Detecting Multiple Structural Breaks in Systems of Linear Regression Equations with Integrated and Stationary Regressors – Supplementary Material A

September 17, 2024

1 Group LARS algorithm

We define some notation used in the exposition of the algorithm. Since our system is vectorized and the columns of Z have a specific structure in the change-point setting, we do not need to extend the correlation criterion as in Similä and Tikka (2006) to account for multiple responses. A simple re-partitioning before the most correlated set is computed allows us to use a modified version of the algorithm proposed by Chan et al. (2014) which itself is a specific adaptation of the group LARS algorithm outlined in Yuan and Lin (2006) to the univariate change-point setting.

We define the $Tq \times d$ matrix $\overline{Z} = I \otimes Z$, where the columns of Z contain the identical regressors for all responses. For $j = 1, \ldots, Tq$, we define the d vector

$$\boldsymbol{B}_j(\boldsymbol{\nu}) = \sum_{l=j}^T \bar{Z}_l' \boldsymbol{\nu}_l.$$

Moreover, we define the $Tq \times d$ matrix $\mathbf{B}(\nu) = (\mathbf{B}'_1(\nu), \dots, \mathbf{B}'_{Tq}(\nu))'$ which has q blocks of dimension $T \times d$. Now, we define the $T \times qd$ matrix $\mathbf{B}^*(\nu)$ re-partitioning $\mathbf{B}(\nu)$ so that the q blocks are concatenated horizontally. $\mathbf{B}^*_j(\nu)$ denotes the j-th row of $\mathbf{B}^*(\nu)$. The matrix $\mathbf{Z}_{\mathcal{A}}$ consists of all columns of \mathbf{Z} that belong to the change-points contained in \mathcal{A} . The implementation of the modified group LARS algorithm on multiple change-points estimation is given below:

1. Initialization: specify K, the maximum number of change-points, and Δ , the minimum distance between change-points. Set $\mu^{[0]} = 0$, k = 1, $\nu^{[0]} = \mathbf{Y}$, $\mathcal{A}_0 = \{\emptyset\}$, and $\mathcal{T} = \{1, \ldots, T\}$.

2. Compute the current "most correlated set"

$$\mathcal{A}_k = \underset{j \in \mathcal{T}}{\operatorname{arg\,max}} \| \boldsymbol{B}_j^*(\boldsymbol{\nu}^{[k-1]}) \|_2.$$

3. Descent direction computation

$$\gamma_{\mathcal{A}_k} = (\mathbf{Z}'_{\mathcal{A}_k} \mathbf{Z}_{\mathcal{A}_k})^{-1} \mathbf{Z}'_{\mathcal{A}_k} \nu^{[k-1]}.$$

4. Descent step search: For $j \in \mathcal{T} \setminus \mathcal{A}_k$ define

$$a_j = \|\boldsymbol{B}_j(\boldsymbol{\nu}^{[k-1]})\|^2, \quad b_j = \boldsymbol{B}'_j(\boldsymbol{Z}_{\mathcal{A}_k}\gamma_{\mathcal{A}_k})\boldsymbol{B}_j(\boldsymbol{\nu}^{[k-1]}),$$

$$c_j = \|\boldsymbol{B}_j(\boldsymbol{Z}_{\mathcal{A}_k}\gamma_{\mathcal{A}_k})\|^2, \quad d_j = \max_{j \in \mathcal{T} \setminus \mathcal{A}_k} a_j.$$

Set $\alpha = \min_{j \in \mathcal{T} \setminus \mathcal{A}_k} a_j \equiv \alpha_{j^*}$, where

$$\alpha_j^+ = \frac{(b_j - d_j) + \sqrt{(b_j - d_j)^2 - (a_j - d_j)(c_j - d_j)}}{c_j - d_j},$$

$$\alpha_j^- = \frac{(b_j - d_j) - \sqrt{(b_j - d_j)^2 - (a_j - d_j)(c_j - d_j)}}{c_j - d_j},$$

and

$$\alpha_j = \begin{cases} \alpha_j^+ & \text{if } \alpha_j^+ \in [0,1], \\ \alpha_j^- & \text{if } \alpha_j^- \in [0,1]. \end{cases}$$

5. If $\alpha \neq 1$ or k < K, update $\mathcal{A}_{k+1} = \mathcal{A}_k \cup \{j^*\}, \mu^{[k]} = \mu^{[k-1]} + \alpha \mathbf{Z}_{\mathcal{A}_k} \gamma_{\mathcal{A}_k}$ and $\nu^{[k]} = Y - \mu^{[k]}$. Set k = k + 1 and go back to step 3. Otherwise, return \mathcal{A}_k as the estimated changepoints.

2 Backward elimination algorithm

The Backward elimination algorithm (BEA) successively eliminates breakpoints until no improvement in terms of the chosen criterion can be reached. For this purpose, we define

$$IC(m, t) = S_T(t_1, \ldots, t_m) + m\omega_T,$$

where $S_T(t_1, \ldots, t_m)$ is the least squares objective function for the pre-selected set of breakpoints and ω_T is the penalty function. The implementation of the BEA is given below:

1. Set
$$K = |\mathcal{A}_T|$$
, $t_K = (t_{K,1}, \dots, t_{K,K}) = \mathcal{A}_T$ and $V_K^* = IC(K, \mathcal{A}_T)$.

- 2. For i = 1, ..., K, compute $V_{K,i} = IC(K 1, t_K \setminus \{t_{K,i}\})$. Set $V_{K-1}^* = \min_i V_{K,i}$.
- 3. If $V_{K-1}^* > V_K^*$, then the estimated changepoints are $\mathcal{A}_T^* = \mathbf{t}_K$.
 - If $V_{K-1}^* \ge V_K^*$ and K = 1, then $\mathcal{A}_T^* = \emptyset$
 - If $V_{K-1}^* \ge V_K^*$ and K > 1, then set $j = \underset{i}{\operatorname{arg\,min}} V_{K,i}$, $t_{K-1} = t_K \setminus \{t_{K-1,j}\}$) and K = K 1. Go to step 2.

3 Additional simulation results

Table S1: Estimation of (multiple) structural breaks in the full model (c = 0.25)

	Panel A: Gro	oup LASS	O with BEA				
	SB1: $(\tau = 0.$	5)					
T	Time [s]	pce	au				
100	0.04	87.4	0.503(0.053)				
200	0.07	95.1	0.499(0.035)				
400	0.28	98.8	0.499(0.019)				
800	1.73	99.8	0.500 (0.001)				
	SB2: $(\tau_1 = 0$.33, $\tau_2 =$	0.67)				
T	Time [s]	pce	$ au_1$	τ_2			
150	0.08	65.5	0.334(0.059)	0.664(0.042)			
300	0.29	86.6	0.332(0.034)	0.668(0.018)			
600	1.45	95.7	0.332(0.021)	0.669(0.009)			
1200	11.29	99.1	$0.331 \ (0.011)$	0.669(0.006)			
	SB4: $(\tau_1 = 0$	$.2, \tau_2 = 0$	$.4, \tau_3 = 0.6, \tau_4 = 0.6$	0.8)			
T	Time [s]	pce	$ au_1$	$ au_2$	$ au_3$	$ au_4$	
250	0.14	40.1	0.232(0.064)	0.425(0.074)	0.609(0.068)	0.795(0.040)	
500	0.74	59.9	0.214(0.048)	0.414(0.049)	0.606(0.037)	0.800 (0.013)	
1000	10.45	82.8	0.200 (0.020)	0.402(0.017)	0.600 (0.015)	0.799(0.007)	
2000	59.55	95.8	0.200(0.008)	0.400(0.007)	0.600(0.005)	0.800(0.004)	
	Panel B: Lik	elihood-b	ased approach				
	Panel B: Like	elihood-b	ased approach				
Т	Panel B: Like SB1: $(\tau = 0.$ Time [s]	elihood-b 5) pce	ased approach τ				
T 100	Panel B: Like SB1: $(\tau = 0.$ Time [s] 0.91	elihood-b 5) pce 93.6	ased approach τ 0.500 (0.126)				
T 100 200	Panel B: Like SB1: $(\tau = 0.$ Time [s] 0.91 3.48	elihood-b 5) <u>pce</u> 93.6 93.8	τ 0.500 (0.126) 0.498 (0.068)				
T 100 200 400	Panel B: Like SB1: $(\tau = 0.$ Time [s] 0.91 3.48 16.50	elihood-ba 5) <u>pce</u> 93.6 93.8 95.9	$\frac{\tau}{0.500\ (0.126)}$ 0.500 (0.126) 0.498 (0.068) 0.500 (0.021)				
T 100 200 400 800	Panel B: Like SB1: $(\tau = 0.$ Time [s] 0.91 3.48 16.50 77.99	elihood-ba 5) <u>pce</u> 93.6 93.8 95.9 95.4	τ 0.500 (0.126) 0.498 (0.068) 0.500 (0.021) 0.500 (0.013)				
T 100 200 400 800	Panel B: Like SB1: $(\tau = 0.$ Time [s] 0.91 3.48 16.50 77.99 SB2: $(\tau_1 = 0.$	elihood-ba 5) pce 93.6 93.8 95.9 95.4 1.33, $\tau_2 =$	τ 0.500 (0.126) 0.498 (0.068) 0.500 (0.021) 0.500 (0.013) 0.67)				
T 100 200 400 800 T	$\begin{array}{c} \mbox{Panel B: Lik} \\ \mbox{SB1: } (\tau = 0. \\ \mbox{Time [s]} \\ \mbox{0.91} \\ \mbox{3.48} \\ \mbox{16.50} \\ \mbox{77.99} \\ \mbox{SB2: } (\tau_1 = 0 \\ \mbox{Time [s]} \end{array}$	elihood-base 5) pce 93.6 93.8 95.9 95.4 1.33, $\tau_2 = pce$	$\begin{array}{c} \hline \\ \hline $	τ2			
T 100 200 400 800 T 150	$\begin{tabular}{ c c c c c } \hline Panel B: Like \\ \hline SB1: (\tau = 0. \\ \hline Time [s] \\ \hline 0.91 \\ 3.48 \\ 16.50 \\ 77.99 \\ \hline SB2: (\tau_1 = 0 \\ Time [s] \\ \hline 3.49 \end{tabular}$	elihood-bo 5) pce 93.6 93.8 95.9 95.4 .33, $\tau_2 = pce$ 35.0	$\begin{tabular}{ c c c c c c c } \hline τ \\ \hline 0.500 (0.126) \\ 0.498 (0.068) \\ 0.500 (0.021) \\ 0.500 (0.013) \\ \hline 0.500 (0.013) \\ \hline $0.67)$ \\ \hline τ_1 \\ \hline 0.347 (0.106) \\ \hline \end{tabular}$	$\frac{\tau_2}{0.661 (0.100)}$			
T 100 200 400 800 T 150 300	$\begin{array}{c} \mbox{Panel B: Likk} \\ \mbox{SB1: } (\tau = 0. \\ \mbox{Time [s]} \\ \mbox{0.91} \\ \mbox{3.48} \\ \mbox{16.50} \\ \mbox{77.99} \\ \mbox{SB2: } (\tau_1 = 0 \\ \mbox{Time [s]} \\ \mbox{3.49} \\ \mbox{3.43} \end{array}$	elihood-b 5) pce 93.6 93.8 95.9 95.4 .33, $\tau_2 = pce$ 35.0 80.6	$ \begin{array}{c} \tau \\ \hline 0.500 \ (0.126) \\ 0.498 \ (0.068) \\ 0.500 \ (0.021) \\ 0.500 \ (0.013) \\ \hline \tau_1 \\ \hline 0.347 \ (0.106) \\ 0.333 \ (0.053) \\ \end{array} $	$ \frac{ au_2}{ 0.661\ (0.100)}$ 0.670 (0.043)			
T 100 200 400 800 T 150 300 600	Panel B: Lik SB1: $(\tau = 0.$ Time [s] 0.91 3.48 16.50 77.99 SB2: $(\tau_1 = 0)$ Time [s] 3.49 13.43 45.49	elihood-b 5) pce 93.6 93.8 95.9 95.4 		$ au_2 \\ 0.661 (0.100) \\ 0.670 (0.043) \\ 0.671 (0.012) \\ \end{array}$			
T 100 200 400 800 T 150 300 600 1200	$\begin{array}{c} \text{Panel B: Lik}\\ \text{SB1: } (\tau = 0.\\ \text{Time [s]}\\ 0.91\\ 3.48\\ 16.50\\ 77.99\\ \text{SB2: } (\tau_1 = 0\\ \text{Time [s]}\\ 3.49\\ 13.43\\ 45.49\\ 316.57\\ \end{array}$	elihood-b 5) pce 93.6 93.8 95.9 95.4 1.33, $\tau_2 = pce$ 35.0 80.6 96.2 95.4	$ \frac{\tau}{0.500 (0.126)} \\ 0.498 (0.068) \\ 0.500 (0.021) \\ 0.500 (0.013) \\ 0.67) \\ \hline \tau_1 \\ \hline 0.347 (0.106) \\ 0.333 (0.053) \\ 0.330 (0.014) \\ 0.330 (0.008) \\ \hline \end{tabular}$	$\begin{array}{c} \tau_2 \\ 0.661 \ (0.100) \\ 0.670 \ (0.043) \\ 0.671 \ (0.012) \\ 0.669 \ (0.007) \end{array}$			
T 100 200 400 800 T 150 300 600 1200	$\begin{array}{c} \text{Panel B: Lik}\\ \text{SB1: } (\tau = 0,\\ \text{Time [s]}\\ 0.91\\ 3.48\\ 16.50\\ 77.99\\ \text{SB2: } (\tau_1 = 0\\ \text{Time [s]}\\ 3.49\\ 13.43\\ 45.49\\ 316.57\\ \text{SB4: } (\tau_1 = 0\\ \end{array}$	elihood-b 5) pce 93.6 93.8 95.9 95.4 .33, $\tau_2 = pce$ 35.0 80.6 96.2 95.4 .2, $\tau_2 = 0$	$\begin{tabular}{ c c c c c c c } \hline τ \\ \hline t 0.500 (0.126) \\ $0.498 (0.068) \\ $0.500 (0.021) \\ $0.500 (0.013) \\ \hline t 0.500 (0.013) \\ \hline t 0.347 (0.106) \\ $0.333 (0.053) \\ $0.330 (0.014) \\ $0.330 (0.008) \\ t 4, $\tau_3 = 0.6, $\tau_4 = 1$ \\ \hline \end{tabular}$	$\begin{array}{c} \tau_2 \\ \hline 0.661 \ (0.100) \\ 0.670 \ (0.043) \\ 0.671 \ (0.012) \\ 0.669 \ (0.007) \\ 0.8 \end{array}$			
T 100 200 400 800 T 150 300 600 1200 T	$\begin{array}{c} \mbox{Panel B: Lik} \\ \mbox{SB1: } (\tau = 0. \\ \mbox{Time [s]} \\ \mbox{0.91} \\ \mbox{3.48} \\ \mbox{16.50} \\ \mbox{77.99} \\ \mbox{SB2: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{3.49} \\ \mbox{3.43} \\ \mbox{45.49} \\ \mbox{316.57} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ \mbox{Time [s]} \\ \mbox{SB4: } (\tau_1 = 0. \\ \mbox{Time [s]} \\ T$	elihood-b 5) pce 93.6 93.8 95.9 95.4 .33, $\tau_2 = pce$ 35.0 80.6 96.2 95.4 .2, $\tau_2 = 0$	$ \begin{array}{c} \hline \tau \\ \hline 0.500 \ (0.126) \\ 0.498 \ (0.068) \\ 0.500 \ (0.021) \\ 0.500 \ (0.013) \\ \hline 0.67) \\ \hline \tau_1 \\ \hline 0.347 \ (0.106) \\ 0.333 \ (0.053) \\ 0.330 \ (0.014) \\ 0.330 \ (0.008) \\ .4, \ \tau_3 = 0.6, \ \tau_4 = t \\ \hline \tau_1 \end{array} $	$\begin{array}{c} \tau_2 \\ \hline 0.661 & (0.100) \\ 0.670 & (0.043) \\ 0.671 & (0.012) \\ 0.669 & (0.007) \\ 0.8) \\ \tau_2 \end{array}$	73		
$\frac{T}{100}$ 200 400 800 $\frac{T}{150}$ 300 600 1200 $\frac{T}{250}$	Panel B: Lik SB1: $(\tau = 0.$ Time [s] 0.91 3.48 16.50 77.99 SB2: $(\tau_1 = 0.$ Time [s] 3.49 13.43 45.49 316.57 SB4: $(\tau_1 = 0.$ Time [s] 2.49 13.43 45.49 316.57	elihood-b 5) pce 93.6 93.8 95.9 95.4 .33, $\tau_2 = pce$ 35.0 80.6 96.2 95.4 .2, $\tau_2 = 0$ pce 6.7	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \tau_2 \\ \hline 0.661 & (0.100) \\ 0.670 & (0.043) \\ 0.671 & (0.012) \\ 0.669 & (0.007) \\ \hline 0.8) \\ \hline \tau_2 \\ \hline 0.403 & (0.034) \\ \end{array}$	73 0.599 (0.033)	74 0.799 (0.028)	
T 100 200 400 800 T 150 300 600 1200 T 250 500	$\begin{array}{c} \mbox{Panel B: Lik} \\ \mbox{SB1: } (\tau = 0. \\ \mbox{Time [s]} \\ \mbox{0.91} \\ \mbox{3.48} \\ \mbox{16.50} \\ \mbox{77.99} \\ \mbox{SB2: } (\tau_1 = 0 \\ \mbox{Time [s]} \\ \mbox{3.49} \\ \mbox{3.43} \\ \mbox{45.49} \\ \mbox{316.57} \\ \mbox{SB4: } (\tau_1 = 0 \\ \mbox{Time [s]} \\ \mbox{Time [s]} \\ \mbox{4.99} \\ \mbox{6.73} \\ \mbox{6.73} \end{array}$	elihood-b 5) pce 93.6 93.8 95.9 95.4 1.33, $\tau_2 = pce$ 35.0 80.6 96.2 95.4 1.2, $\tau_2 = 0$ pce 6.7 52.4	$ \begin{array}{c} \hline \tau \\ \hline \hline 0.500 \ (0.126) \\ 0.498 \ (0.068) \\ 0.500 \ (0.021) \\ 0.500 \ (0.013) \\ \hline 0.67) \\ \hline \tau_1 \\ \hline 0.347 \ (0.106) \\ 0.333 \ (0.053) \\ 0.330 \ (0.014) \\ 0.330 \ (0.008) \\ \hline .4, \ \tau_3 = 0.6, \ \tau_4 = 1 \\ \hline \tau_1 \\ \hline 0.204 \ (0.031) \\ 0.199 \ (0.018) \\ \hline \end{array} $	$\begin{array}{c} \tau_2 \\ \hline 0.661 \ (0.100) \\ 0.670 \ (0.043) \\ 0.671 \ (0.012) \\ 0.669 \ (0.007) \\ \hline \end{array}$	73 0.599 (0.033) 0.600 (0.017)	74 0.799 (0.028) 0.800 (0.016)	
T 100 200 400 800 T 150 300 600 1200 T 250 500 1000	$\begin{array}{c} \text{Panel B: Lik} \\ \text{SB1: } (\tau = 0. \\ \text{Time [s]} \\ 0.91 \\ 3.48 \\ 16.50 \\ 77.99 \\ \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ 3.49 \\ 13.43 \\ 45.49 \\ 316.57 \\ \text{SB4: } (\tau_1 = 0 \\ \text{Time [s]} \\ 14.99 \\ 61.73 \\ 316.27 \\ \end{array}$	elihood-b 5) pce 93.6 93.8 95.9 95.4 	$ \frac{\tau}{100000000000000000000000000000000000$	$\begin{array}{c} \tau_2 \\ \hline 0.661 \ (0.100) \\ 0.670 \ (0.043) \\ 0.671 \ (0.012) \\ 0.669 \ (0.007) \\ \hline \end{array}$		74 0.799 (0.028) 0.800 (0.016) 0.800 (0.007)	

Note: We use 1,000 replications of the data-generating process given in Equation (10) of the main text with c = 0.25. The variance of the error terms is $\sigma_{\xi}^2 = \sigma_e^2 = \sigma_u^2 = 1$. The first subpanel reports the results for one active breakpoint at $\tau = 0.5$, the second subpanel considers two active breakpoints at $\tau_1 = 0.33$ and $\tau_2 = 0.67$ and the third subpanel has four active breakpoints at $\tau_1 = 0.2$, $\tau_2 = 0.4$, $\tau_3 = 0.6$, and $\tau_4 = 0.8$. Time [s] denotes the average computing time for one replication in seconds. Standard deviations are given in parentheses. We conduct the $\sup(l + 1|l)$ test at the 5% level to determine the number of breaks.

Table S2: Estimation of (multiple) structural breaks in the full model (c = 0.5)

	Panel A: Group LASSO with BEA						
	SB1: $(\tau = 0.4$	5)					
T	Time [s]	pce	τ				
100	0.04	99.2	0.501(0.027)				
200	0.08	99.4	0.500(0.012)				
400	0.25	99.9	0.500(0.008)				
800	1.86	100	0.500(0.004)				
	SB2: $(\pi = 0$	33 m -	0.67)				
T	Time $[s]$	nce	0.01) T1	79			
150	0.07	01.0	0.227 (0.028)	0.050 (0.030)			
150	0.07	91.9	0.337 (0.038) 0.225 (0.010)	0.059 (0.030)			
500 600	0.24	91.2	0.333(0.019) 0.222(0.010)	0.000(0.010) 0.667(0.008)			
1200	1.44	99.9 100	0.332(0.010) 0.224(0.006)	0.007 (0.008) 0.667 (0.004)			
1200	11.00	100	0.334 (0.000)	0.007 (0.004)			
	SB4: $(\tau_1 = 0$	$.2, \tau_2 = 0$	$.4, \tau_3 = 0.6, \tau_4 = 0.6$	0.8)			
T	Time [s]	pce	$ au_1$	τ_2	τ_3	$ au_4$	
250	0.14	68.3	0.215(0.038)	0.407(0.034)	0.598(0.031)	0.792(0.028)	
500	0.66	91.6	0.202(0.018)	0.403(0.014)	0.598(0.012)	0.801(0.014)	
1000	10.11	99.7	0.200(0.008)	$0.401 \ (0.007)$	0.598(0.005)	0.800(0.007)	
2000	77.21	99.8	0.200(0.004)	$0.401 \ (0.004)$	$0.601 \ (0.003)$	$0.798\ (0.005)$	
	Donol D. Lik	libood b	acad approach				
	Tallel D. Like	ennood-b	ased approach				
	SB1: $(\tau = 0)$	5)					
Т		<i>,</i>					
	Time s	nce	τ				
100	Time [s]	<i>pce</i>	τ				
100	0.76 3.50	90.9	au 0.500 (0.030) 0.500 (0.010)				
100 200 400	0.76 3.50 17.50	pce 90.9 93.2 95.7	$\begin{array}{c} \tau \\ \hline 0.500 \ (0.030) \\ 0.500 \ (0.010) \\ 0.500 \ (0.005) \end{array}$				
100 200 400 800	0.76 3.50 17.50 79.67	pce 90.9 93.2 95.7 95.4	$\begin{array}{c} \tau \\ \hline 0.500 \ (0.030) \\ 0.500 \ (0.010) \\ 0.500 \ (0.005) \\ 0.500 \ (0.003) \end{array}$				
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \end{array} $	0.76 3.50 17.50 79.67	<i>pce</i> 90.9 93.2 95.7 95.4	$\begin{array}{c} \tau \\ \hline 0.500 \; (0.030) \\ 0.500 \; (0.010) \\ 0.500 \; (0.005) \\ 0.500 \; (0.003) \end{array}$				
100 200 400 800	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \text{SB2: } (\tau_1 = 0 \end{array}$	pce 90.9 93.2 95.7 95.4 .33, $\tau_2 =$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ 0.67) \end{array}$				
100 200 400 800 <i>T</i>	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \end{array}$	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ .33, \ \tau_2 = \\ pce \end{array}$	$\begin{array}{c} \tau \\ 0.500 \ (0.030) \\ 0.500 \ (0.010) \\ 0.500 \ (0.005) \\ 0.500 \ (0.003) \end{array}$	72			
	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ 3.63 \end{array}$	$\frac{pce}{90.9}$ 93.2 95.7 95.4 .33, $\tau_2 = \frac{pce}{94.1}$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline 0.67) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \end{array}$	$\frac{\tau_2}{0.667 (0.016)}$			
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \\ \hline T \\ 150 \\ 300 \\ \end{array} $	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ 3.63 \\ 13.98 \end{array}$	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline .33, \ \tau_2 = \\ pce \\ \hline 94.1 \\ 93.4 \end{array}$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ \end{array}$				
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \\ \hline T \\ 150 \\ 300 \\ 600 \\ \hline 600 \\ \hline $	$\frac{11me [s]}{0.76}$ $\frac{3.50}{17.50}$ $\frac{17.50}{79.67}$ $\frac{SB2: (\tau_1 = 0}{Time [s]}$ $\frac{3.63}{13.98}$ 62.03	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline .33, \ \tau_2 = \\ pce \\ \hline 94.1 \\ 93.4 \\ 95.8 \\ \end{array}$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ 0.330 & (0.004) \\ \end{array}$	$ \frac{ au_2}{ 0.667\ (0.016)} \\ 0.670\ (0.007) \\ 0.670\ (0.003) \\ ext{}$			
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \\ \hline T \\ 150 \\ 300 \\ 600 \\ 1200 \\ \end{array} $	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \end{array}$ SB2: $(\tau_1 = 0 \\ \text{Time [s]} \\ \begin{array}{c} 3.63 \\ 13.98 \\ 62.03 \\ 328.30 \\ \end{array}$	$\begin{array}{c} \hline pce \\ 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline 333, \tau_2 = \\ \hline pce \\ \hline 94.1 \\ 93.4 \\ 95.8 \\ 95.3 \\ \end{array}$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ 0.330 & (0.004) \\ 0.330 & (0.002) \\ \end{array}$	$\begin{array}{c} \tau_2 \\ 0.667 \ (0.016) \\ 0.670 \ (0.007) \\ 0.670 \ (0.003) \\ 0.669 \ (0.002) \end{array}$			
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \\ \hline T \\ 150 \\ 300 \\ 600 \\ 1200 \\ \end{array} $	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ \hline 3.63 \\ 13.98 \\ 62.03 \\ 328.30 \\ \end{array}$	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline \\$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ 0.330 & (0.004) \\ 0.330 & (0.002) \\ \hline \end{array}$	$\begin{array}{c} \tau_2 \\ \hline 0.667 \ (0.016) \\ 0.670 \ (0.007) \\ 0.670 \ (0.003) \\ 0.669 \ (0.002) \\ \end{array}$			
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \\ \hline T \\ 150 \\ 300 \\ 600 \\ 1200 \\ T \end{array} $	Time [s] 0.76 3.50 17.50 79.67 SB2: $(\tau_1 = 0$ Time [s] 3.63 13.98 62.03 328.30 SB4: $(\tau_1 = 0$ Time [c]	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline \\$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.004) \\ 0.330 & (0.002) \\ 0.330 & (0.002) \\ 0.44, \tau_3 = 0.6, \tau_4 = 0 \\ \hline \tau_1 \\ \hline \tau_$	$\frac{\tau_2}{0.667 (0.016)}$ 0.670 (0.007) 0.670 (0.003) 0.669 (0.002) 0.8)			
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \end{array} $ $ \begin{array}{r} T \\ 150 \\ 300 \\ 600 \\ 1200 \end{array} $ $ \begin{array}{r} T \\ T \\ \hline \end{array} $	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \end{array}$ $\begin{array}{c} \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ 3.63 \\ 13.98 \\ 62.03 \\ 328.30 \\ \end{array}$ $\begin{array}{c} \text{SB4: } (\tau_1 = 0 \\ \text{Time [s]} \\ \end{array}$	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline .33, \tau_2 = \\ pce \\ \hline 94.1 \\ 93.4 \\ 95.8 \\ 95.3 \\ .2, \tau_2 = 0 \\ pce \\ \hline \end{array}$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline 0.67) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ 0.330 & (0.004) \\ 0.330 & (0.002) \\ 0.4, \tau_3 = 0.6, \tau_4 = t \\ \hline \tau_1 \\ \hline 0.000 & (0.015) \\ \hline 0.000 & (0.015) \\ \hline \end{array}$	$\frac{\tau_2}{0.667 (0.016)}$ 0.667 (0.007) 0.670 (0.003) 0.669 (0.002) 0.8) τ_2 0.400 (0.012)	T3	74	
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \end{array} $ $ \begin{array}{r} T \\ 150 \\ 300 \\ 600 \\ 1200 \end{array} $ $ \begin{array}{r} T \\ 250 \\ 500 \end{array} $	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \end{array}$ $\begin{array}{c} \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ 3.63 \\ 13.98 \\ 62.03 \\ 328.30 \\ 328.30 \\ \end{array}$ $\begin{array}{c} \text{SB4: } (\tau_1 = 0 \\ \text{Time [s]} \\ 15.60 \\ 15.60 \\ \end{array}$	$\begin{array}{c} pce \\ \hline 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline 33, \tau_2 = \\ pce \\ \hline 94.1 \\ 93.4 \\ 95.8 \\ 95.3 \\ \hline .2, \tau_2 = 0 \\ pce \\ \hline 94.9 \\ \hline 94.$	$\begin{array}{c} \tau \\ \hline 0.500 \ (0.030) \\ 0.500 \ (0.010) \\ 0.500 \ (0.005) \\ 0.500 \ (0.003) \\ \hline \tau_1 \\ \hline 0.326 \ (0.023) \\ 0.330 \ (0.009) \\ 0.330 \ (0.009) \\ 0.330 \ (0.002) \\ 0.430 \ (0.002) \\ 1.4, \ \tau_3 = 0.6, \ \tau_4 = 0 \\ \hline \tau_1 \\ \hline 0.200 \ (0.012) \\ 0.0151 \\ \hline \end{array}$	$\begin{array}{c} \tau_2 \\ \hline 0.667 & (0.016) \\ 0.670 & (0.007) \\ 0.670 & (0.003) \\ 0.669 & (0.002) \\ \hline 0.80 \\ \hline \tau_2 \\ \hline 0.401 & (0.012) \\ 0.402 & (0.022) \\ \hline \end{array}$			
$ \begin{array}{r} 100 \\ 200 \\ 400 \\ 800 \\ \hline T \\ 150 \\ 300 \\ 600 \\ 1200 \\ \hline T \\ 250 \\ 500 \\ 500 \\ 1000 \\ \hline 1000 \\ 1000 $	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \end{array}$ $\begin{array}{c} \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ 3.63 \\ 13.98 \\ 62.03 \\ 328.30 \\ \end{array}$ $\begin{array}{c} \text{SB4: } (\tau_1 = 0 \\ \text{Time [s]} \\ 15.60 \\ 64.05 \\ 905 \ \text{cc} \end{array}$	$\begin{array}{c} pce \\ \hline pce \\ 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ \hline 33, \tau_2 = \\ pce \\ \hline 94.1 \\ 93.4 \\ 95.8 \\ 95.3 \\ 25.8 \\ 95.3 \\ 25.7 \\ \tau_2 = 0 \\ pce \\ \hline 94.9 \\ 100 \\ 00.7 \\ \hline \end{array}$	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ 0.330 & (0.009) \\ 0.330 & (0.009) \\ 0.330 & (0.002) \\ 0.44, \tau_3 = 0.6, \tau_4 = t \\ \hline \tau_1 \\ \hline 0.200 & (0.012) \\ 0.200 & (0.012) \\ 0.200 & (0.022) \\ \hline \end{array}$	$\begin{array}{c} \hline \tau_2 \\ \hline 0.667 & (0.016) \\ 0.670 & (0.007) \\ 0.670 & (0.003) \\ 0.669 & (0.002) \\ \hline 0.8) \\ \hline \tau_2 \\ \hline 0.401 & (0.012) \\ 0.400 & (0.005) \\ 0.400 & (0.005) \\ \hline \end{array}$	$ \frac{ au_3}{ 0.600\ (0.011)}$		
$\begin{array}{c} 100\\ 200\\ 400\\ 800\\ \hline T\\ 150\\ 300\\ 600\\ 1200\\ \hline T\\ \hline 250\\ 500\\ 1000\\ 9000\\ \hline \end{array}$	$\begin{array}{c} \text{Time [s]} \\ 0.76 \\ 3.50 \\ 17.50 \\ 79.67 \\ \end{array}$ $\begin{array}{c} \text{SB2: } (\tau_1 = 0 \\ \text{Time [s]} \\ \hline 3.63 \\ 13.98 \\ 62.03 \\ 328.30 \\ \end{array}$ $\begin{array}{c} \text{SB4: } (\tau_1 = 0 \\ \text{Time [s]} \\ \hline 15.60 \\ 64.05 \\ 305.63 \\ 055.63 \\ 055.64 \\ 15.60 \\ \end{array}$	$\begin{array}{c} pce \\ \hline pcg \\ 90.9 \\ 93.2 \\ 95.7 \\ 95.4 \\ 95.4 \\ 95.8 \\ 95.3 \\ 2, \ \tau_2 = 0 \\ pce \\ \hline 94.1 \\ 93.4 \\ 95.8 \\ 95.3 \\ 2, \ \tau_2 = 0 \\ pce \\ \hline 94.9 \\ 100 \\ 96.7 \\ 95.9 \\ \hline 05.9 \\ 96.7 \\ 95.9 \\ \hline 05.9 \\ 96.7 \\ 95.9 \\ \hline 05.9 \\ \hline $	$\begin{array}{c} \tau \\ \hline 0.500 & (0.030) \\ 0.500 & (0.010) \\ 0.500 & (0.005) \\ 0.500 & (0.003) \\ \hline \tau_1 \\ \hline 0.326 & (0.023) \\ 0.330 & (0.009) \\ 0.330 & (0.009) \\ 0.330 & (0.004) \\ 0.330 & (0.002) \\ 1.4, \tau_3 = 0.6, \tau_4 = t \\ \hline \tau_1 \\ \hline 0.200 & (0.012) \\ 0.200 & (0.012) \\ 0.200 & (0.003) \\ 0.200 & (0.021) \\ 0.200 &$	$\begin{array}{c} \tau_2 \\ \hline 0.667 & (0.016) \\ 0.670 & (0.007) \\ 0.670 & (0.003) \\ 0.669 & (0.002) \\ \hline 0.8) \\ \hline \tau_2 \\ \hline 0.401 & (0.012) \\ 0.400 & (0.005) \\ 0.400 & (0.005) \\ 0.400 & (0.005) \\ \hline 0.400 $	$ au_3$ 0.600 (0.011) 0.600 (0.004) 0.600 (0.002)	$ au_4$ 0.800 (0.009) 0.800 (0.004) 0.800 (0.002)	

Note: We use 1,000 replications of the data-generating process given in Equation (10) of the main text with c = 0.5. The variance of the error terms is $\sigma_{\xi}^2 = \sigma_e^2 = \sigma_u^2 = 1$. The first subpanel reports the results for one active breakpoint at $\tau = 0.5$, the second subpanel considers two active breakpoints at $\tau_1 = 0.33$ and $\tau_2 = 0.67$ and the third subpanel has four active breakpoints at $\tau_1 = 0.2$, $\tau_2 = 0.4$, $\tau_3 = 0.6$, and $\tau_4 = 0.8$. Time [s] denotes the average computing time for one replication in seconds. Standard deviations are given in parentheses. We conduct the $\sup(l + 1|l)$ test at the 5% level to determine the number of breaks.

Table S3: Estimation of (multiple) structural breaks in the full model (c = 1.5)

	Panel A: Group LASSO with BEA						
	SB1: $(\tau = 0.4$	5)					
T	Time [s]	pce	au				
100	0.04	99.9	0.501(0.010)				
200	0.08	99.9	0.500(0.004)				
400	0.24	100	0.500(0.002)				
800	1.45	100	0.500(0.001)				
	CD9 (- 0	99 -	0.67)				
T	5B2: $(\tau_1 = 0$ Time [c]	.33, $\tau_2 =$	0.07)	~			
	Time [s]	pce	71	72			
150	0.06	97.6	0.335(0.031)	0.660 (0.025)			
300	0.22	99.8	0.332(0.017)	0.667 (0.013)			
600	1.36	99.9	0.332(0.009)	0.668(0.007)			
1200	10.30	100	0.331(0.005)	0.669(0.003)			
	SB4: $(\tau_1 = 0$	2. $\tau_2 = 0$	$.4, \tau_3 = 0.6, \tau_4 = 0$	0.8)			
T	Time [s]	pce	τ_1	τ2	τ_3	$ au_4$	
250	0.13	91.8	0.218(0.031)	0.404(0.020)	0.596(0.017)	0.787 (0.028)	
500	0.64	98.1	0.203(0.017)	0.402(0.012)	0.598(0.009)	0.803(0.012)	
1000	10.14	99.8	0.199(0.008)	0.401(0.005)	0.599(0.005)	0.800 (0.008)	
2000	77.41	100	0.200 (0.004)	0.402(0.002)	0.600(0.003)	0.800 (0.003)	
	Panel B: Like	elihood-b	ased approach				
	SB1: $(\tau = 0.4)$	5)					
T	Time [s]	pce	τ				
100	1.40	90.0	0.500(0.003)				
200	3.61	93.0	0.500(0.002)				
400	18.35	95.7	0.500(0.001)				
800	99.73	95.6	0.500(0.001)				
	SP2: (0)	22 ~ -	0.67)				
T	Time $[s]$	nce	0.01) TI	79			
150	4.49	04.0	0.207 (0.002)	0.007 (0.009)			
150	4.42	94.0	0.327 (0.003) 0.221 (0.001)	0.667 (0.002) 0.670 (0.001)			
500 600	79.65	92.9	0.331(0.001) 0.330(0.001)	0.070(0.001) 0.670(0.001)			
1900	261 57	95.8	0.330(0.001) 0.220(0.001)	0.070(0.001) 0.670(0.001)			
1200	501.57	90.4	0.330 (0.001)	0.070 (0.001)			
	SB4: $(\tau_1 = 0$	$.2, \tau_2 = 0$	$0.4, \tau_3 = 0.6, \tau_4 = 0$	0.8)			
T	Time [s]	pce	$ au_1$	τ_2	$ au_3$	$ au_4$	
250	17.83	99.9	0.200 (0.008)	0.400(0.013)	0.601(0.032)	0.801 (0.038)	
500	76.36	100	0.200(0.001)	0.400(0.001)	0.600(0.001)	0.800(0.001)	
1000	329.93	97.8	0.200(0.001)	0.400(0.001)	0.600(0.001)	0.800(0.001)	
2000	1890.99	96.2	$0.200\ (0.001)$	$0.400\ (0.001)$	0.600(0.001)	0.800(0.001)	

Note: We use 1,000 replications of the data-generating process given in Equation (10) of the main text with c = 1.5. The variance of the error terms is $\sigma_{\xi}^2 = \sigma_e^2 = \sigma_u^2 = 1$. The first subpanel reports the results for one active breakpoint at $\tau = 0.5$, the second subpanel considers two active breakpoints at $\tau_1 = 0.33$ and $\tau_2 = 0.67$ and the third subpanel has four active breakpoints at $\tau_1 = 0.2$, $\tau_2 = 0.4$, $\tau_3 = 0.6$, and $\tau_4 = 0.8$. Time [s] denotes the average computing time for one replication in seconds. Standard deviations are given in parentheses. We conduct the $\sup(l + 1|l)$ test at the 5% level to determine the number of breaks.

Table S4: Estimation of (multiple) structural breaks in the full model using the group LASSO with BEA (c = 0.5). Correlated errors.

	Panel A:	Cross-correlated e	errors ($\rho = 0.95$)				
	GD1 (0.5)					
T	SB1: $(\tau =$	= 0.5)					
1	pce	τ					
100	90.6	$0.500 \ (0.017)$					
200	96.6	$0.500 \ (0.009)$					
400	98.9	$0.500 \ (0.006)$					
	SB2: $(\tau_1$	$= 0.33, \tau_2 = 0.67)$					
T	pce	$ au_1$	$ au_2$				
150	91.2	0.336 (0.032)	0.661(0.027)				
300	94.4	0.332 (0.018)	0.668 (0.014)				
600	98.5	0.331 (0.009)	0.669(0.008)				
	SB4: $(\tau_1$	$= 0.2, \tau_2 = 0.4, \tau_3$	$\tau_4 = 0.6, \tau_4 = 0.8)$				
T	pce	τ_1	$ au_2$	τ_3	$ au_4$		
250	78.6	$0.212 \ (0.030)$	0.403(0.024)	0.598(0.020)	$0.792 \ (0.026)$		
500	95.6	0.203(0.017)	$0.401 \ (0.013)$	0.598(0.010)	$0.801 \ (0.014)$		
1000	98.9	$0.200 \ (0.008)$	$0.400 \ (0.006)$	$0.599\ (0.005)$	$0.800 \ (0.007)$		
	Panol B.	Cross correlated (a = 0.05 and corr	ally correlated (d	= 0.8) orrors		
	Panel B:	Cross-correlated ($\rho = 0.95$) and series	ally correlated (ϕ	= 0.8) errors		
	Panel B:	Cross-correlated ($\rho = 0.95$) and series	ally correlated (ϕ	= 0.8) errors		
Т	Panel B: SB1: $(\tau = nce)$	Cross-correlated ($= 0.5$)	$(\rho = 0.95)$ and series	ally correlated (ϕ	= 0.8) errors		
<u>T</u>	Panel B: SB1: $(\tau = pce)$	Cross-correlated ($= 0.5$) τ	$\rho = 0.95$) and series	ally correlated (ϕ	= 0.8) errors		
T 100	Panel B: SB1: $(\tau = pce)$ 92.9	$\frac{\text{Cross-correlated (}}{\tau}$	$\rho=0.95)$ and series of the s	ally correlated (ϕ	= 0.8) errors		
T 100 200 400	Panel B: SB1: $(\tau = pce)$ 92.9 99.5	Cross-correlated (= 0.5) $\frac{\tau}{0.502 (0.067)}$ 0.501 (0.029) 0.501 (0.014)	$\rho = 0.95$) and seri	ally correlated (ϕ	= 0.8) errors		
$\begin{array}{c} T\\100\\200\\400\end{array}$	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100	Cross-correlated (= 0.5) $\frac{\tau}{0.502 \ (0.067)}$ 0.501 (0.029) 0.501 (0.014)	$\rho = 0.95$) and seri	ally correlated (ϕ	= 0.8) errors		
$\begin{array}{c} T\\ 100\\ 200\\ 400 \end{array}$	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: (τ_1)	Cross-correlated (= 0.5) τ 0.502 (0.067) 0.501 (0.029) 0.501 (0.014) = 0.33, $\tau_2 = 0.67$)	$\rho = 0.95$) and seri	ally correlated (ϕ	= 0.8) errors		
T 100 200 400 T	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 = pce)$	Cross-correlated (= 0.5) τ 0.502 (0.067) 0.501 (0.029) 0.501 (0.014) = 0.33, $\tau_2 = 0.67$) τ_1	$\rho = 0.95$) and seri	ally correlated (ϕ	= 0.8) errors		
T 100 200 400 T 150	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 pce)$ 89.0	Cross-correlated (= 0.5) τ 0.502 (0.067) 0.501 (0.029) 0.501 (0.014) = 0.33, $\tau_2 = 0.67$) τ_1 0.333 (0.042)	$\rho = 0.95$) and seri	ally correlated (φ	= 0.8) errors		
T 100 200 400 T 150 300	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 + pce)$ 89.0 98.7	$\begin{array}{c} \text{Cross-correlated (}\\ = 0.5) \\ \hline \\ \tau \\ \hline \\ 0.502 \ (0.067) \\ 0.501 \ (0.029) \\ 0.501 \ (0.014) \\ = 0.33, \ \tau_2 = 0.67) \\ \hline \\ \hline \\ \hline \\ \hline \\ 0.333 \ (0.042) \\ 0.330 \ (0.023) \end{array}$	$r_{p} = 0.95$) and series r_{2} 0.665 (0.037) 0.670 (0.021)	ally correlated (φ	= 0.8) errors		
T 100 200 400 T 150 300 600	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 pce)$ 89.0 98.7 100	Cross-correlated (τ_2 0.665 (0.037) 0.6670 (0.021) 0.669 (0.010)	ally correlated (φ	= 0.8) errors		
T 100 200 400 T 150 300 600	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 pce)$ 89.0 98.7 100	Cross-correlated (τ_2 0.665 (0.037) 0.670 (0.021) 0.669 (0.010)	ally correlated (ϕ	= 0.8) errors		
T 100 200 400 T 150 300 600	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 pce)$ 89.0 98.7 100 SB4: (τ_1)	Cross-correlated (τ_2 0.665 (0.037) 0.669 (0.010) τ_2 0.669 (0.010) τ_2 0.669 (0.010) τ_2	ally correlated (ϕ	= 0.8) errors		
T 100 200 400 T 150 300 600 T	Panel B: SB1: $(\tau = pce)$ 92.9 99.5 100 SB2: $(\tau_1 pce)$ 89.0 98.7 100 SB4: $(\tau_1 pce)$	$\begin{array}{c} \text{Cross-correlated (}\\ \hline \\ = 0.5) \\ \hline \\ \hline \\ 0.502 \ (0.067) \\ 0.501 \ (0.029) \\ 0.501 \ (0.014) \\ \hline \\ = 0.33, \ \tau_2 = 0.67) \\ \hline \\ \hline \\ \hline \\ \hline \\ 0.333 \ (0.042) \\ 0.330 \ (0.023) \\ 0.331 \ (0.011) \\ \hline \\ = 0.2, \ \tau_2 = 0.4, \ \tau_3 \\ \hline \\ $	τ_2 0.665 (0.037) 0.670 (0.021) 0.669 (0.010) τ_2	ally correlated (φ	= 0.8) errors		
$\begin{array}{c} T \\ 100 \\ 200 \\ 400 \\ \end{array}$ $\begin{array}{c} T \\ 150 \\ 300 \\ 600 \\ \end{array}$ $\begin{array}{c} T \\ T \\ 250 \end{array}$	Panel B: $SB1: (\tau = pce)$ 92.9 99.5 100 $SB2: (\tau_1 pce)$ 89.0 98.7 100 $SB4: (\tau_1 pce)$ $SB4: (\tau_1 pce)$ 73.4	Cross-correlated ($\frac{\tau_2}{0.665 (0.037)}$ 0.665 (0.037) 0.670 (0.021) 0.669 (0.010) $s = 0.6, \tau_4 = 0.8)$ $\frac{\tau_2}{0.402 (0.024)}$	ally correlated (ϕ τ_3 0.599 (0.024)	= 0.8) errors τ_4		
$\begin{array}{c} T \\ 100 \\ 200 \\ 400 \\ \end{array} \\ \hline T \\ 150 \\ 300 \\ 600 \\ \hline T \\ 250 \\ 500 \\ \end{array}$	Panel B: SB1: $(\tau = \frac{pce}{92.9} + \frac{99.5}{100}$ SB2: $(\tau_1 + \frac{pce}{89.0} + \frac{89.0}{98.7} + \frac{98.7}{100}$ SB4: $(\tau_1 + \frac{pce}{73.4} + \frac{95.1}{95.1})$	Cross-correlated (= 0.5) τ 0.502 (0.067) 0.501 (0.029) 0.501 (0.014) = 0.33, $\tau_2 = 0.67$) τ_1 0.333 (0.042) 0.330 (0.023) 0.331 (0.011) = 0.2, $\tau_2 = 0.4$, τ_3 τ_1 0.212 (0.029) 0.202 (0.016)	τ_2 τ_2 0.665 (0.037) 0.670 (0.021) 0.669 (0.010) τ_2 0.402 (0.024) 0.400 (0.014)	ally correlated (ϕ τ_3 0.599 (0.024) 0.599 (0.012)	= 0.8) errors τ_4 0.795 (0.027) 0.800 (0.014)		

Note: We use 1,000 replications of the data-generating process given in Equation (10) of the main text with c = 0.5. The variance of the error terms is $\sigma_{\xi}^2 = \sigma_e^2 = \sigma_u^2 = 1$. In the first panel, we set the cross-correlation coefficient to $\rho = 0.95$ and in the second panel, we additionally use AR(1) processes with autoregressive coefficient $\phi = 0.8$ to generate the error terms. The first subpanel reports the results for one active breakpoint at $\tau = 0.5$, the second subpanel considers two active breakpoints at $\tau_1 = 0.33$ and $\tau_2 = 0.67$ and the third subpanel has four active breakpoints at $\tau_1 = 0.2$, $\tau_2 = 0.4$, $\tau_3 = 0.6$, and $\tau_4 = 0.8$. Standard deviations are given in parentheses.

	Group L	ASSO with BEA				
	SB1: $(\tau$	= 0.5)				
T	pce	τ				
100	97.3	0.501 (0.016)				
200	99.3	0.500(0.006)				
400	100	0.500(0.005)				
800	100	0.500(0.001)				
	SB2: $(\tau_1$	$= 0.33, \tau_2 = 0.67)$				
T	pce	$ au_1$	$ au_2$			
150	79.0	0.339(0.034)	0.661 (0.028)			
300	95.4	0.333(0.018)	0.666(0.017)			
600	99.5	0.332(0.010)	0.668(0.008)			
1200	100	0.333~(0.006)	0.667(0.004)			
	SB4: $(\tau_1$	$= 0.2, \tau_2 = 0.4, \tau_3$	$= 0.6, \tau_4 = 0.8)$			
T	pce	$ au_1$	τ_2	$ au_3$	$ au_4$	
250	52.6	0.249(0.054)	0.421(0.061)	0.602(0.049)	0.778(0.043)	
500	78.7	0.208(0.031)	0.405(0.027)	0.600(0.026)	$0.799 \ (0.019)$	
1000	97.6	0.200(0.011)	$0.401 \ (0.009)$	0.599(0.008)	$0.800 \ (0.009)$	
2000	99.9	$0.200 \ (0.005)$	$0.401 \ (0.005)$	0.599(0.004)	0.800(0.004)	

Table S5: Estimation of (multiple) partial structural breaks in the full model (c = 0.5)

Note: We use 1,000 replications of the data-generating process given in Equation (10) of the main text with c = 0.5 but only the coefficients of the first equation change. Those changes are adjusted to ensure that the break magnitude is identical to the common break specification used to create Table Table S2. The variance of the error terms is $\sigma_{\xi}^2 = \sigma_e^2 = \sigma_u^2 = 1$. The first panel reports the results for one active breakpoint at $\tau = 0.5$, the second panel considers two active breakpoints at $\tau_1 = 0.33$ and $\tau_2 = 0.67$ and the third panel has four active breakpoints at $\tau_1 = 0.2$, $\tau_2 = 0.4$, $\tau_3 = 0.6$, and $\tau_4 = 0.8$. Standard deviations are given in parentheses.

Table S6: Estimation of (multiple) structural breaks in the full model (c = 0.5) with endogenous regressors

	Group L	ASSO with BEA				
	SB1: $(\tau$	= 0.5)				
Т	pce	au				
100	92.7	$0.501 \ (0.026)$				
200	94.4	0.500(0.012)				
400	96.8	0.500(0.007)				
	GDA (0.00				
	SB2: $(\tau_1$	$= 0.33, \tau_2 = 0.67)$				
T	pce	$ au_1$	$ au_2$			
150	86.3	0.336(0.036)	0.660(0.029)			
300	98.7	0.335(0.022)	0.665(0.017)			
600	100	0.332(0.011)	0.668(0.008)			
	SB4: $(\tau_1$	$= 0.2, \tau_2 = 0.4, \tau_3$	$\tau = 0.6, \tau_4 = 0.8)$			
Т	pce	$ au_1$	$ au_2$	$ au_3$	$ au_4$	
250	68.8	0.215(0.038)	0.407(0.034)	0.597(0.033)	0.793(0.032)	
500	91.9	0.201 (0.017)	0.403(0.013)	0.597(0.011)	0.801 (0.014)	
1000	99.9	0.200(0.009)	$0.401 \ (0.007)$	$0.598\ (0.006)$	0.799(0.008)	

Note: We use 1,000 replications of the data-generating process given in Equation (10) of the main text with c = 0.5. The variance of the error terms is $\sigma_{\xi}^2 = \sigma_e^2 = \sigma_u^2 = 1$. The error terms are correlated with the innovations of the first (second) integrated regressor with coefficient 0.5 (0.25). The first panel reports the results for one active breakpoint at $\tau = 0.5$, the second panel considers two active breakpoints at $\tau_1 = 0.33$ and $\tau_2 = 0.67$ and the third panel has four active breakpoints at $\tau_1 = 0.2$, $\tau_2 = 0.4$, $\tau_3 = 0.6$, and $\tau_4 = 0.8$. Standard deviations are given in parentheses.

References

- Chan, N.H., Yau, C.Y., Zhang, R.M., 2014. Group LASSO for Structural Break Time Series. Journal of the American Statistical Association 109, 590–599. doi:10.1080/01621459. 2013.866566.
- Similä, T., Tikka, J., 2006. Common subset selection of inputs in multiresponse regression. Proceedings of the 2006 International Joint Conference on Neural Networks, 1908–1915.
- Yuan, M., Lin, Y., 2006. Model selection and estimation in regression with grouped variables. Journal of the Royal Statistical Society. Series B: Statistical Methodology 68, 49–67. doi:10. 1111/j.1467-9868.2005.00532.x.