0.018	-0.003	-0.012	0.092	0.174
NVDA	0.028	-0.004	0.005	-0.008	-0.019	0.061	0.13	0.027	-0.005	0.003	-0.012	-0.017	0.052	0.115
PG	0.046	0.004	0.024	0.02	0.008	0.152	0.306	0.041	-0.002	0.024	0.005	0	0.114	0.222
SHW	0.143	0.536	0.007	0.272	0.402	1.986	2.629	0.094	0.198	0.073	0.171	0.151	0.663	1.331
TRV	0.085	0.323	0.032	0.782	0.282	1.926	0.998	0.05	0.037	0.044	0.07	0.036	0.244	0.49
UNH	0.026	0.048	0.029	0.068	0.057	0.293	0.543	0.041	0.021	0.032	0.198	0.033	0.239	0.486
V	0.023	-0.005	0.013	-0.003	0.006	0.141	0.263	0.036	0.008	0.02	0.014	0.033	0.149	0.283
VZ	0.043	-0.002	0.016	-0.006	-0.014	0.111	0.208	0.037	-0.002	0.017	-0.009	-0.015	0.079	0.152
WMT	0.029	-0.013	0.012	0.706	-0.011	0.131	0.26	0.032	-0.006	0.017	0.004	-0.004	0.114	0.213

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). The squared daily return of the estimated efficient price serves as the proxy for the daily integrated variance. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.23: Tests of equal accuracy for DJIA stocks (QLIKE): mixed frequencies

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	PRV_{N}	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}
AAPL	△	△	▽	▽	▲	▲	△	△	▽	▽	▽	▲	▲
AMGN	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	△	△	▽	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲
AXP	△	△	△	▽	▲	▲	△	▽	△	△	▽	▲	▲
BA	▲	△	△	△	▲	▲	▲	▽	△	△	▽	▲	▲
CAT	△	△	▽	▽	▲	▲	△	▽	△	△	▽	▲	▲
CRM	▲	△	▽	▽	▲	▲	▲	▽	△	△	▽	▲	▲
CSCO	▲	△	▽	▽	▲	▲	▲	△	△	▽	△	▲	▲
CVX	▲	△	△	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
DIS	▲	△	△	▽	▲	▲	▲	△	△	△	▽	▲	▲
GS	▲	△	△	▽	▲	▲	▲	▽	△	△	△	▲	▲
HD	▲	△	▽	▽	▲	▲	▲	△	△	△	▽	▲	▲
HON	△	△	△	▽	▲	▲	△	△	△	△	△	▲	▲
IBM	△	△	△	▽	▲	▲	△	△	△	△	▽	▲	▲
JNJ	△	△	△	▽	▲	▲	△	▽	△	△	▽	▲	▲
JPM	▲	△	△	▽	▲	▲	▲	△	△	△	▽	▲	▲
KO	△	△	▽	▽	△	▲	△	▽	△	▽	▽	△	▲
MCD	▲	△	△	△	▲	▲	△	△	△	△	△	▲	▲
MMM	△	△	△	▽	▲	▲	▲	△	△	△	△	▲	▲
MRK	▲	△	△	▽	▲	▲	▲	△	△	△	▽	▲	▲
MSFT	△	△	▽	▽	△	▲	△	▽	△	▽	△	▲	▲
NKE	▲	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲
NVDA	△	▽	▽	▽	▲	▲	▲	△	△	△	△	▲	▲
PG	▲	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲
SHW	△	△	△	▽	▲	▲	△	△	▲	△	△	▲	▲
TRV	△	△	△	▽	▲	▲	△	△	△	△	△	▲	▲
UNH	▲	△	△	▽	▲	▲	▲	△	△	△	△	▲	▲
V	▲	△	▽	▽	▲	▲	▲	△	△	△	▽	▲	▲
VZ	▲	△	▽	▽	▲	▲	▲	△	△	▽	▽	▲	▲
WMT	△	▽	▽	▽	▲	▲	△	▽	▽	▽	▽	▲	▲
23	8	30	22	28	8	8	12	21	27	24	24	8	5

	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	PRV_{T}	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	PRV_{K}	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}
AAPL	△	△	△	▽	▽	▲	▲	△	▽	△	▽	▽	▲	▲
AMGN	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	△	▽	△	▽	▽	▲	▲	△	▽	△	△	▽	▲	▲
AXP	△	△	△	△	▽	▲	▲	△	▲	△	▲	▲	▲	▲
BA	▲	▽	△	△	▽	▲	▲	▲	▽	▲	△	▽	▲	▲
CAT	△	▽	△	▽	▽	▲	▲	△	▲	△	▲	▲	▲	▲
CRM	▲	▽	△	▽	▽	▲	▲	▲	▲	△	▲	▲	▲	▲
CSCO	▲	△	△	▽	▽	▲	▲	▲	△	△	△	▽	▲	▲
CVX	▲	△	△	△	▽	▲	▲	▲	△	△	△	△	▲	▲
DIS	▲	▽	△	△	▽	▲	▲	▲	△	△	▲	△	▲	▲
GS	▲	△	△	△	△	▲	▲	△	▲	▲	▲	▲	▲	▲
HD	▲	▽	△	▽	▽	▲	▲	▲	▲	△	▲	▲	▲	▲
HON	△	△	△	△	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲
IBM	▲	△	△	△	▽	▲	▲	△	▲	△	▲	▲	▲	▲
JNJ	▲	▽	△	△	▽	▲	▲	▲	△	△	△	▲	▲	▲
JPM	▲	▽	△	△	▽	▲	▲	▲	△	△	△	△	▲	▲
KO	△	△	△	▽	▽	△	▲	△	▽	△	▽	▽	△	▲
MCD	▲	△	△	▲	△	▲	▲	△	▲	△	▲	▲	▲	▲
MMM	△	△	△	▽	△	▲	▲	△	△	△	▲	△	▲	▲
MRK	▲	△	△	△	▽	▲	▲	▲	▲	△	▲	▲	▲	▲
MSFT	△	▽	△	▽	▽	△	▲	△	▽	△	▽	▽	△	▲
NKE	▲	▽	△	▽	▽	▲	▲	▲	△	△	△	△	▲	▲
NVDA	△	▽	▽	▽	▽	▲	▲	△	▽	△	▽	▽	▲	▲
PG	▲	▽	△	▽	▽	▲	▲	▲	△	△	△	▲	▲	▲
SHW	△	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
TRV	△	△	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲
UNH	▲	△	△	▲	△	▲	▲	▲	▲	△	▲	▲	▲	▲
V	▲	▽	△	△	▽	▲	▲	▲	△	△	△	▲	▲	▲
VZ	▲	△	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
WMT	△	▽	▽	▽	▽	▲	▲	△	△	▽	▽	△	▲	▲
	6	15	28	15	17	6	4	12	24	28	23	26	6	5

	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	PRV_{P}	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	PRV_{Z}	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	△	▽	△	▽	▽	▲	▲	△	▽	▽	▽	▽	▲	▲
AMGN	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	△	▽	△	△	▽	▲	▲	△	△	▽	△	△	▲	▲
AXP	△	△	△	△	△	▲	▲	△	▽	△	△	▽	▲	▲
BA	△	▽	△	△	▽	▲	▲	△	▽	△	▽	▽	▲	▲
CAT	▲	▲	△	▲	▲	▲	▲	△	△	△	△	△	▲	▲
CRM	△	△	△	△	△	▲	▲	▲	△	△	△	▲	▲	▲
CSCO	▲	△	△	△	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
CVX	▲	△	△	▲	△	▲	▲	▲	△	△	△	△	▲	▲
DIS	▲	▽	△	△	▽	▲	▲	▲	▽	△	△	▽	▲	▲
GS	▲	▲	△	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲
HD	△	▲	▽	▽	▲	▲	▲	▲	△	△	▽	▽	▲	▲
HON	▲	▲	△	▲	▲	▲	▲	▲	▲	△	△	▲	▲	▲
IBM	△	△	△	△	△	▲	▲	▲	△	△	△	▽	▲	▲
JNJ	△	△	△	△	△	▲	▲	△	△	△	△	△	▲	▲
JPM	▲	▽	△	△	△	▲	▲	▲	▽	△	△	▽	▲	▲
KO	△	△	△	▽	▽	△	▲	△	▽	△	▽	▽	△	▲
MCD	△	△	△	△	▲	▲	▲	▲	△	△	▲	△	▲	▲
MMM	△	▲	△	▲	▲	▲	▲	△	△	△	△	△	▲	▲
MRK	▲	△	△	▲	△	▲	▲	▲	▽	△	▽	▽	▲	▲
MSFT	△	▽	△	▽	▽	△	▲	△	▽	△	▽	▽	△	▲
NKE	△	▽	△	▽	▽	▲	▲	△	▽	△	△	▽	▲	▲
NVDA	△	▽	▽	▽	▽	▲	▲	△	▽	▽	▽	▽	▲	▲
PG	▲	△	△	△	△	▲	▲	▲	△	△	△	△	▲	▲
SHW	▲	▲	△	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲
TRV	▲	▲	△	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲
UNH	▲	▲	△	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲
V	▲	△	△	▽	△	▲	▲	▲	△	△	△	▲	▲	▲
VZ	▲	△	△	▽	▽	▲	▲	▲	▽	△	▽	▽	▲	▲
WMT	△	▽	▽	▲	▽	▲	▲	△	△	▽	▽	△	▲	▲
	5	16	27	13	21	6	4	3	8	26	10	6	5	4

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The squared daily return of the estimated efficient price serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.24: Forecast evaluation: inclusion in MCS (QLIKE), $h = 1, \ldots, 5$ (mixed frequencies)

	$RV_{GRT}^{(390)}$	$RV_{GRT}^{(78)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$
AAPL	5	5	5	5	5	5	5	5	5	5
AMGN	2	0	5	0	0	0	0	1	4	1
AMZN	0	0	5	1	3	2	2	5	5	2
AXP	3	3	5	4	4	3	4	1	0	1
BA	5	4	5	5	5	5	5	5	5	5
CAT	5	3	5	5	5	3	3	3	3	5
CRM	5	5	5	4	4	1	5	5	4	5
CSCO	2	2	5	0	3	1	3	0	0	3
CVX	5	4	5	5	5	5	5	4	4	5
DIS	4	5	5	5	5	4	5	4	4	5
GS	1	5	0	5	5	2	2	4	4	3
HD	5	5	5	4	5	5	5	4	4	5
HON	3	5	5	5	5	5	5	2	3	0
IBM	3	2	5	2	2	1	2	2	2	4
JNJ	2	2	5	1	1	2	2	1	0	2
JPM	4	5	4	5	5	5	2	4	4	4
KO	3	5	5	5	5	3	5	1	1	4
MCD	2	5	5	5	5	5	5	1	0	4
MMM	3	0	5	0	1	0	0	4	4	3
MRK	3	5	5	5	5	5	5	2	0	4
MSFT	5	4	5	5	5	5	5	5	5	5
NKE	1	0	5	5	5	0	2	0	0	2
NVDA	5	5	5	5	5	5	5	5	5	5
PG	0	2	2	5	5	1	2	0	0	0
SHW	1	2	5	0	0	0	4	0	0	0
TRV	5	5	5	5	5	5	4	4	4	5
UNH	5	4	5	4	5	5	5	3	2	5
V	0	1	5	2	0	4	4	0	0	0
VZ	5	5	5	5	5	3	2	5	1	5
WMT	3	2	5	2	1	2	2	2	1	3
	0.633	0.667	0.940	0.727	0.760	0.613	0.700	0.547	0.493	0.667

	$RV_{N}^{(390)}$	$RBPV_{N}$	PRV_{N}	$PBPV_{N}$	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}
AAPL	5	5	5	5	5	5	5	5	5	5	5	4	4	5	5	5	5
AMGN	4	4	0	1	4	4	4	4	2	0	5	0	0	0	0	1	4
AMZN	0	0	5	5	0	0	5	5	5	5	4	5	5	2	5	5	5
AXP	3	5	4	4	2	4	0	0	3	3	1	3	3	1	1	1	0
BA	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5	5
CAT	3	2	4	5	3	3	4	4	5	2	2	5	5	2	3	3	2
CRM	5	4	4	4	2	5	4	4	5	5	4	4	4	3	5	4	4
CSCO	5	0	0	5	5	5	0	2	2	3	5	0	3	2	3	0	0
CVX	5	5	5	5	5	5	4	4	5	4	5	5	5	5	5	4	4
DIS	5	5	5	5	4	5	5	4	3	5	5	5	5	5	5	4	4
GS	4	0	5	5	4	4	4	4	0	3	0	5	5	3	0	1	4
HD	5	5	4	5	5	5	4	3	2	5	0	5	5	4	3	4	2
HON	4	5	0	1	0	4	0	0	3	5	5	4	5	5	5	2	3
IBM	2	5	1	2	1	2	3	2	3	3	5	1	1	1	3	2	2
JNJ	2	5	1	1	2	2	0	0	3	5	1	1	1	2	3	0	0
JPM	5	4	5	5	5	4	4	4	4	5	0	4	3	4	1	4	4
KO	5	5	5	5	5	5	3	3	4	4	5	5	5	1	4	1	0
MCD	5	4	5	5	5	5	1	0	0	0	2	5	5	4	3	0	0
MMM	0	4	1	1	0	0	4	4	4	0	5	1	1	0	0	4	4
MRK	5	5	5	5	5	5	1	0	3	5	4	5	5	5	3	1	0
MSFT	5	0	5	5	5	5	5	3	5	5	5	5	5	5	5	5	5
NKE	0	5	5	5	0	2	0	0	4	0	5	5	5	2	5	1	2
NVDA	5	4	5	5	5	5	5	5	5	5	5	5	5	5	0	5	5
PG	1	5	4	4	1	3	0	0	0	2	3	4	5	1	1	0	0
SHW	0	0	0	0	0	2	0	0	4	3	3	0	2	0	3	1	0
TRV	5	4	5	5	5	5	4	4	2	0	1	5	5	0	0	0	0
UNH	4	4	0	0	5	5	3	1	5	4	3	5	5	5	4	3	3
V	2	5	1	0	4	4	0	0	0	1	2	1	0	1	4	0	0
VZ	5	5	5	5	4	1	5	1	5	5	3	5	5	2	2	5	2
WMT	2	5	0	1	2	2	1	0	4	3	2	0	1	2	2	1	0
	0.707	0.760	0.660	0.727	0.653	0.740	0.553	0.473	0.667	0.660	0.667	0.713	0.753	0.547	0.587	0.480	0.460

	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$
AAPL	5	5	5	5	5	5	5	5	5	5	5
AMGN	3	3	4	0	0	0	4	4	4	2	0
AMZN	1	5	4	0	0	0	5	5	5	5	4
AXP	4	5	4	4	5	5	5	0	0	5	3
BA	5	5	5	5	5	5	5	5	5	5	5
CAT	5	3	2	5	5	3	3	4	3	3	3
CRM	5	4	3	5	5	5	4	5	4	5	5
CSCO	1	2	5	0	3	0	3	0	0	3	3
CVX	5	5	4	5	5	5	5	4	4	5	5
DIS	5	5	5	5	5	5	5	5	4	5	5
GS	0	4	0	5	5	5	5	4	4	0	0
HD	5	5	3	5	5	5	5	3	1	5	5
HON	5	4	3	5	5	1	4	4	4	3	2
IBM	5	5	3	4	5	5	5	2	3	3	5
JNJ	5	5	5	5	5	5	5	0	0	3	3
JPM	4	5	2	5	5	5	5	1	4	4	5
KO	5	5	0	5	5	5	5	3	1	4	5
MCD	5	5	5	5	5	5	5	2	0	0	2
MMM	4	5	5	1	1	2	5	4	4	4	5
MRK	4	4	2	5	5	5	4	0	0	3	5
MSFT	5	5	5	5	5	5	5	5	5	5	0
NKE	5	5	5	5	5	5	5	1	5	5	1
NVDA	5	5	5	5	5	5	5	5	5	5	5
PG	2	5	0	5	5	5	2	0	0	0	0
SHW	4	5	5	5	5	4	5	1	0	3	1
TRV	4	4	5	5	5	4	5	1	0	1	1
UNH	5	5	5	5	5	5	5	4	2	4	4
V	1	5	0	5	5	1	3	0	0	1	0
VZ	5	5	3	5	5	5	5	5	2	5	5
WMT	5	5	5	5	5	5	5	0	0	4	3
	0.813	0.920	0.713	0.860	0.893	0.800	0.913	0.547	0.493	0.700	0.633

	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
AMGN	4	0	0	0	0	2	4	3	0	4	0	0	3	4	3	4
AMZN	0	4	5	5	0	5	5	5	4	0	3	5	5	0	5	5
AXP	3	4	4	4	5	3	2	3	2	0	3	3	2	1	1	0
BA	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
CAT	0	5	5	2	2	3	2	2	0	0	2	2	0	0	2	2
CRM	0	5	5	4	4	4	4	5	4	4	5	5	1	5	4	4
CSCO	5	0	3	0	3	0	0	2	2	5	0	3	2	3	0	0
CVX	4	5	5	5	5	4	4	5	4	0	5	5	5	2	4	4
DIS	5	5	5	5	5	4	3	5	5	5	5	5	3	5	4	4
GS	0	3	3	4	0	4	4	2	0	0	4	4	0	0	3	4
HD	3	5	5	5	5	3	2	4	4	3	5	5	5	5	4	3
HON	3	5	5	3	2	0	0	4	5	5	0	5	4	5	0	1
IBM	4	3	3	1	5	2	2	2	0	0	2	2	0	0	2	1
JNJ	0	1	1	3	1	0	0	0	1	0	1	1	0	1	0	0
JPM	4	5	5	5	5	4	4	4	5	3	5	5	5	3	4	4
KO	5	5	5	5	5	3	1	2	5	4	5	5	4	5	1	1
MCD	4	5	5	5	5	0	0	0	1	4	5	5	3	2	0	0
MMM	5	1	4	4	5	4	4	2	3	5	0	0	0	0	4	4
MRK	4	5	5	4	5	1	0	2	1	0	5	5	4	1	0	0
MSFT	3	5	5	4	4	5	5	5	5	5	5	5	5	5	5	5
NKE	5	5	5	2	5	2	5	2	1	5	5	5	4	5	1	2
NVDA	5	5	5	5	5	5	5	5	5	3	5	5	5	5	5	5
PG	0	5	5	0	0	0	0	0	2	0	5	5	1	1	0	0
SHW	2	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
TRV	2	5	5	0	1	0	0	2	0	0	5	5	0	0	0	0
UNH	5	5	5	4	5	2	0	4	4	5	5	5	4	5	3	3
V	0	3	3	0	0	0	0	0	0	0	1	3	0	0	0	0
VZ	0	5	5	5	5	5	1	5	5	5	5	5	3	3	5	0
WMT	0	4	4	3	3	1	1	1	2	0	1	1	1	0	1	0
	0.567	0.787	0.833	0.653	0.667	0.513	0.453	0.573	0.533	0.500	0.680	0.760	0.527	0.507	0.473	0.440

This table reports the number of inclusions in the model confidence set. The HAR model is used to forecast the squared daily return of the estimated efficient price which acts as the unbiased volatility proxy for day $t + h$. The forecast error are evaluated with the QLIKE loss function. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.25: Tests of equal accuracy for DJIA stocks (QLIKE): mixed frequencies, GRT only

	$RV_{GRT}^{(78)}$	PRV_{GRT}	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}
AAPL	△	△	▽	▼	▲	▲
AMGN	△	△	△	△	△	△
AMZN	△	△	▽	▽	△	△
AXP	△	△	△	▽	▲	▲
BA	▲	▲	△	△	▲	▲
CAT	△	△	▽	▽	▲	▲
CRM	▲	△	▽	▽	▲	▲
CSCO	▲	△	▽	▽	▲	▲
CVX	▲	△	△	▽	▲	▲
DIS	▲	▲	△	▽	▲	▲
GS	▲	▲	△	▽	▲	▲
HD	▲	△	▽	▽	▲	▲
HON	△	△	△	▽	▲	▲
IBM	△	△	△	▽	▲	▲
JNJ	△	△	△	▽	▲	▲
JPM	▲	△	△	▽	▲	▲
KO	△	△	▽	▽	△	▲
MCD	▲	△	△	△	▲	▲
MMM	▲	▲	△	▽	▲	▲
MRK	▲	△	△	▽	▲	▲
MSFT	△	△	▽	▽	△	▲
NKE	▲	△	▽	▽	▲	▲
NVDA	△	▽	▽	▽	▲	▲
PG	▲	△	▽	▽	▲	▲
SHW	△	△	△	▽	▲	▲
TRV	△	△	△	▽	▲	▲
UNH	▲	△	△	▽	▲	▲
V	▲	△	▽	▽	▲	▲
VZ	▲	△	▽	▽	▲	▲
WMT	▲	▽	▽	▽	▲	▲

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The squared daily return of the estimated efficient price serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days.

Table F.26: Tests of equal accuracy for DJIA stocks (QLIKE): mixed frequencies, including jump-robust estimators

	$RV_{GRT}^{(78)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	$RBPV_{N}$	PRV_{N}	$PBPV_{N}$
AAPL	△	▽	△	▲	▽	▽	▲	▲	△	△	▲	▽	▲
AMGN	▲	△	▲	▲	△	△	▲	▲	▲	▲	▲	▲	▲
AMZN	△	△	△	▲	▽	▽	▲	▲	▲	▲	▲	▲	▲
AXP	△	△	△	▲	△	▽	▲	▲	△	▽	△	△	▲
BA	▲	▲	△	▲	△	△	▲	▲	△	▽	△	△	▲
CAT	△	△	△	▲	▽	▽	▲	▲	△	▽	△	△	▲
CRM	△	△	△	▲	▽	▽	▲	▲	△	▽	△	△	▲
CSCO	▲	▲	△	▲	▽	▽	▲	▲	▲	△	▲	△	▲
CVX	△	△	△	▲	△	▽	▲	▲	△	▽	△	△	▲
DIS	▲	▲	△	▲	△	▽	▲	▲	▲	△	▲	△	▲
GS	▲	▲	△	▲	△	▽	▲	▲	▲	▽	▲	△	▲
HD	△	△	△	▲	▽	▽	▲	▲	△	△	△	△	▲
HON	△	△	△	▲	△	▽	▲	▲	△	△	▲	△	▲
IBM	△	△	△	▲	△	▽	▲	▲	△	△	△	△	▲
JNJ	△	△	△	▲	△	▽	▲	▲	△	▽	▲	△	▲
JPM	△	▲	△	▲	△	▽	▲	▲	△	△	▲	△	▲
KO	△	▲	△	▲	▽	▽	△	▲	△	▽	▲	△	▲
MCD	▲	▲	△	▲	△	△	▲	▲	△	△	▲	△	▲
MMM	△	△	△	▲	△	▽	▲	▲	▲	△	△	△	▲
MRK	▲	▲	△	▲	△	▽	▲	▲	▲	△	▲	△	▲
MSFT	△	▲	△	▲	▽	▽	△	▲	△	▽	▲	△	▲
NKE	△	△	△	▲	▽	▽	▲	▲	△	△	▲	△	▲
NVDA	△	▲	▽	▲	▽	▽	▲	▲	▲	△	▲	△	▲
PG	△	△	△	▲	▽	▽	▲	▲	△	△	▲	△	▲
SHW	△	△	△	▲	△	▽	▲	▲	△	△	▲	▲	▲
TRV	△	△	△	▲	△	▽	▲	▲	△	△	△	△	▲
UNH	▲	▲	△	▲	△	▽	▲	▲	▲	△	▲	△	▲
V	▲	△	△	▲	▽	▽	▲	▲	▲	△	▲	△	▲
VZ	▲	▲	△	▲	▽	▽	▲	▲	▲	△	▲	△	▲
WMT	△	▲	▽	▲	▽	▽	▲	▲	△	▽	▲	▽	▲
26	27	26	28	29	27	27	27	27	27	25	25	27	28

	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV_{T}^{AC}	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}
AAPL	▽	▽	▲	▲	△	△	△	△	▲	▽	▽	▲	▲
AMGN	▲	▲	▲	▲	▲	△	△	▲	▲	▲	△	▲	▲
AMZN	▲	▲	▲	▲	△	▽	△	△	▲	▽	▽	▲	▲
AXP	△	▽	▲	▲	△	△	△	△	▲	△	▽	▲	▲
BA	△	▽	▲	▲	△	▽	▽	△	▲	△	▽	▲	▲
CAT	△	▽	▲	▲	△	▽	△	△	▲	▽	▽	▲	▲
CRM	△	▽	▲	▲	△	▽	△	△	▲	▽	▽	▲	▲
CSCO	▽	△	▲	▲	▲	△	▲	△	▲	▽	▽	▲	▲
CVX	▽	▽	▲	▲	△	△	△	△	▲	△	▽	▲	▲
DIS	△	▽	▲	▲	▲	▽	▲	△	▲	△	▽	▲	▲
GS	△	△	▲	▲	▲	△	▲	△	▲	△	△	▲	▲
HD	△	▽	▲	▲	△	▽	△	△	▲	▽	▽	▲	▲
HON	△	△	▲	▲	△	△	△	△	▲	△	▽	▲	▲
IBM	△	▽	▲	▲	△	△	▲	△	▲	△	▽	▲	▲
JNJ	△	▽	▲	▲	△	▽	△	△	▲	△	▽	▲	▲
JPM	△	▽	▲	▲	△	▽	△	△	▲	△	▽	▲	▲
KO	▽	▽	△	▲	△	△	▲	△	▲	▽	▽	△	▲
MCD	△	△	▲	▲	▲	△	▲	△	▲	▲	△	▲	▲
MMM	△	△	▲	▲	△	△	▲	△	▲	▽	△	▲	▲
MRK	△	▽	▲	▲	▲	△	▲	△	▲	△	▽	▲	▲
MSFT	▽	△	▲	▲	△	▽	△	△	▲	▽	▽	△	▲
NKE	▽	▽	▲	▲	△	▽	△	△	▲	▽	▽	▲	▲
NVDA	△	△	▲	▲	△	▽	△	▽	▲	▽	▽	▲	▲
PG	▽	▽	▲	▲	△	▽	△	△	▲	▽	▽	▲	▲
SHW	△	△	▲	▲	△	▲	▲	△	▲	▲	▲	▲	▲
TRV	△	△	▲	▲	△	△	▲	△	▲	▲	△	▲	▲
UNH	△	△	▲	▲	▲	△	▲	△	▲	▲	△	▲	▲
V	△	▽	▲	▲	▲	▽	▲	△	▲	△	▽	▲	▲
VZ	▽	▽	▲	▲	▲	△	▲	△	▲	▽	▽	▲	▲
WMT	▽	▽	▲	▲	△	▽	▲	▽	▲	▽	▽	▲	▲
	28	25	27	27	27	27	29	28	30	26	25	27	27

	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$
AAPL	△	▽	▽	△	▲	▽	▽	▲	▲	△	▽	▽	△	▲
AMGN	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	△	▽	▽	△	▲	△	▽	▲	▲	△	▽	▲	△	▲
AXP	△	△	▲	△	▲	▲	▲	▲	▲	△	△	▲	△	▲
BA	△	▽	△	△	▲	△	▽	▲	▲	△	▽	▲	△	▲
CAT	△	△	▲	△	▲	△	△	▲	▲	△	▲	▲	△	▲
CRM	△	▲	▲	△	▲	△	▲	▲	▲	△	△	▲	△	▲
CSCO	▲	△	▲	△	▲	△	▽	▲	▲	▲	△	▲	△	▲
CVX	▲	△	▲	△	▲	△	△	▲	▲	△	△	▲	△	▲
DIS	▲	△	▲	△	▲	▲	△	▲	▲	▲	▽	▲	△	▲
GS	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲
HD	△	△	▲	△	▲	▲	▲	▲	▲	△	△	▲	▽	▲
HON	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲
IBM	△	▲	▲	△	▲	▲	▲	▲	▲	△	△	▲	△	▲
JNJ	△	△	▲	△	▲	△	△	▲	▲	△	△	▲	△	▲
JPM	△	△	▲	△	▲	△	△	▲	▲	△	▽	▲	△	▲
KO	△	▽	▲	△	▲	▽	▽	△	▲	△	△	▲	△	▲
MCD	△	▲	▲	△	▲	▲	▲	▲	▲	△	△	▲	△	▲
MMM	△	△	▲	△	▲	▲	△	▲	▲	△	▲	▲	△	▲
MRK	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	△	▲	△	▲
MSFT	△	▽	△	△	▲	▽	▽	△	▲	△	▽	△	△	▲
NKE	△	△	▲	△	▲	△	△	▲	▲	△	▽	▲	△	▲
NVDA	△	▽	△	△	▲	▽	▽	▲	▲	△	▽	△	▽	▲
PG	△	△	▲	△	▲	△	▲	▲	▲	△	△	▲	△	▲
SHW	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲
TRV	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲
UNH	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	△	▲
V	▲	△	▲	△	▲	△	▲	▲	▲	▲	△	▲	△	▲
VZ	▲	▽	▲	△	▲	▽	▽	▲	▲	▲	△	▲	△	▲
WMT	△	△	▲	▽	▲	▽	△	▲	▲	△	▽	▲	▽	▲
	27	25	26	26	28	28	23	27	27	27	26	27	28	28

	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	▽	▽	▲	▲	△	▽	▽	▽	▲	▽	▽	▲	▲
AMGN	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	△	▽	▲	▲	△	△	▲	▽	▲	▲	△	▲	▲
AXP	△	△	▲	▲	△	▽	▲	△	▲	△	▽	▲	▲
BA	△	▽	▲	▲	△	▽	▲	△	▲	▽	▽	▲	▲
CAT	▲	▲	▲	▲	△	△	▲	△	▲	△	△	▲	▲
CRM	△	△	▲	▲	△	△	▲	△	▲	△	△	▲	▲
CSCO	△	▽	▲	▲	▲	▽	▲	△	▲	▽	▽	▲	▲
CVX	▲	△	▲	▲	▲	△	△	△	▲	△	△	▲	▲
DIS	△	▽	▲	▲	▲	▽	△	△	▲	△	▽	▲	▲
GS	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲
HD	▽	△	▲	▲	△	△	▲	△	▲	▽	▽	▲	▲
HON	▲	▲	▲	▲	▲	▲	▲	△	▲	△	▲	▲	▲
IBM	△	△	▲	▲	△	△	▲	△	▲	△	▽	▲	▲
JNJ	△	△	▲	▲	△	△	△	△	▲	△	△	▲	▲
JPM	△	△	▲	▲	△	▽	△	△	▲	△	▽	▲	▲
KO	▽	▽	△	▲	△	▽	▲	△	▲	▽	▽	△	▲
MCD	△	▲	▲	▲	▲	△	▲	△	▲	▲	△	▲	▲
MMM	▲	▲	▲	▲	△	△	▲	△	▲	△	△	▲	▲
MRK	▲	△	▲	▲	▲	▽	▲	△	▲	▽	▽	▲	▲
MSFT	▽	▽	△	▲	△	▽	△	△	▲	▽	▽	△	▲
NKE	▽	▽	▲	▲	△	▽	△	△	▲	△	▽	▲	▲
NVDA	▽	▽	▲	▲	△	▽	▽	▽	▲	▽	▽	▲	▲
PG	△	△	▲	▲	△	△	▲	△	▲	△	△	▲	▲
SHW	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲
TRV	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲
UNH	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲
V	▽	△	▲	▲	▲	△	▲	△	▲	△	▲	▲	▲
VZ	▽	▽	▲	▲	▲	▽	▲	△	▲	▽	▽	▲	▲
WMT	▲	▽	▲	▲	△	△	▲	▽	▲	▽	△	▲	▲
	27	25	27	26	28	27	26	27	28	28	26	27	27

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The squared daily return of the estimated efficient price serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

F.7 Robustness check – mixed frequencies 5-min/1-min/30-sec/1-sec RV estimators

Table F.27: Tests of equal accuracy for DJIA stocks (QLIKE): mixed frequencies

	$RV_{GRT}^{(78)}$	$RV_{GRT}^{(30sec)}$	$RV_{GRT}^{(1sec)}$	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	$RV_{N}^{(30sec)}$	$RV_{N}^{(1sec)}$	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RV_{T}^{(30sec)}$	$RV_{T}^{(1sec)}$	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RV_{K}^{(30sec)}$	$RV_{K}^{(1sec)}$	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$	$RV_{P}^{(30sec)}$	$RV_{P}^{(1sec)}$	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RV_{Z}^{(30sec)}$	$RV_{Z}^{(1sec)}$
AAPL	▲	▽	▼	▲	▲	△	▲	▲	△	▽	▼	▲	▽	▽	▽	▲	▽	▽	▼	▲	▽	▽	▼
AMGN	▲	▲	▽	▲	▲	▲	▲	▲	▽	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
AMZN	▲	△	▽	▲	▲	▲	▲	▲	▽	▽	▽	▲	▽	△	▲	▲	▲	△	▲	▲	▲	▲	▲
AXP	▲	△	▽	▲	▽	△	▲	▲	▽	△	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	△	△	▲
BA	▲	△	▽	△	▽	△	▽	△	▽	△	▲	▲	▽	△	▲	△	△	▽	▲	△	▽	▽	▲
CAT	▲	▲	△	▲	▽	▲	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲
CRM	▲	△	▽	▲	▽	△	▲	▲	▽	△	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	△	▽	▲
CSCO	▲	▽	▽	▲	△	△	▲	▲	▽	▽	▽	▲	▽	▽	△	▲	▽	▽	▽	▲	▽	▽	▽
CVX	▲	▽	▽	▲	△	▽	△	▲	▽	▽	△	▲	△	▽	▲	▲	▽	▽	▲	▲	▽	▽	▲
DIS	▲	△	▽	▲	△	△	▽	▲	▽	△	▽	▲	△	△	▲	▲	▽	△	▲	▲	▽	△	△
GS	▲	▲	△	▲	▽	△	▲	▲	▽	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
HD	▲	▲	△	▲	▽	△	▲	▲	▽	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	△	▲
HON	▲	△	△	▲	△	▲	▲	▲	△	△	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	▲
IBM	▲	△	▽	▲	△	△	△	▲	▽	△	▲	▲	▲	▲	▲	▲	△	△	▲	▲	▽	△	▲
JNJ	▲	△	▽	▲	▽	△	▽	▲	▽	△	▲	▲	△	△	▲	▲	△	△	▲	▲	△	△	▲
JPM	▲	▽	▽	▲	△	▽	▽	▲	▽	▽	▽	▲	▽	▽	▲	▲	▽	▽	▲	▲	▽	▽	△
KO	▲	△	▽	▲	▽	△	▽	▲	△	△	△	▲	▽	△	▲	▲	▽	▽	▲	▲	▽	△	▲
MCD	▲	△	△	▲	△	△	△	▲	△	△	▲	▲	▲	△	▲	▲	▲	△	▲	▲	△	△	▲
MMM	▲	▲	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
MRK	▲	△	▽	▲	△	△	▽	▲	▽	△	▽	▲	▲	▲	▲	▲	▽	△	▲	▲	▽	△	▲
MSFT	▲	▽	▽	▲	▽	▽	▲	▲	▽	▽	▽	▲	▽	▽	△	▲	▽	▽	▽	▲	▽	▽	△
NKE	▲	△	▽	▲	△	△	▽	▲	▽	△	▲	▲	△	△	▲	▲	▽	▽	▲	▲	▽	▽	▲
NVDA	▲	▽	▽	▲	▲	▲	▲	▲	▽	▽	▽	▲	▽	▽	▲	▲	▽	▽	▲	▲	▽	▽	▲
PG	▲	△	△	▲	△	△	▲	▲	△	△	▲	▲	▲	△	▲	▲	△	△	▲	▲	▽	△	▲
SHW	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
TRV	▲	▲	△	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
UNH	▲	▲	▲	▲	▽	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲
V	▲	△	▽	▲	△	△	△	▲	▽	△	▲	▲	△	△	▲	▲	▽	▽	▲	▲	△	△	▲
VZ	▲	▽	▽	▲	△	▽	▽	▲	▽	▽	▽	▲	▽	▽	▲	▲	▽	▽	▽	▲	▽	▽	▽
WMT	▲	△	▽	▲	△	△	▽	▲	▽	▽	▽	▲	▽	▽	▲	▲	▽	▽	▲	▲	▽	▽	▲
27	21	29	27	19	27	28	23	8	14	16	12	13	21	23	23	8	13	19	14	6	5	7	5

This table reports the QLIKE distances from the benchmark estimator ($RV_{GRT}^{(390)}$). △ (▲) denotes a (significant) positive distance and ▽ (▼) denotes a (significant) negative distance, respectively. For example, ▲ means that the RV_{GRT} estimator significantly outperforms a given competitor. The sparsely sampled (20-minute) RV obtained from the defragmented prices serves as the proxy for the daily integrated variance. The test is conducted at the 5% significance level and $k = 1$ (number of false rejections). We employ a stationary bootstrap (1000 draws) for the evaluation of our test statistics with an average block length of ten days. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

F.8 Panel Regression – Relative loss GRT vs univariate models (5-min/1-min)

In the following, we evaluate the performance differences between the single market measures and the GRT-based estimator by way of a panel analysis. This allows us to regress the QLIKE loss differences on a set of conditioning variables that provide further insights on how the exchange specific microstructure influences the relative performance.

We focus on the standard RV estimator sampled at the 1-minute frequency which has performed well in the in-sample and out-of-sample analyses and evaluate against the one-day lead of RV_{GRT} sampled every 20 minutes as the proxy. This means we compare $RV_{GRT}^{(390)}$ as the benchmark to the $RV^{(390)}$ estimators computed from the returns observed at the respective markets. The relative losses are denoted as $\Delta L_i = L(\theta_t, RV_{GRT}^{(390)}) - L(\theta_t, RV_i^{(390)})$. We repeat the analysis for 5-minute data as a robustness check.

For the set of conditioning variables, we primarily focus on high-frequency variables that characterize the market-specific liquidity and overall trading environment. The first variable is *avgDuration* measuring the daily average time between ticks in seconds. To compute this variable, we utilize quote data at the tick level. Then, we also measure the relative daily trading volume, *relTradevol*, allocated to a given market. We consider three intraday liquidity measures. The effective spread (*effectiveSpread*) is defined as $ES_t = 2D_t \times [PRICE_t - (BID_t + OFR_t)/2]$. The variable D_t is the sign of the trade, indicating whether the buyer (seller) is the initiator of the trade $D_t = 1$ ($D_t = -1$). The depth imbalance difference (*depthImbalanceDifference*) is defined as $DId_t = D_t \times (OFRSIZ_t - BIDSIZ_t)/(OFRSIZ_t + BIDSIZ_t)$ and price impact (*priceImpact*) is defined as $PI_t = (ES_t - RS_t)/2$, where $RS_t = 2D_t \times [PRICE_t - (BID_{t+300} + OFR_{t+300})/2]$ indicates the relative spread. Furthermore, we obtain the daily *ReMeDI* estimates of the microstructure noise level at the 1-second frequency. Choosing a relatively high sampling frequency generally improves the precision of a microstructure noise estimator. Finally, we include *jumpprop* computed as $\max\{RV_t - RBPV_t, 0\}/RV_t$ to measure the proportion of the quadratic variation attributable to jumps.

The panel is unbalanced as not all stocks of the DJIA trade on the same day in all exchanges. The cross-sectional dimension is $N = 30$ with maximum length of the sampling period amounting to $T = 1961$ trading days. We use entity fixed effects and cluster the standard errors at the stock level.

We first run regressions for each exchange with the results in Table F.28 indicated by the exchange ticker symbol. While no clear pattern emerges how the average duration, trading volume, and the liquidity measures affect the relative performance, we clearly see a negative effect of *ReMeDI* and *jumpprop*. A negative effect means that the relative losses tend to become more negative if those variables increase which in turn means that the GRT-based estimator performs well if the single market data is noisy or marked by substantial jumps. Additionally, the coefficient of price impact is significantly negative for Nasdaq, Arca, and Cboe EDGX but insignificant and positive for NYSE and Cboe BZX.

Next, we repeat this analysis with a panel model that combines the data from all five exchanges and controls for the market-specific losses by setting additional market fixed effects. The standard errors are again clustered on the stock level. We estimate the model for 1-minute and 5-minute data checking the robustness of our results across sampling frequencies (Table F.30). The effects of *ReMeDI* and *jumpprop* remain significantly negative. The individual liquidity measures do not have significant explanatory power for the relative losses. Only *priceImpact* has a significant positive coefficient in the model using 1-minute data.

Table F.28: Relative loss GRT vs univariate models, unbalanced panel, entity fixed effects, clustered SE (stock level), 1-min

	ΔL_N	ΔL_T	ΔL_P	ΔL_K	ΔL_Z
	(1)	(2)	(3)	(4)	(5)
avgDuration	2.516	-0.043^{**}	-2.700^{***}	-0.071^{***}	-0.039
	(1.939)	(0.021)	(0.924)	(0.022)	(0.029)
relTradevol	1.483	0.004	-8.846	1.865^{**}	-0.295^{***}
	(1.573)	(0.023)	(6.368)	(0.796)	(0.112)
effectiveSpread	-0.959^{***}	0.055^{*}	4.015	0.207	-0.004
	(0.218)	(0.029)	(2.676)	(0.200)	(0.062)
depthImbalanceDifference	0.117	-0.055^{**}	-0.275	-0.071	-0.293^{***}
	(0.488)	(0.023)	(0.381)	(0.103)	(0.078)
priceImpact	0.167	-0.146^{***}	-1.220^{***}	-0.054^{***}	0.002
	(0.110)	(0.046)	(0.227)	(0.007)	(0.004)
ReMeDI	-1.446^{***}	-0.380^{***}	-2.619^{***}	-1.691^{***}	-0.410^{***}
	(0.291)	(0.065)	(0.223)	(0.211)	(0.066)
jumpprop	-4.837	-0.356^{***}	-2.718^{**}	-1.647^{***}	-0.981^{***}
	(3.883)	(0.092)	(1.141)	(0.376)	(0.185)
Observations	52,607	52,758	52,735	52,692	52,602
R^2	0.473	0.050	0.402	0.262	0.149
Adjusted R^2	0.472	0.049	0.402	0.262	0.148
Note:					*p<0.1; **p<0.05; ***p<0.01

This table reports the relative QLIKE distances from the benchmark estimator $RV_{GRT}^{(390)}$ to the $RV^{(390)}$ estimators computed from the returns observed at the respective markets, $\Delta L_i = L(\theta_t, RV_{GRT}^{(390)}) - L(\theta_t, RV_i^{(390)})$. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.29: Relative loss GRT vs univariate models, unbalanced panel, entity fixed effects, clustered SE (stock level), 5-min

	ΔL_N	ΔL_T	ΔL_P	ΔL_K	ΔL_Z
	(1)	(2)	(3)	(4)	(5)
avgDuration	0.565	-0.002	-0.824^{***}	0.001	0.009
	(0.407)	(0.001)	(0.318)	(0.007)	(0.032)
relTradevol	0.343	-0.001	-2.926	0.428^{*}	-0.059
	(0.353)	(0.012)	(1.885)	(0.239)	(0.065)
effectiveSpread	-0.266^{***}	0.038^{**}	1.186	0.063	-0.014
	(0.040)	(0.018)	(0.767)	(0.052)	(0.025)
depthImbalanceDifference	0.324	-0.021^{**}	-0.034	0.013	-0.097^{**}
	(0.405)	(0.009)	(0.119)	(0.017)	(0.041)
priceImpact	-0.001	-0.080^{**}	-0.377^{***}	-0.012^{**}	-0.002
	(0.022)	(0.033)	(0.060)	(0.005)	(0.003)
ReMeDI	-0.307^{***}	-0.030^{***}	-0.684^{***}	-0.402^{***}	0.010
	(0.055)	(0.011)	(0.063)	(0.082)	(0.024)
jumpprop	-1.575^{*}	-0.227^{***}	-0.492^{*}	-0.244^{***}	-0.463^{***}
	(0.897)	(0.074)	(0.281)	(0.066)	(0.094)
Observations	52,607	52,758	52,735	52,692	52,602
R^2	0.427	0.021	0.365	0.128	0.055
Adjusted R^2	0.427	0.021	0.365	0.127	0.055
Note:					*p<0.1; **p<0.05; ***p<0.01

This table reports the relative QLIKE distances from the benchmark estimator $RV_{GRT}^{(78)}$ to the $RV^{(78)}$ estimators computed from the returns observed at the respective markets, $\Delta L_i = L(\theta_t, RV_{GRT}^{(78)}) - L(\theta_t, RV_i^{(78)})$. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.30: Relative loss GRT vs univariate models, unbalanced panel, entity and market fixed effects, clustered SE (stock level)

	Dependent variable:	
	ΔL	
	5-min	1-min
avgDuration	−0.068	−0.151
	(0.168)	(0.672)
relTradevol	−0.548	−2.073
	(0.520)	(1.732)
effectiveSpread	−0.102	−0.401
	(0.158)	(0.509)
depthImbalanceDifference	0.011	−0.300
	(0.102)	(0.200)
priceImpact	0.045	0.306***
	(0.040)	(0.107)
ReMeDI	−0.387***	−1.665***
	(0.101)	(0.314)
jumpprop	−0.929**	−3.450***
	(0.373)	(1.228)
N	0.141	0.590
	(0.124)	(0.396)
P	0.165	0.540
	(0.129)	(0.402)
T	0.147	0.591
	(0.145)	(0.469)
Z	0.109	0.492*
	(0.073)	(0.259)
Observations	263,394	263,394
R^2	0.372	0.434
Adjusted R^2	0.372	0.434
Note:	*p<0.1; **p<0.05; ***p<0.01	

This table reports the relative QLIKE distances from the benchmark estimator RV_{GRT} to the RV estimators computed from the returns observed at the respective markets, $\Delta L_i = L(\theta_t, RV_{GRT}) - L(\theta_t, RV_i)$. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

F.9 Economic benefits: artificial option straddle market

We design an artificial option straddle market analogously to Bandi and Russell (2008) and Bandi et al. (2008). Each agent selects one volatility forecasting method and, based on this method, prices a 1-day at-the-money option on a \$1 share of a given stock. The exercise price of the option is \$1 and the risk-free rate is zero. Agents then plug these inputs into the Black-Scholes formula to compute what they regard as the "fair" option price. Under these assumptions, the Black-Scholes call price simplifies to

$$\text{call}_t = 2\Phi\left(\frac{1}{2}\widehat{X}_{s,t}\right) - 1, \tag{F.1}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution, $\widehat{X}_{s,t}$ is the out-of-sample volatility forecast delivered by method s using the estimates from the HAR model in (38), and call_t is the call option price.

The artificial option straddle market operates as follows. First, during the trading phase, agents submit their individual Black–Scholes prices. Agents are then paired, and trades are executed at the midpoint of their quoted prices. If an agent's volatility forecast exceeds this midpoint, the option is perceived as underpriced, prompting the agent to buy. Conversely, if the forecast is below the midpoint, the agent sells.

All trades involve straddles—one call and one put. Agents with higher variance forecasts effectively bet on volatility: they regard the straddle as undervalued at the midpoint and anticipate that large price swings will push either the call or the put deep into the money. Positions are subsequently hedged. The daily profit for a trader who buys the straddle is given by

$$|\Delta P_{n,t}| - 2\text{call}_t + \Delta P_{n,t}\Big(1 - 2\Phi\big(\tfrac{1}{2}\widehat{X}_{s,t}\big)\Big), \tag{F.2}$$

where $\Delta P_{n,t}$ denotes the stock's return on market $n = 1, \ldots, N+1$ on day t and market $N+1$ denotes the defragmented returns. The daily profit for a seller is

$$2\text{call}_t - |\Delta P_{n,t}| - \Delta P_{n,t}\Big(1 - 2\Phi\big(\tfrac{1}{2}\widehat{X}_{s,t}\big)\Big). \tag{F.3}$$

Second, for each trading day, profits and losses are computed for every agent. Finally, we report the median of each agent's daily profits across all trading days. Stock-specific results are presented in Table F.32, whereas aggregate results by exchange and class of estimator are reported in Table F.31. Overall, the pre-averaged GRT estimator delivers the largest profit, whereas most estimators yield median losses — particularly those that fail to account for additional microstructure noise. Moreover, it is important to highlight that the GRT-based estimators uniformly outperform their univariate counterparts for all estimators, except for the five-minute RV estimator computed from prices sampled from exchanges K and Z.

Finally, since our primary objective is not to identify the single best estimator but to assess whether formally addressing fragmentation noise through our GRT-based class of estimators improves performance relative to the standard univariate approach adopted in the literature, we select the best-performing estimator for each trading day and evaluate maximum profits in Table F.33. The results clearly show that maximum profits occur when the GRT-based class of estimators is applied, with gains over single-market sources ranging from 0.88 to 1.66 cents on average.

Table F.31: Average annualized daily profits (in cents) for each market/estimator

	$RV^{(78)}$	$RV^{(390)}$	$RBPV$	PRV	$PBPV$	RV_{AC}	RV_{NW}	RK_{MTH}	RK_P
GRT	-2.142	-1.474	-1.137	**0.579**	0.448	-1.295	0.011	-3.595	-6.007
N	-4.539	-4.123	-16.267	0.071	0.123	-3.027	-5.164	-5.287	-8.013
T	-2.469	-1.865	-7.163	0.019	0.056	-1.618	-0.686	-4.307	-6.218
K	-2.011	-6.742	-18.319	0.160	0.115	-3.699	-7.009	-3.560	-6.042
P	-2.961	-6.966	-27.293	-0.178	0.055	-2.667	-7.388	-4.444	-7.455
Z	-2.097	-2.168	-16.32	0.318	0.299	-1.466	-2.131	-4.503	-6.259

This tables reports the sample average of median annualized daily profits that can be realized from the set of estimators computed for each data source. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.32: Annualized daily profits (in cents)

	$RV_{GRT}^{(78)}$	$RV_{GRT}^{(390)}$	$RBPV_{GRT}$	PRV_{GRT}	$PBPV_{GRT}$	RV_{GRT}^{AC}	RV_{GRT}^{NW}	RK_{GRT}^{MTH}	RK_{GRT}^{P}	$RV_{N}^{(78)}$	$RV_{N}^{(390)}$	$RBPV_{N}$	PRV_{N}
AAPL	-0.469	-2.099	-0.654	0.279	0.592	1.279	3.148	-4.507	-8.891	-9.204	-4.528	-57.313	-2.216
AMGN	-2.756	-0.619	10.519	-2.098	-2.311	-2.729	7.322	-0.659	-0.823	-11.337	-9.386	-7.892	-2.935
AMZN	-2.345	1.377	3.577	-2.782	-3.139	-2.939	-1.239	-0.996	0.41	-16.932	-41.114	-93.233	-5.256
AXP	-0.996	-2.043	-0.227	2.893	2.301	1.879	0.862	-5.138	-12.269	-5.828	-3.118	-3.885	3.289
BA	-3.663	-5.637	-1.267	1.051	1.641	-2.662	-0.68	-3.847	-11.984	-5.673	-5.142	-8.467	1.581
CAT	-0.249	-1.358	-1.822	1.992	2.603	-1.168	-0.253	-0.522	-2.999	0.081	-2.486	-7.324	2.394
CRM	-2.107	-1.829	-4.569	-0.606	-0.94	-1.686	1.972	0.1	-3.016	-4.029	-1.839	-9.06	-2.386
CSCO	-3.084	0.11	1.773	-1.919	-1.116	0.932	8.688	-6.563	-6.522	-9.615	-1.328	-35.348	-0.697
CVX	-3.986	-0.69	-1.32	-0.12	0.048	0.406	0.598	-5.006	-6.436	-3.315	-0.082	-1.494	-0.85
DIS	-0.014	-1.242	0.74	-0.614	-0.764	-1.118	-0.858	-2.422	-7.054	-1.063	-1.385	-0.154	0.085
GS	-0.941	0.076	-9.901	2.453	1.468	0.995	2.116	-6.261	-9.427	-1.827	-3.242	-23.28	1.883
HD	-2.153	0.801	-5.589	-0.473	0.143	-0.717	-0.761	-1.602	-4.227	-3.366	0.896	-10.587	-0.842
HON	-0.241	0.112	3.831	1.649	1.815	0.81	4.472	-4.959	-6.918	-6.197	-6.488	-27.712	-0.638
IBM	-2.012	-0.51	-1.001	1.146	0.769	-0.687	0.827	-1.127	-3.473	-1.741	-0.763	-5.797	0.328
JNJ	-0.921	2.867	-3.324	-0.383	-0.773	-1.341	-0.647	-1.169	-1.846	0.428	3.101	-3.099	-0.844
JPM	0.258	0.108	-9.004	0.021	-0.007	-2.862	-1.829	-2.98	-2.084	0.014	0.369	-11.946	1.245
KO	-2.426	1.043	-8.491	0.704	0.379	1.391	1.393	-5.175	-11.188	-3.044	0.959	-6.926	1.22
MCD	-1.408	-1.743	2.638	3.649	3.961	-4.339	-2.683	-4.225	-3.118	-2.922	-1.957	-2.846	3.108
MMM	-3.817	-2.641	4.51	-1.359	-1.397	-3.92	0.483	-8.997	-12.857	-3.569	-1.232	-0.775	-1.551
MRK	-5.526	0.936	-3.834	1.811	2.072	0.972	1.312	-4.228	-8.654	-4.611	1.314	-6.837	1.797
MSFT	-3.676	-0.802	-0.801	-1.36	-1.257	-2.113	0.698	1.765	2.06	-6.041	-4.8	-70.433	-1.275
NKE	-2.615	-21.368	14.849	5.685	4.77	-8.212	-10.538	-13.843	-16.463	-2.457	-18.996	19.04	3.519
NVDA	-4.89	-5.313	-2.642	1.372	-1.416	-2.267	-4.012	-0.885	-2.306	-8.803	-13.249	-69.237	-1.348
PG	-4.601	0.67	-3.931	1.192	1.18	2.114	-0.059	-5.457	-9.058	-4.837	-0.719	-3.835	1.799
SHW	-2.714	-0.723	-0.14	-4.447	-4.373	-2.401	0.699	-11.95	-12.065	-4.709	-3.125	-5.958	-4.503
TRV	-1.102	-1.315	-2.333	-0.467	-0.082	-1.61	0.166	-1.675	-2.851	-1.264	-0.969	-11.018	-0.812
UNH	-1.5	4.014	0.482	-0.169	0.494	1.302	3.377	-2.932	-9.776	-2.627	1.045	-11.459	-0.876
V	-2.302	-1.271	0.667	2.719	2.775	-4.329	-5.257	-0.145	-0.887	-6.119	-2	1.798	2.898
VZ	0.311	-1.709	-14.844	1.776	0.658	1.208	-2.02	-0.243	-1.025	0.947	0.713	-14.293	2.16
WMT	-2.308	-3.426	-1.994	3.775	3.334	-5.033	-6.958	-2.188	-4.455	-6.519	-4.128	1.349	1.846
Average	-2.142	-1.474	-1.137	**0.579**	0.448	-1.295	0.011	-3.595	-6.007	-4.539	-4.123	-16.267	0.071

	$PBPV_{N}$	RV_{N}^{AC}	RV_{N}^{NW}	RK_{N}^{MTH}	RK_{N}^{P}	$RV_{T}^{(78)}$	$RV_{T}^{(390)}$	$RBPV_{T}$	PRV_{T}	$PBPV_{T}$	RV_{T}^{AC}
AAPL	-1.875	-0.359	-7.797	-11.699	-8.815	-0.363	-2.338	1.643	-2.983	-2.417	1.116
AMGN	-2.676	-0.046	-5.593	-3.839	-4.573	-3.132	-0.695	5.853	-1.156	-1.134	-2.978
AMZN	-6.473	-30.819	-66.919	-10.664	-15.359	-2.381	2.835	-1.674	-0.832	-0.9	-0.732
AXP	3.415	-1.224	0.708	-13.113	-22.405	-0.661	-3.28	-11.179	2.636	2.04	1.137
BA	1.174	-2.326	-1.079	-5.27	-13.989	-4.522	-4.945	-4.509	0.666	1.66	-4.864
CAT	2.552	-3.421	-1.627	-0.496	-3.636	-0.887	-1.636	-13.627	1.442	1.342	-2.562
CRM	-2.355	-3.183	-4.445	-1.387	-3.991	-4.472	-1.433	-4.878	-1.81	-1.637	-0.278
CSCO	0.049	3.38	8.488	-7.88	-10.593	-5.058	5.124	2.61	-3.447	-1.622	3.385
CVX	-0.298	-0.396	1.861	-4.909	-6.297	-1.888	-0.152	-3.874	-0.695	-0.561	0.338
DIS	0.249	-1.652	-0.236	-5.385	-8.718	-1.728	-0.481	-6.236	-1.251	-0.936	-0.305
GS	1.686	0.644	-3.261	-3.531	-7.033	-2.704	-0.682	-18.58	2.553	2.322	0.303
HD	-0.493	-2.158	-0.401	-2.919	-5.508	-2.994	1.776	-16.947	0.716	0.648	1.102
HON	-0.49	-6.929	-7.835	-8.019	-12.279	-0.463	0.507	-0.046	2.161	2.073	-0.023
IBM	0.018	-0.095	0.506	-2.427	-3.87	-1.219	-2.329	-14.254	-0.651	-0.399	-1.326
JNJ	-0.064	-1.522	-1.004	-1.707	-1.933	-0.41	2.01	-11.012	-1.703	-1.932	-0.771
JPM	1.259	-2.125	-0.584	-2.202	-1.879	1.029	0.266	-13.488	-0.676	-0.998	-3.58
KO	0.856	1.581	0.814	-4.813	-9.735	-1.735	-1.591	-9.663	0.342	0.026	0.212
MCD	2.93	-4.486	-4.037	-6.329	-9.852	-1.632	-5.027	-8.404	1.662	2.088	-6.031
MMM	-0.546	-4.603	0.525	-8.251	-9.848	-4.647	-5.568	-0.331	-2.405	-2.237	-5.197
MRK	1.712	0.867	0.821	-5.463	-6.476	-6.469	0.241	-10.562	0.913	0.823	-0.689
MSFT	-1.226	-2.661	-5.087	3.86	-0.004	-6.411	-1.857	-0.421	-2.836	-1.994	-1.49
NKE	2.455	-13.018	-14.364	-12.547	-14.018	-0.668	-15.233	9.308	3.912	1.013	-7.115
NVDA	-0.354	-4.183	-30.119	-3.09	-7.173	1.178	-6.455	-4.849	-0.56	-0.868	-6.154
PG	1.707	0.571	-0.02	-6.425	-6.331	-6.229	-0.23	-11.394	1.792	2.211	2.044
SHW	-4.453	-5.149	-1.372	-12.607	-17.458	0.572	-4.15	-16.06	-1.01	-0.757	-2.205
TRV	-0.688	-0.755	-0.619	-0.086	-1.68	-2.816	-2.474	-7.385	-0.866	-0.14	-3.041
UNH	-0.478	0.43	1.272	-6.995	-13.5	-1.944	1.828	-10.824	-0.771	-0.316	1.146
V	2.55	-3.604	-3.733	-3.016	-2.253	-5.433	-2.442	-4.78	3.339	2.45	-3.491
VZ	1.868	1.618	-2.235	-0.823	-2.519	-2.851	-3.24	-18.561	-0.184	-0.835	-1.29
WMT	1.682	-5.178	-7.555	-6.594	-8.661	-3.143	-4.305	-10.759	2.284	2.677	-5.186
Average	0.123	-3.027	-5.164	-5.287	-8.013	-2.469	-1.865	-7.163	0.019	0.056	-1.618

	RV_{T}^{NW}	RK_{T}^{MTH}	RK_{T}^{P}	$RV_{K}^{(78)}$	$RV_{K}^{(390)}$	$RBPV_{K}$	PRV_{K}	$PBPV_{K}$	RV_{K}^{AC}	RV_{K}^{NW}	RK_{K}^{MTH}	RK_{K}^{P}	$RV_{P}^{(78)}$	$RV_{P}^{(390)}$
AAPL	2.686	-4.462	-6.526	-1.142	-1.11	-0.555	0.001	0.748	-0.153	3.843	-6.068	-6.732	1.585	-0.984
AMGN	3.943	-0.484	-1.571	1.61	-15.039	-13.159	-2.585	-2.736	2.059	-3.328	-1.138	-3.795	-4.236	-15.599
AMZN	0.211	-1.284	-1.319	-4.55	2.888	-12.282	-6.297	-4.161	-4.32	3.314	-0.775	-0.907	-1.771	1.769
AXP	-2.068	-7.175	-12.863	-2.147	-16.281	-27.309	2.099	2.623	-1.589	-14.543	-10.609	-12.086	-2.242	-9.587
BA	-1.15	-3.802	-8.051	-2.464	-2.372	-8.465	3.852	2.985	0.049	0.487	-2.87	-11.629	-3.446	-1.795
CAT	-2.054	-1.356	-1.954	-3.528	-21.581	-39.46	-2.864	-2.523	-12.214	-23.62	-1.426	-4.387	-3.416	-20.193
CRM	2.028	-2.774	-6.296	-4.285	-18.594	-32.255	-0.936	-1.322	-0.334	-20.066	-3.246	-4.732	-1.088	-4.812
CSCO	9.316	-9.569	-9.06	-3.914	0.748	0.95	-1.506	-0.685	1.452	3.993	-10.616	-12.725	-5.737	-0.542
CVX	0.786	-3.137	-9.125	0.151	-2.699	-23	-0.749	-0.678	-0.449	-3.554	-3.313	-7.468	-3.19	-1.812
DIS	0.722	-4.781	-7.574	-1.823	-1.25	-3.388	-0.067	0.14	-1.287	1.724	-3.325	-6.491	0.699	-0.563
GS	0.224	-6.89	-9.185	-3.193	-15.559	-44.847	2.887	3.215	-6.768	-15.081	-10.226	-9.146	-4.239	-28.183
HD	-1.412	-3.998	-6.132	-0.372	-11.719	-22.663	1.827	1.143	-3.296	-12.5	-0.732	-4.223	-5.363	-4.135
HON	2.288	-4.534	-7.978	-3.721	-16.85	-21.163	-3.11	-3.6	-10.057	-14.422	-2.183	-4.617	-13.628	-33.762
IBM	-2.66	-4.305	-6.059	-1.559	-10.633	-28.092	0.559	0.631	-3.927	-14.65	0.374	-2.331	-1.349	-2.808
JNJ	-0.588	-1.112	-1.026	1.71	-2.619	-25.842	-0.001	-0.062	-4.183	-8.196	-2.361	-4.226	-1.531	1.497
JPM	-3.58	-2.509	-3.336	-2.555	-0.632	-22.886	0.365	-0.221	-0.687	-4.062	-3.983	-9.388	0.506	-1.297
KO	1.345	-4.901	-9.106	-0.608	-0.758	-33.229	2.114	2.054	1.172	-3.559	-5.028	-7.853	-2.505	1.143
MCD	-3.802	-5.275	-4.331	0.778	-18.266	-8.402	-0.975	-0.8	-12.327	-7.601	-0.049	-2.815	-0.41	-2.247
MMM	-1.496	-7.646	-4.769	-1.067	3.082	-6.1	-0.795	0.388	-3.785	-0.038	-5.811	-4.684	-0.237	3.949
MRK	-2.004	-4.447	-9.362	-9.091	-5.538	-34.177	1.526	1.027	-3.618	-17.119	-9.863	-14.485	-6.338	-2.206
MSFT	-0.049	-0.34	2.042	-3.974	1.053	-13.727	-0.53	-0.749	-0.709	0.487	1.102	2.611	-3.665	-1.062
NKE	-0.167	-15.286	-11.355	5.025	0.159	1.792	3.241	2.872	0.845	6.082	-0.57	-0.92	-0.109	-2.901
NVDA	-2.983	-0.764	-5.383	-1.09	-7.406	3.04	-3.444	-4.542	-4.858	2.327	-3.702	-4.876	-0.811	-5.527
PG	-1.75	-7.641	-10.523	-2.051	-4.568	-25.28	3.974	3.168	2.627	-9.431	-6.306	-10.036	-7.548	-3.367
SHW	-3.147	-3.188	-7.496	-10.725	-9.995	-9.175	-6.753	-6.205	-14.844	-7.596	-3.109	-6.368	-5.95	-18.139
TRV	-0.868	-1.583	-3.724	-5.01	-6.051	-12.415	0.115	0.426	-4.612	-2.663	-7.2	-7.206	-12.048	-33.317
UNH	3.287	-4.382	-11.999	-3.551	-7.993	-8.773	0.216	0.258	-0.911	-3.718	-5.522	-14.023	-3.142	-4.475
V	-4.708	-5.299	-3.956	0.099	-3.545	-29.01	5.302	5.572	-18.489	-17.238	-0.679	-1.01	0.768	-10.681
VZ	-3.789	-0.18	-2.067	-0.755	-7.15	-37.104	1.438	0.143	2.738	-12.18	1.011	-1.917	0.469	-4.388
WMT	-9.131	-6.113	-6.467	3.468	-1.986	-12.583	5.883	4.348	-8.51	-17.363	1.421	-2.787	1.138	-2.944
Average	-0.686	-4.307	-6.218	-2.011	-6.742	-18.319	0.160	0.115	-3.699	-7.009	-3.560	-6.042	-2.961	-6.966

	$RBPV_{P}$	PRV_{P}	$PBPV_{P}$	RV_{P}^{AC}	RV_{P}^{NW}	RK_{P}^{MTH}	RK_{P}^{P}	$RV_{Z}^{(78)}$	$RV_{Z}^{(390)}$	$RBPV_{Z}$	PRV_{Z}	$PBPV_{Z}$	RV_{Z}^{AC}	RV_{Z}^{NW}	RK_{Z}^{MTH}	RK_{Z}^{P}
AAPL	-3.658	-1.72	-0.312	0.675	3.463	-4.229	-4.224	-0.312	-1.255	-0.102	0.729	0.975	1.406	3.287	-5.104	-5.943
AMGN	-12.509	-7.308	-6.416	-7.072	-5.321	-0.608	-7.217	5.17	8.82	1.689	-4.135	-2.624	6.942	15.566	1.004	-2.478
AMZN	-49.551	-2.012	-0.459	1.785	-17.386	0.073	-2.801	1.143	-5.272	-58.556	0.471	0.695	-0.042	-20.056	-2.45	-4.428
AXP	-26.553	2.897	2.896	-2.186	-4.377	-9.202	-11.978	-2.136	-3.53	-21.572	1.356	1.618	-0.201	-0.275	-10.975	-8.74
BA	-79.112	1.318	1.431	-3.033	-1.839	-7.855	-14.647	-0.47	-2.562	-22.218	2.439	1.748	-2.309	-3.152	-3.808	-10.378
CAT	-52.403	0.282	0.405	-2.364	-17.767	-5.183	-10.827	-5.234	-3.364	-18.873	0.724	0.304	-3.409	-4.348	-5.224	-5.622
CRM	-63.505	-1.838	-1.915	-0.883	-13.568	-1.607	-8.877	-3.3	-5.376	-14.624	-0.777	-0.299	-5.568	-0.302	-5.069	-10.837
CSCO	4.274	-2.228	-1.159	-0.39	8.575	-10.249	-9.788	-0.892	1.761	0.041	-2.808	-2.085	4.485	8.44	-3.635	-4.225
CVX	-15.513	-1.114	-0.499	-1.097	-0.477	-7.025	-10.169	-1.814	0.485	-18.504	-0.804	-0.791	1.437	-0.806	-6.618	-5.321
DIS	-22.29	1.157	1.065	-2.433	-3.1	-5.627	-7.899	-1.876	-1.239	-7.511	0.085	-0.806	-2.34	-1.322	-7.541	-9.55
GS	-68.869	1.903	2.362	-3.842	-21.871	-8.697	-7.829	-0.859	-3.103	-21.302	1.408	1.397	-4.004	-5.137	-9.04	-9.839
HD	-23.465	-1.78	-1.985	-3.214	-10.255	-0.518	-6.822	-3.72	-1.143	-8.4	-0.545	-0.783	-2.539	-3.165	-0.738	-3.236
HON	-52.505	-0.939	-1.281	-10.004	-35.041	-9.74	-16.719	-0.083	2.013	-2.599	1.356	2.139	0.748	3.816	-6.003	-8.034
IBM	-19.019	-0.244	0.863	-0.637	-2.592	-2.759	-9.305	-3.097	-1.186	-18.063	1.443	1.352	-0.97	-1.663	-4.12	-5.496
JNJ	-27.016	-0.978	-1.21	-0.626	-7.686	3.1	-0.399	-1.877	0.653	-23.337	-0.349	-0.553	-2.004	-2.923	-1.846	-4.001
JPM	-34.358	-0.52	-0.454	-1.046	-4.634	-3.254	-4.192	-0.916	1.552	-8.818	0.322	0.565	-0.617	-0.544	-2.874	-6.23
KO	-8.64	1.58	1.216	0.505	0.773	-5.191	-8.746	-1.581	1.478	-8.438	0.951	0.3	1.143	1.65	-5.94	-12.669
MCD	-7.478	2.854	2.759	-4.67	-3.939	-3.68	-7.808	-1.683	-5.222	-10	2.21	1.88	-2.408	-1.602	-9.145	-5.6
MMM	-17.393	2.789	3.339	-0.307	3.575	-9.486	-7.519	-3.764	-0.808	-3.896	0.224	0.893	-2.458	2.793	-7.57	-4.873
MRK	-12.819	0.574	0.8	-2.099	-3.315	-7.077	-14.002	-9.377	-1.186	-10.429	2.289	1.964	0.987	-2.601	-6.429	-10.259
MSFT	-14.059	-1.733	-1.679	-3.088	-3.361	1.651	1.771	-7.245	-2.607	-8.46	-0.095	-0.693	-2.844	0.511	3.488	3.769
NKE	6.717	2.983	2.192	-1.819	-0.763	-13.805	-8.099	2.306	-7.444	-2.131	0.619	0.979	-0.008	-1.648	-4.355	-3.947
NVDA	-12.966	-1.784	-2.101	-3.923	-4.736	-0.81	-6.667	-1.309	-5.508	-58.381	0.374	0.561	-2.663	-6.086	-0.156	-2.364
PG	-25.062	0.082	-0.416	-0.699	-6.279	-5.444	-8.279	-1.593	-0.678	-22.161	1.523	1.07	-0.474	-2.493	-8.016	-11.216
SHW	-23.415	-2.858	-2.103	-2.107	-12.389	-0.746	-2.369	-8.554	-10.221	-28.853	-2.442	-2.682	-4.38	-10.518	-6.413	-8.391
TRV	-45.773	-3.275	-2.41	-11.747	-19.596	-3.351	-4.72	-1.943	-1.143	-15.535	-1.327	-1.243	-3.297	-3.606	-5.923	-7.205
UNH	-16.45	-0.441	-0.101	-1.449	-4.718	-8.276	-15.569	-2.657	-2.335	-16.109	0.402	-0.304	-3.729	-2.022	-7.778	-12.811
V	-38.434	2.454	2.258	-9.948	-17.722	-2.852	-4.199	-2.677	-9.168	-23.521	0.643	0.658	-9.854	-9.852	-2.397	-3.533
VZ	-34.575	2.395	2.613	-0.039	-9.361	-1.163	-2.755	-1.091	-2.054	-21.389	0.913	0.681	-0.279	-4.702	0.043	-2.639
WMT	-22.393	2.177	1.962	-2.241	-5.939	0.301	-0.999	-1.456	-5.394	-17.544	2.339	2.05	-4.72	-11.158	-0.456	-1.671
Average	-27.293	-0.178	0.055	-2.667	-7.388	-4.444	-7.455	-2.097	-2.168	-16.320	0.318	0.299	-1.466	-2.131	-4.503	-6.259

This table reports the median annualized daily profits (in cents) from the artificial straddle option market. The pre-averaging and kernel-based estimators are computed using data sampled at 1-second intervals. The trading venues are denoted with their ticker symbol as the subscript: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

Table F.33: Maximum annualized daily profits (in cents) for each market

	GRT	N	T	K	P	Z
AAPL	3.148	−0.359	2.686	3.843	3.463	3.287
AMGN	10.519	−0.046	5.853	2.059	−0.608	15.566
AMZN	3.577	−5.256	2.835	3.314	1.785	1.143
AXP	2.893	3.415	2.636	2.623	2.897	1.618
BA	1.641	1.581	1.660	3.852	1.431	2.439
CAT	2.603	2.552	1.442	−1.426	0.405	0.724
CRM	1.972	−1.387	2.028	−0.334	−0.883	−0.299
CSCO	8.688	8.488	9.316	3.993	8.575	8.440
CVX	0.598	1.861	0.786	0.151	−0.477	1.437
DIS	0.740	0.249	0.722	1.724	1.157	0.085
GS	2.453	1.883	2.553	3.215	2.362	1.408
HD	0.801	0.896	1.776	1.827	−0.518	−0.545
HON	4.472	−0.490	2.288	−2.183	−0.939	3.816
IBM	1.146	0.506	−0.399	0.631	0.863	1.443
JNJ	2.867	3.101	2.010	1.710	3.100	0.653
JPM	0.258	1.259	1.029	0.365	0.506	1.552
KO	1.393	1.581	1.345	2.114	1.580	1.650
MCD	3.961	3.108	2.088	0.778	2.854	2.210
MMM	4.510	0.525	−0.331	3.082	3.949	2.793
MRK	2.072	1.797	0.913	1.526	0.800	2.289
MSFT	2.060	3.860	2.042	2.611	1.771	3.769
NKE	14.849	19.040	9.308	6.082	6.717	2.306
NVDA	1.372	−0.354	1.178	3.04	−0.810	0.561
PG	2.114	1.799	2.211	3.974	0.082	1.523
SHW	0.699	−1.372	0.572	−3.109	−0.746	−2.442
TRV	0.166	−0.086	−0.140	0.426	−2.410	−1.143
UNH	4.014	1.272	3.287	0.258	−0.101	0.402
V	2.775	2.898	3.339	5.572	2.454	0.658
VZ	1.776	2.160	−0.180	2.738	2.613	0.913
WMT	3.775	1.846	2.677	5.883	2.177	2.339
Average	3.130	1.878	2.251	2.011	1.468	2.020

This tables reports the median maximum annualized daily profit that can be realized from the set of estimators computed for each data source. The trading venues are denoted with their ticker symbol: NYSE ("N"), Nasdaq ("T"), Arca ("P"), Cboe EDGX ("K"), and Cboe BZX ("Z").

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