Are gold and silver cointegrated? New evidence from quantile cointegrating regressions

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Abstract

This paper revisits the study on the long-run relationship between gold and silver by Escribano and Granger [1998]. We apply a quantile cointegration model to gold and silver prices and to prices of the corresponding futures contracts. Whereas cointegration models, assuming a constant cointegrating vector, fail to detect a cointegration relationship between gold and silver, we are able to show that a nonlinear long-run relationship exits. The cointegrating vector is modelled as state-dependent and time-varying in our framework and the quantile cointegration estimates reveal substantial asymmetry in the relationship. The results suggest that the pronounced role of precious metals as investment opportunities particularly in bubble-like episodes and times of financial turmoil leads to comovement of gold and silver in these periods.

Keywords: Gold; silver; quantile cointegration; time-varying; state-dependence
JEL Classification: C32; C58; G11; G15

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1 Introduction

Gold and silver share a long-standing relationship that goes back to the first issuance of gold and silver coins that were used as currency. The monetary use of gold and silver was facilitated by their unique characteristics. They are rare, easily transportable, malleable and do not corrode so that they serve as a perfect store of value. The monetary system of, for example, Germany was backed by silver until 1873 and the gold-backed Bretton Woods system de facto ended in 1971 with the change to a system of national fiat monies. Subsequently, the relationship between gold and silver changed drastically with the transformation from commodity money to fiat money.

Although precious metals are still seen as stores of value, their commercial uses have gained importance. Gold is used, among others, in restorative dentistry and, since it is highly conductive, for high quality electrical connectors. Silver is the most reflective known metal and therefore used in photography, optics, as well as the solar energy industry. Both metals are also used in jewellery (demand for jewellery accounted for around 50 per cent of world gold demand and 20 percent of global silver demand in 2014[1]).

Gold and silver also play a prominent role as investments. In times of financial turmoil which are characterized by rapidly decreasing values of stock indices, the prices of precious metals tend to move in the opposite direction. Investors are interested in assets which are uncorrelated or ideally negatively correlated with the general market developments to hedge against adverse financial events. Evidence for a safe haven role of gold has recently been found by Baur and Lucey [2010] and Baur and McDermott [2010]. Further, Agyei-Ampomah et al. [2014] report that other precious metals, including silver, may present even better investment alternatives than gold in financial crises

[1] The estimates are taken from the World Gold Survey 2016 and World Silver Survey 2016 (GFMS [2016a] and GFMS [2016b]).

periods. Lucey and Li [2015] find similar results specifically for the US equity market.

It is of considerable interest to market participants to know whether a long-run relationship between gold and silver prices exists for the following reasons: First, knowing that the prices form a cointegration relationship may be used for forecasting purposes. Maintaining an equilibrium relationship over an extended period of time implies that at least one variable adjusts to disequilibrium states. The adjustment behaviour could then help to predict future returns of the adjusting variable(s). Second, a cointegrated gold and silver portfolio would be a suitable long-term hedge and could qualify for a market-neutral pairs trading strategy (Alexander [1999]).[2] Third, as gold and silver are seen as substitutes to reduce similar types of risks in portfolios (Ciner [2001] and Hillier et al. [2006]), finding evidence of cointegration provides statistical support that gold and silver follow a common stochastic trend. Fourth, additional information about the trajectory of gold prices might reduce uncertainty for central banks and other major institutions.

The question of whether gold and silver are cointegrated has already drawn some attention in the literature: Wahab et al. [1994] study the long-run relationship between gold and silver spot and futures markets using daily prices from 1982 to 1992. They apply cointegration and error correction models to constrained and unconstrained gold-silver spreads and find a long-run relationship. However, no profitable trading strategy can be based on the intercommodity spread. Escribano and Granger [1998] investigate the relationship between gold and silver prices after the collapse of the Bretton Woods system using monthly data from 1971 to 1990. They have to pre-specify regimes in

[2] In pairs trading, two or more assets are identified that share similar characteristics and for which prices should be similar, i.e. they hold a long-run relationship. Then if the relative pricing between the assets indicates a mispricing, the trading strategy consists of buying the lower-priced asset and selling the higher-priced asset leading to a statistical arbitrage in the short-run. However, it is assumed that the mispricings will be corrected in the long-run. Prices are usually modelled as a random walk so that a cointegration analysis has to be employed to capture the long-run relationship between prices. If evidence for a cointegration relationship between the assets can be established, the disequilibrium series is mean-reverting and mispricings have to be corrected to maintain the long-run equilibrium.

order to find evidence for cointegration and the null hypothesis of no cointegration cannot be rejected for the full sample. They argue that the cointegration relationship only holds for the well-known Hunt brothers episode ('silver bubble') from June 1979 to March 1980 and the post-bubble period in the 1980s, but markets begin to separate at the end of their sample. The authors encourage further research to focus on the potential nonlinearity in the data, particularly on the time-varying dependence between the prices. Adrangi et al. [2000] find that gold leads the gold-silver long-run relationship using high frequency futures data. Ciner [2001] responds to the claim of a long-run relationship between gold and silver and uses daily closing prices of gold and silver futures contracts traded on the Tokyo Commodity Exchange (TOCOM) to verify whether markets indeed became separate. The results do not support a stable long-run relationship between gold and silver futures for the period from 1992 to 1998. Lucey and Tully [2006] use a dynamic cointegration approach which involves a recursive or rolling window estimation and identify periods of weak and strong dependence. They use a sample of Friday closing prices from 1978 to 2002 for their analysis and conclude that overall a cointegration relationship has been maintained. Baur and Tran [2014] revisit the dataset used by Escribano and Granger [1998] and expand the time period to July 2011. They find evidence for cointegration in the full sample but the results suggest that the cointegrating vector changes during bubble and crisis periods. They conclude that the long-run relationship between gold and silver is not stable. The results point to a comovement only in specific episodes in which the store of value aspect of precious metals is particularly important.

Potential nonlinearity in the long-run relationship between gold and silver has so far been treated either as a structural break in the cointegrating vector or as a recursive/rolling window estimation to identify periods of stronger and weaker dependence. On the one hand, a division of the sample period into subperiods requires that dummy variables have to be specified arbitrarily. On the other hand, an application of dynamic cointegration models might identify a number of subperiods with stronger dependence but estimation requires specifying the appropriate length of the estimation window which influences the result. Moreover, nonlinearities cannot be quantified.

In this paper, we propose a quantile cointegration approach which enables to model a state-dependent and time-varying cointegrating vector. The values of the cointegrating vector may vary over the innovation quantile. Thereby, the degree of comovement between gold and silver does not depend on regime specifications based on prior knowledge about market conditions (e.g. bubble and crisis periods). Rather, a potentially nonlinear dependence can be revealed from the data based on the state of the individual prices without any prior separation into regimes. Specifically, this allows to measure the response of silver prices to gold prices, if silver prices are high and vice versa. The effects of speculative bubbles and financial turmoil on the prices is implicitly modelled since prices of precious metals tend to increase and reach local maxima in these periods thereby altering the state of the prices. To determine whether gold and silver are cointegrated under the quantile cointegration framework, we use a CUSUM cointegration test developed by Xiao [2009].

This paper contributes to the empirical literature by modelling the state- and time-dependence of the long-run relationship between gold and silver prices and attempts to explain why gold and silver move together in the long-run. First, we revisit an extended gold and silver dataset in a monthly frequency to allow a comparison to the Escribano and Granger [1998] and Baur and Tran [2014] studies. Furthermore, we also conduct the analysis using observations at a daily frequency as well as using prices of futures contracts from 1980 to 2017 to examine the robustness of our results to different frequencies and whether our results are driven by unique characteristics of the spot market. We are able to reveal an asymmetric pattern in the long-run relationship characterized by a stronger response of silver prices to gold prices when silver prices are high and of gold prices to silver prices when gold prices are high.

The remainder of the paper is organized as follows: Section 2 discusses economic reasons why gold and silver might share a common stochastic trend, Section 3 introduces a CUSUM test for linear cointegration models and describes the quantile cointegration methodology by Xiao [2009]. In Section 4, we apply these techniques to the gold and silver relationship and Section 5 concludes on our results.

2 Why should gold and silver share a common stochastic trend?

Although gold and silver possess similar characteristics, their differing commercial uses suggest that their markets are separated and hence no long-run relationship between them exists. Granger [1986] states that prices generated on a jointly efficient, speculative market cannot be cointegrated since this would violate the efficient market hypothesis. However, the findings on whether gold and silver markets are efficient are mixed. For example, Smith [2002] investigates London gold prices and finds autocorrelated returns of the twice-daily fixing prices, speaking against the random walk hypothesis. The closing prices, by contrast, seem to be generated randomly. Pierdzioch et al. [2014] account for transaction costs and show that a trading rule which incorporates publicly available information does not outperform a simple buy-and-hold strategy, implying that the gold market is informationally efficient. Ntim et al. [2015] extend their analysis of gold price efficiency to different markets. They report a higher probability of rejecting the weak-form efficiency in emerging gold markets than developed ones. Charles et al. [2015] find that return predictability of precious metals markets has been changing over time. Gold seems to have a higher degree of market efficiency over silver and platinum.

The exact mechanisms of the price formation of precious metals prices is still little understood. Precious metals are seen both as a commodities as well as financial assets. While financial asset returns are strongly correlated with macroeconomic indicators and each other, commodity returns are typically less correlated with financial assets returns and returns of other commodities (Tang and Xiong [2012]).

As a distinctive feature of precious metals, and in contrast to other commodities like crude oil, prices are largely unaffected by annual production since the life span is practically infinite and stockpile outweighs annual production. The price formation is therefore most likely determined on the demand side. Nonetheless, several changes on the supply side since 1970 have taken place: While mine production became less important for overall gold supply, recovering gold from scrap has gained importance. Instead, for silver the contribution of mining to overall production has increased.

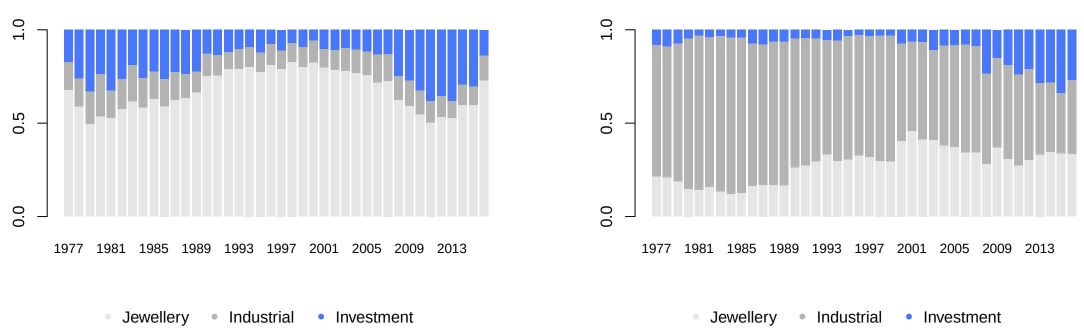

Figure 1: Relative contribution to global gold (left) and silver (right) demand from 1977 to 2016. Data taken from the GFMS gold and silver surveys (1991 - 2017).

The relative contribution of industrial fabrication and retail investment to annual demand for gold and silver is depicted in Figure 1.[3] In 2016, around 14% of total gold demand and 40% of total silver demand was attributed to industrial fabrication. Taking

[3]While industrial fabrication includes electronics, medical uses and other fabrications for gold, it includes electronics, photography and other industrial applications for silver (The Silver Institute [1990]). Silverware demand is added to silver jewellery demand.

into consideration that jewellery items are often seen as stores of value, gold seems to be mainly used as a cash-like asset, while silver prices are determined largely by industrial demand. Nevertheless, gold and silver show a visible comovement in historical price series (see Figure 2). A closer inspection of the time series plot reveals that gold and

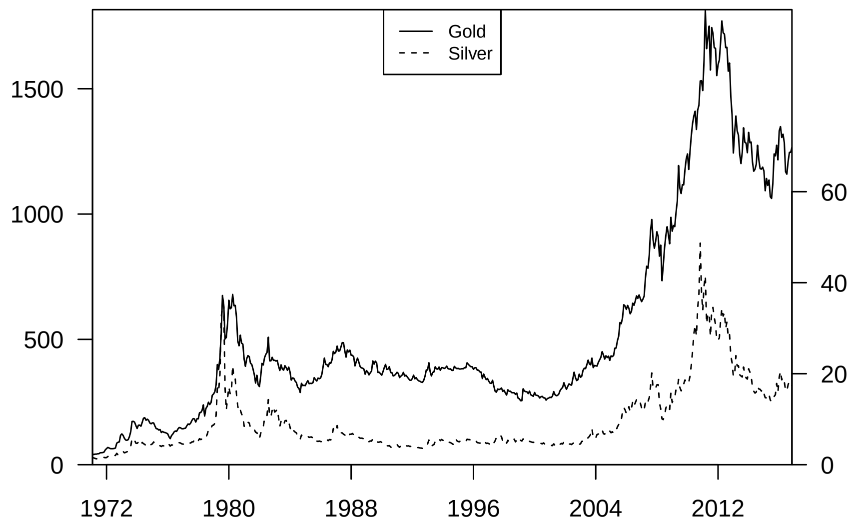

Figure 2: Historic gold and silver spot prices. The left (right) axis describes the gold (silver) prices in USD.

silver boom and bust during the same time periods. However, the behaviour in tranquil times is far less synchronized. The long-run relationship, if it exists, might therefore be characterized by episodes of stronger and weaker dependence. Although gold and silver are no industrial substitutes, their use on financial markets, especially as a safe haven asset in crisis periods, could translate to periods in which the store of value aspect of gold and silver is pronounced and might be the reason why the individual prices follow a similar trajectory. In Figure 3 the amount of investment in gold and silver is displayed. At least in the later part of the sample, both variables follow a similar trend. The strong increase in silver retail investment after 2006 can in parts be explained by the inception of a silver ETF which reduces the costs of investment (Fassas [2012]). The lack of silver retail investment despite a rally in silver prices during the Hunt brothers episode indicates that this supply side event should be treated as an anomaly.

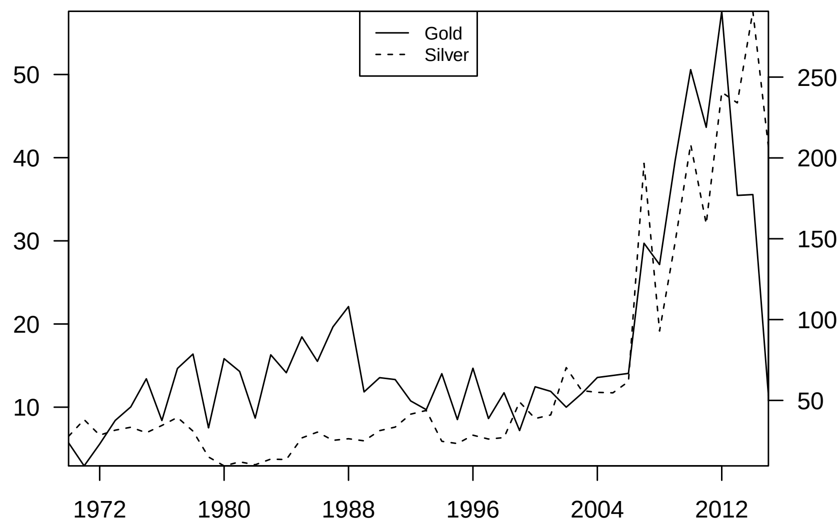

Figure 3: Retail investment in gold and silver in million ounces.

To formalize the idea of time-varying comovement depending on whether precious metals are used as assets or commodities, we consider a structural model for gold and silver prices in the spirit of Roll [1984]. We denote the price of gold as p_G and the price of silver as p_S and express the observed prices as functions of unobserved efficient prices p^*_G and p^*_S. The model is then given as follows,

$$\begin{aligned} p_{i,t} &= p^*_{i,t} + \epsilon_{i,t} \qquad i = G, S \\ p^*_{i,t} &= p^*_{i,t-1} + \beta_{i\tau} c_t \\ \beta_{i\tau} &= \beta_i(c_{\tau t}, r_{G,\tau t}, r_{S,\tau t}), \end{aligned} \tag{1}$$

where c_t is a permanent shock and ϵ_t is a transitory shock. The quantile-dependent parameters $\beta_{i\tau}$ are given as a function of a common news factor c_t and gold and silver-specific news, $r_{G,t}$ and $r_{S,t}$. Further, the parameters fulfil the restriction,

$$\beta_{S\tau} - \beta_\tau \beta_{G\tau} = 0. \tag{2}$$

Under these conditions, the prices of gold and silver are nonlinearly cointegrated with a time-varying and quantile-dependent cointegrating vector β_τ. It is therefore possible in our structural model to allow that the response β_τ is weaker when market-specific news drive gold and silver prices and stronger when gold and silver are used as financial assets. Market-specific news, $r_{G,t}$ and $r_{S,t}$, are thought of as being determined by, for example, industrial demand and supply changes. The common news factor c_t is related to financial market activity. It is associated with the relative attractiveness of precious metals as investment opportunities. Particularly, gold and silver prices are assumed to be similarly affected if investor's demand for hedges against inflationary and equity risks increases. We assume that prices of precious metals respond strongly to each other if the relative attractiveness of precious metals increases. We model this situation as an upper tail realization of c_t and a correspondingly small realization of $r_{G,t}$ and $r_{S,t}$.[4] This nonlinear relationship is then captured by a cointegrating vector which depends on the innovation quantile. Since precious metals are seen as safe haven assets, we expect to observe a stronger response in periods of financial turmoil and a weaker response in tranquil periods. Hence, linking gold and silver prices by a single linear cointegrating vector would not result in a stationary spread.

Adrangi et al. [2000] offer another explanation for a time-varying long-run relationship of gold and silver. They suggest that gold-silver spread trading might contribute to

[4] We assume a mean zero distribution for the news variables c_t, $r_{G,t}$ and $r_{S,t}$.

the periodical comovement of both precious metals. This well-known strategy consists of simultaneously taking a long position in one precious metal and a short position on the other. The rationale for this type of trading rests on the perception that a 'normal' difference between the prices exists. Traders would then drive the seemingly misspriced price up or down by investing accordingly. In case of unusually large differences, we would expect a stronger response of both prices to each other. In the remainder of the paper, we investigate a potentially time-varying comovement using quantile cointegrating regressions.

3 Econometric framework

The quantile cointegration model builds on the residual-based cointegration approach proposed in Engle and Granger [1987]. The long-run equilibrium equation is specified as

$$y_t = \alpha + \beta x_t + u_t, \tag{3}$$

where y_t and x_t are $I(1)$ variables. In the following empirical application, the price of gold takes the role of y_t and the prices of silver takes the role of x_t. We distinguish between linear cointegration where both variables are connected by constant coefficients $\theta = (\alpha, \beta)$ and quantile cointegration where the coefficients $\theta_t = (\alpha_t, \beta_t)$ are time-varying. In both cases, the existence of a long-run equilibrium requires the error term u_t to be mean zero stationary.

3.1 Linear cointegration

The linear cointegration model is estimated by least squares assuming constancy of the parameters. Thereby, we estimate the conditional expected value of y_t as a function of the variables x_t,

$$\widehat{E}(y_t|x_t) = \widehat{\alpha} + \widehat{\beta}x_t. \tag{4}$$

Estimating equation (3) using a pair of potentially cointegrated variables $(y_t, x_t)'$ introduces an endogeneity problem if x_t is not weakly exogenous.[5] Although the least squares estimator is still super-consistent, it is second order biased due to the dependence of x_t and u_t. Two modifications are proposed in the literature to restore the statistical properties of the least squares estimator in cointegrating regressions. The first approach adds leads and lags of x_t to the long-run equation (3) so that we arrive at

$$y_t = \alpha + \beta x_t + \sum_{j=-K}^{K} \Pi_j \Delta x_{t-j} + \epsilon_t. \tag{5}$$

In this dynamic OLS (DOLS) method, originally proposed by Saikkonen [1991], the error term u_t is decomposed into a component related to Δx_t and a pure innovation term ϵ_t. Using equation (5) instead of (3) guarantees that the least squares estimator is asymptotically unbiased. From a practical perspective, the drawback of this approach is the uncertainty regarding the dynamic specification as the number of leads and lags is generally unknown. However, standard model selection criteria can be used to determine the lag length (Choi and Kurozumi [2012]).

The second approach involves a nonparametric correction of the original estimator, known as fully modified OLS (FM-OLS) estimation. We define the FM-OLS estimator of β as

$$\hat{\beta}^{+}_{LS} = \left[\sum_t y_t^+ x_t - n\widehat{\lambda}^+_{vu}\right]\left[\sum_t x_t^2\right]^{-1}, \tag{6}$$

where $y_t^+ = y_t - v_t\widehat{\Omega}_{vv}^{-1}\widehat{\Omega}_{vu}$, $v_t = \Delta x_t$, $\widehat{\lambda}^+_{vu} = \widehat{\lambda}_{vu} - \widehat{\lambda}_{vv}\widehat{\Omega}_{vv}^{-1}\widehat{\Omega}_{vu}$ and n is the sample size. The relevant long-run (co-)variances are estimated by applying a kernel estimator to

[5] This is of particular interest since we do not know which variable drives the gold and silver long-run relationship.

the residuals obtained by estimating the cointegrating regression (3) with least squares. We choose a Bartlett kernel $k(\cdot)$ with the plug-in bandwidth $M = 1.1447(\phi(1)n)^{1/3}$ according to Andrews [1991], where $\phi(1) = 4\hat{\rho}^2/(1-\hat{\rho}^2)^2$ and $\hat{\rho}$ is the estimated first order autocorrelation of the least squares residual $\hat{u}_t$.[6] We arrive at the kernel estimates,

$$\begin{aligned}
\widehat{\lambda}_{\upsilon u} &= \sum_{h=0}^{\lfloor M \rfloor} k\left(\tfrac{h}{M}\right) C_{\upsilon u}(h), & \widehat{\lambda}_{\upsilon\upsilon} &= \sum_{h=0}^{\lfloor M \rfloor} k\left(\tfrac{h}{M}\right) C_{\upsilon\upsilon}(h), \\
\widehat{\Omega}_{\upsilon u} &= \sum_{h=-\lfloor M \rfloor}^{\lfloor M \rfloor} k\left(\tfrac{h}{M}\right) C_{\upsilon u}(h), & \widehat{\Omega}_{\upsilon\upsilon} &= \sum_{h=-\lfloor M \rfloor}^{\lfloor M \rfloor} k\left(\tfrac{h}{M}\right) C_{\upsilon\upsilon}(h), \\
\widehat{\omega}_u^2 &= \sum_{h=-\lfloor M \rfloor}^{\lfloor M \rfloor} k\left(\tfrac{h}{M}\right) C_{uu}(h),
\end{aligned} \tag{7}$$

where $C_{\upsilon\psi}(h)$, $C_{\upsilon\upsilon}(h)$ and $C_{uu}(h)$ are sample covariances defined by $C_{\upsilon u}(h) = n^{-1}\sum \upsilon_t \hat{u}_{t+h}$, $C_{\upsilon\upsilon}(h) = n^{-1}\sum \upsilon_t \upsilon_{t+h}$, $C_{uu}(h) = n^{-1}\sum \hat{u}_t \hat{u}_{t+h}$, respectively. For a more comprehensive discussion of fully modified least squares, see Hansen [1992] and Xiao and Phillips [2002].

Testing for cointegration is based on the residuals obtained by estimating the long-run equilibrium equation. In contrast to the Engle-Granger procedure where the null hypothesis is no cointegration, we follow Xiao and Phillips [2002] and test the null hypothesis of cointegration directly. If y_t and x_t are cointegrated, the residuals should reflect this by displaying fluctuations that resemble a stationary process. A substantial stochastic trend in the residuals would lead to inflated variation over time and would point to the alternative of no cointegration.

The cointegration test proposed in Xiao and Phillips [2002] uses a partial sum process (related to the CUSUM test literature, see Shin [1994]) to measure the fluctuation

[6] Xiao and Phillips [2002] discuss different choices of the bandwidth parameter M. Several data-dependent choices have been considered for the empirical application which yielded qualitatively similar results.

in the residuals. The test statistic is constructed as

$$CS_n = \max_{k=1,\ldots,n} \frac{1}{\widehat{\omega}_{vu}\sqrt{n}} |\sum_{j=1}^{k} \hat{u}_j^+| \tag{8}$$

where $\widehat{\omega}_{vu}^2 = \widehat{\omega}_u^2 - \widehat{\Omega}_{uv}\widehat{\Omega}_{vv}^{-1}\widehat{\Omega}_{vu}$ and $\hat{u}^+$ is the vector of fully modified residuals. Under the null hypothesis of cointegration and for $n \to \infty$, the test statistic has a stable distribution and critical values can be found by way of Monte Carlo simulation. In the alternative of no cointegration, both numerator and denominator diverge for $n \to \infty$. However, Xiao and Phillips [2002] show that the denominator diverges at a slower rate so that the test statistic as a whole diverges.

3.2 Quantile cointegration

In the following, we consider a relaxation of the assumption about constant coefficients made in the linear cointegration model. The long-run equilibrium equation is now specified as a random coefficient model,

$$y_t = \alpha_t + \beta_t x_t + u_t. \tag{9}$$

In particular, the value of the coefficients are allowed to vary over the innovation quantile. Hence, we estimate $\theta(\tau) = (\alpha(\tau), \beta(\tau))$ using quantile regressions. The quantile regression estimator for each quantile $\tau \in \mathcal{T}$ is obtained by solving

$$\widehat{\theta}(\tau) = \arg\min_{\theta \in \mathbb{R}^2} \sum_{t=1}^{n} \rho_\tau(y_t - \alpha(\tau) - \beta(\tau)x_t), \tag{10}$$

where $\rho_\tau(u) = u(\tau - \mathbb{1}\{u < 0\})$ is the asymmetric weights function as in Koenker and Bassett [1978] and $\mathbb{1}\{\cdot\}$ is a Heaviside indicator function. In contrast to least squares estimation, where the conditional expected value of y_t is estimated, quantile regression expresses the τth quantile of y_t conditional on the information set $\mathcal{F}_t$ in period t,

$$\widehat{Q}_{y_t}(\tau|\mathcal{F}_t) = \widehat{\alpha}(\tau) + \widehat{\beta}(\tau)x_t + F_u^{-1}(\tau). \tag{11}$$

The residual weights are computed as $\psi_\tau(u) = \tau - \mathbb{1}\{u < 0\}$ and the τth residual series as $u_{t\tau} = y_t - \widehat{\alpha}(\tau) - \widehat{\beta}(\tau)x_t$.

Similar to the least squares estimator, the quantile regression estimator has to be modified by dynamic augmentation or nonparametric correction to restore its statistical properties. We refer to Xiao [2009] for a detailed discussion of fully-modified quantile regression estimators. The null hypothesis of quantile-dependent cointegration is tested by evaluating the fluctuation of the residual weights. Xiao [2009] proposes the Kolmogoroff-Smirnoff type test statistic for the τth quantile regression,

$$Y_{n\tau}^* = \max_{k=1,\ldots,n} \frac{1}{\widehat{\omega}_\psi^* \sqrt{n}} \sum_{j=1}^{k} \psi_\tau(\hat{\epsilon}_{j\tau}), \tag{12}$$

where $\widehat{\omega}_\psi^{*2}$ is a consistent estimate of the long-run variance of $\psi_\tau(\hat{\epsilon}_{j\tau})$. The test is based on the residual weights which are mean-zero instead of the residuals for which the τth quantile is zero. The quantile regression residual $u_{t\tau}$ and residual weights $\psi_\tau(\hat{\epsilon}_{j\tau})$ are obtained from the lead-lag augmented regression in equation (5). Under the alternative of no cointegration for a given quantile τ, the test statistic $Y_{n\tau}^*$ diverges to infinity for $n \to \infty$. In this case the variables would not be cointegrated at the τth quantile of the dependent variable's conditional distribution.

4 Empirical Analysis

We analyze gold and silver spot prices at a monthly frequency from August 1971 to May 2017 and daily spot and futures prices from April 1980 to May 2017. The London

OTC market and New York COMEX are considered major gold and silver markets. We use the morning official fixing price at the London Bullion market for the daily price series and build a monthly price series from the first price reported in each month. The futures prices are obtained for COMEX 100 ounces gold contracts and COMEX 5000 ounces silver contracts. We denote the spot prices of gold and silver as p_G and p_S, respectively. The futures prices are denoted as p_G^F and p_S^F. Gold prices are denominated in USD per troy ounce whereas silver is denominated in USD cents per troy ounce.

Although the quantile cointegrating regression model is able to account for time-varying coefficients in the long-run equation, it does, however, not account for exogenous regime shifts. Our sample of monthly spot prices includes the Hunt brothers' attempt to corner the silver market in the late 1970s and early 1980s. The Hunt brothers and their collaborators tried to restrict the supply of silver on the market so that it became difficult for investors who sold short to deliver at the end of the contract. The price of silver subsequently increased dramatically and this peak appears as a striking anomaly in the data. However, they only acted in the silver market and did not act on the gold market in the same fashion. It has to be assumed that the potential long-run relationship between gold and silver was exogenously altered during the Hunt brothers episode. This means, we should model this period as a separate regime.[7] Later, we see that accounting for this period specifically does not have a significant effect on our results.

We start the analysis by testing all price series for their order of integration. For this matter, we apply Dickey-Fuller tests to the prices and to the returns. The null hypothesis of a unit root cannot be rejected for the prices but it can be rejected at all conventional significance levels for the returns. Hence, each series is determined to

[7] In contrast, we do not model the financial crisis in 2008 as a separate regime since gold and silver markets were both affected. Prices of precious metals increased due to a higher demand of investors for safe haven assets without necessarily changing the relationship between them.

be integrated of order one and we proceed to conduct the cointegration analysis. The results of the unit root tests are depicted in Table 1.

Table 1: Augmented Dickey-Fuller tests for gold and silver prices

	drift	lags	*trend*	lags		*drift*	lags
$p_{G,m}$	-0.519	1	-1.431	1	$\Delta p_{G,m}$	-18.20^{***}	1
$p_{S,m}$	-2.365	2	-2.855	2	$\Delta p_{S,m}$	-18.50^{***}	1
$p_{G,d}$	-0.376	1	-1.804	1	$\Delta p_{G,d}$	-71.29^{***}	1
$p_{S,d}$	-1.886	1	-2.838	1	$\Delta p_{S,d}$	-34.83^{***}	6
$p^F_{G,d}$	-0.427	1	-1.819	1	$\Delta p^F_{G,d}$	-70.15^{***}	1
$p^F_{S,d}$	-1.855	1	-2.730	1	$\Delta p^F_{S,d}$	-68.86^{***}	1

The subscript m denotes monthly observations and d denotes daily observations, respectively. The superscript F denotes the futures prices. Including an intercept in the ADF test equation is indicated with *drift*, including an additional linear trend term with *trend*. The lag selection was achieved via Bayesian Information Criterion (BIC).
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

4.1 Monthly spot prices

In the first part of the empirical analysis, we revisit a data set similar to the one found in Escribano and Granger [1998]. The long-run equilibrium model between gold and silver prices is expressed as

$$p_{S,t} = \alpha + \beta\, p_{G,t} + u_t, \quad (13)$$ [8]

assuming that gold leads the long-run relationship. The cointegrating vector is estimated by FM-OLS and the CUSUM cointegration test is applied to the residuals $\hat{u}_t^+$. The CS_n statistic amounts to 2.301 for the monthly series such that the null hypothesis of linear cointegration can be rejected at the 1% significance level. This means we find strong evidence that gold and silver are not cointegrated in the Engle-Granger

[8] Specifically modelling the Hunt brothers episode using a dummy variable and an interaction term to account for the 'silver bubble' period from June 1979 to March 1980 leaves the results virtually unchanged.

framework assuming a constant cointegrating vector. The FM-OLS estimator for β amounts to 1.537 and the DOLS estimator takes the value 1.704. This result supports the findings in Escribano and Granger [1998] and Baur and Tran [2014] who cannot find a cointegration relationship for the full sample as well.

We now test for quantile cointegration. The $Y^*_{n\tau}$ statistic is computed for each quantile $\tau \in \{0.01, \ldots, 0.99\}$ and is plotted in Figure 4. The null hypothesis of quantile cointegration cannot be rejected at the 5% significance level for any quantile τ. The point estimates for the quantile-dependent estimator are depicted in Figure 5. It can be inferred from the plots that significant asymmetry is present in the quantile regression estimates. The slope parameter largely coincides with the conditional-mean benchmark (DOLS) with the exception that the lower tail estimates are slightly smaller than the DOLS estimate. However, the point estimates for upper quantiles (above the 80% quantile) are significantly larger than the benchmark.

The quantile cointegration estimates suggest that silver prices respond more strongly to gold prices if silver prices are high. The shape of the quantile-dependent response curve matches the predictions from gold-silver spread trading (Adrangi et al. [2000]) where the response to corrections should be strongest if silver prices are relatively high. Unfortunately, the quantile cointegration methodology does not allow us to study the adjustment behaviour in more detail. A plot of the historic time series (Figure 2) shows that relatively high gold and silver prices occurred during the Hunt brothers episode and during the recent price run of precious metals from 2006 to 2012 which includes the financial crisis of 2008. In general, the quantile cointegration framework is not able to identify periods with stronger responses directly, since conditional quantiles are estimated. However, we are able to indicate the periods in which the residuals were assigned a higher weight to find the most influential observations for a given conditional quantile. This is depicted in Figure 6, where we mark the higher weighted residuals

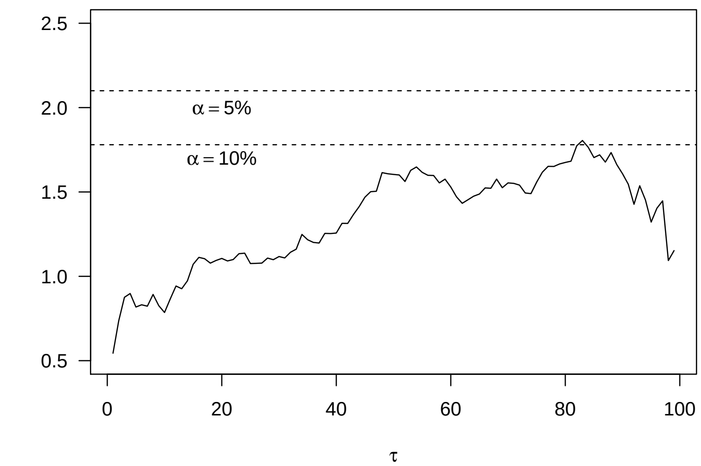

Figure 4: The estimated $Y^*_{n\tau}$ statistics for the 1% to 99% quantile (monthly series). The critical value, dashed line, is 1.78 for the 10% significance level (2.1 for the 5% significance level).

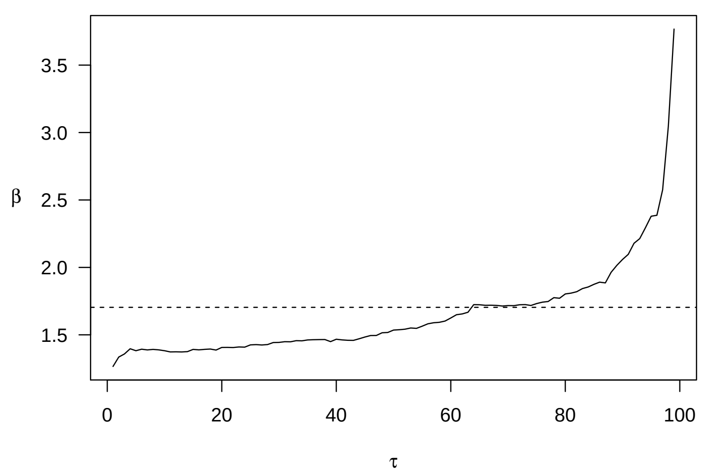

Figure 5: Estimation results for the slope coefficient in the lead-lag augmented quantile cointegration regression (monthly series). The DOLS estimate serves as a benchmark (dashed line).

for the lower tail (25% quantile) and the upper tail (75% quantile) with a blue and red rhombus, respectively.

The indicated periods of weaker dependence match the results of Escribano and

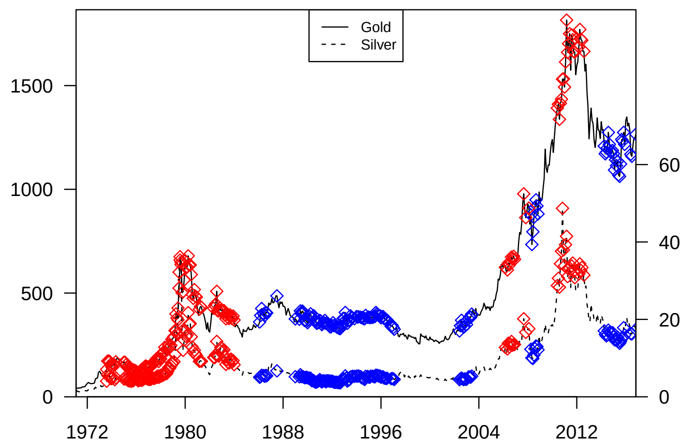

Figure 6: Periods of conditional 25% (75%) quantile monthly silver prices. Observations are marked with a blue (red) rhombus if they received a higher weight in the loss function of the 25% (75%) quantile regression.

Granger [1998] who claim that the cointegration relationship dissolves towards the end of their sample in 1990. Periods of stronger dependence are found during the 'silver bubble', during the financial crisis and at the peak of the European sovereign debt crisis. Lucey and Tully [2006] find a different pattern but their sample period is shorter and excludes the Hunt brothers episode as well as the financial crisis. In general our data-driven framework finds a state-dependence of the long-run relationship between gold and silver that resemble the pre-specified conditional-mean results obtained by Baur and Tran [2014] who also find a change of the cointegrating vector during bubble-like episodes and financial crises.

Since we do not know with certainty which variable leads the potential nonlinear gold and silver long-run relationship, we re-estimate the long-run equation assuming that silver leads the pricing process. The results, which are not reported, show a stronger response of gold prices to silver price changes for upper conditional quantiles of gold.

4.2 Daily spot and futures prices

We now consider daily spot and futures prices as a robustness check for our results in the previous section and relate our results to the findings in Ciner [2001]. The historic price series for gold and silver futures contracts starts in March 1980. Hence, we do not need to model the Hunt Brothers episode. We also restrict the sample for the daily spot prices to the same time span as the futures series to allow for a direct comparison. The results for the daily spot prices series are largely in accordance with the monthly series, we obtain conditional-mean estimates 1.783 (DOLS) and 1.646 (FM-OLS). The quantile-dependent estimates are depicted in Figure 7. The response is weaker compared to the benchmark value in lower quantiles and stronger for upper quantiles. The CUSUM test statistic based on the fully modified residuals is 2.771 and the null hypothesis of linear cointegration is not rejected at the 0.1% significance level. Thus, we find only weak evidence against the null hypothesis of cointegration considering the sample size of 9688 for daily prices compared to the sample size of 550 for the monthly series. The quantile cointegration test statistics are depicted in Figure 8. We observe generally larger $Y^*_{n\tau}$ statistics for the daily series and have to reject the null hypothesis for some upper quantiles. However, the results for the daily series are not unexpected since the power of the quantile cointegration test naturally increases with sample size which is accounted for by the 0.1% significance level. The periods of conditional 25% (75%) quantile daily silver prices, depicted in Figure 9, are identified only slightly different compared to the quantile cointegration model for monthly prices although the Hunt brothers episode is excluded from the sample.

We now turn to the relationship between prices of gold and silver futures contracts. The long-run equilibrium (13) is estimated and subsequently the linear cointegration test is conducted. The CS_n statistic amounts to 2.794 which does not lead to a rejection of the null hypothesis of linear cointegration at the 0.1% significance level. Again,

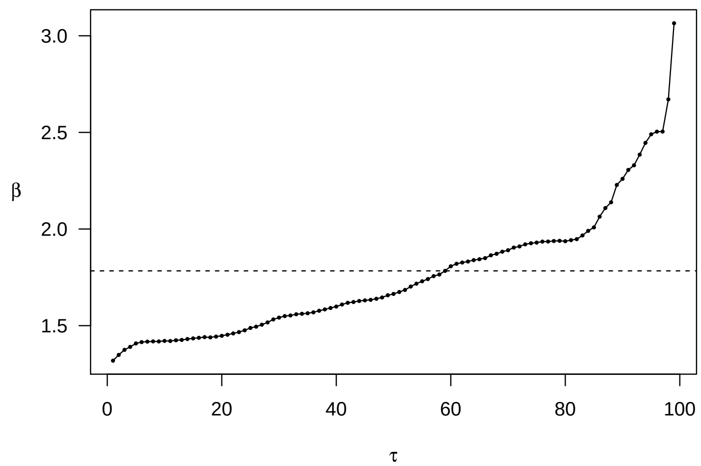

Figure 7: Estimation results for the slope coefficient in the lead-lag augmented quantile cointegration regression (daily series). The DOLS estimate serves as a benchmark (dashed line).

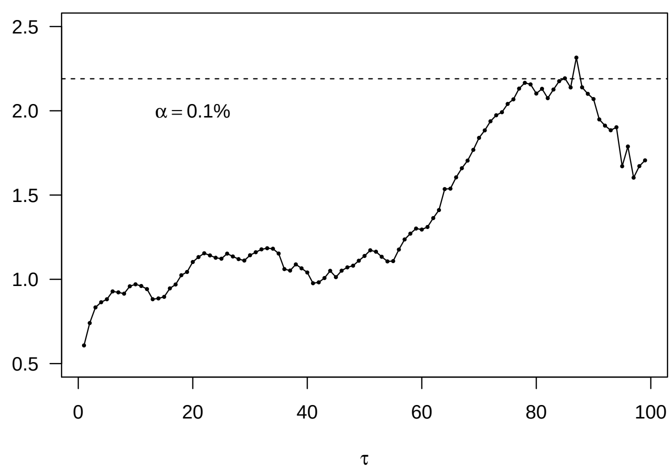

Figure 8: The estimated $Y^*_{n\tau}$ statistic for the 1% to 99% quantile (daily series). The critical value for the 0.1% significance level is 2.19.

evaluating the $Y^*_{n\tau}$ process points to no cointegration relationship for upper quantiles of silver. The quantile-dependent estimates of β display an increasing response to gold futures prices for upper quantiles of the conditional distribution. The indicated periods

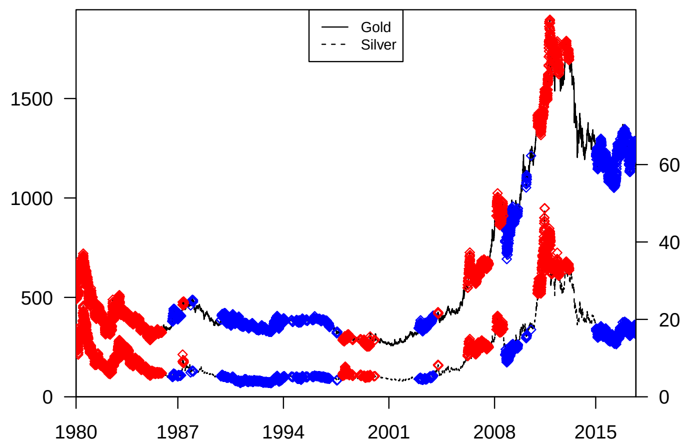

Figure 9: Periods of conditional 25% (75%) quantile daily silver prices. Observations are marked with a blue (red) rhombus if they received a higher weight in the loss function of the 25% (75%) quantile regression.

of conditional 25% (75%) quantile daily prices of silver futures are virtually identical to the daily spot price series and hence not reported. The data span from 1992 to 1998 for which Ciner [2001] could not find evidence for linear cointegration corresponds to a period of weak dependence in our quantile cointegration model.

The results for daily spot and futures prices are very similar which means that the asymmetrical response pattern is not a unique feature of the price discovery in spot markets. The comovement in bubble and crisis periods is not necessarily created by distinct features of the gold fixing process but rather could be generated by a general need of investors for safe haven assets. Both precious metals share store of value characteristics which are most sought after during times of financial market turbulence. In tranquil times, the individual (industrial) demand for gold or silver seems to drive the individual prices.

5 Conclusion

In this paper, we estimate a time-varying cointegrating vector for the gold and silver long-run relationship depending on the innovation quantile. Our empirical results point to an asymmetric dependence between silver prices and gold prices. We observe a stronger response of silver prices to gold price changes when silver prices are at a relatively high level and a stronger response of gold prices to silver price changes when gold prices are at a relatively high level. The long-run relationship between gold and silver is therefore best characterized by a state-dependence.

More specifically, after the prices were deregulated in 1971, high gold and silver prices can generally be found in times of financial stress and bubble-like episodes. Only in those periods, we find a strong dependence between prices which results in a visible comovement. It can be suspected that one of the key properties of gold and silver – the store of value aspect – plays a more prominent role in periods of financial turbulence where other assets lose value and the investors' search for safe haven assets increases demand for gold and silver. This in turn increases prices for gold and silver simultaneously. Moreover, the analysis over a post-'silver bubble' sample and at a different frequency shows that the asymmetrical pattern is remarkably stable and the results can easily be transferred to the futures market.

In general, we emphasize the abilities of the quantile cointegration framework to detect nonlinearities in a cointegration relationship. Considering our empirical results, it is now possible to understand the difficulties, described in previous studies, to find a stable long-run relationship between the two precious metals. Although we observe a comovement of both prices over decades, we fail to estimate a single constant cointegrating coefficient that connects both prices. Allowing for a more general time-varying and quantile-dependent cointegrating vector enables us to capture the time- and state-dependence of the long-run relationship. We find that the intercommodity spread is only then stationary if the nonlinear cointegrating vector varies across conditional quantiles. The estimated relationship cannot directly be used for forecasting, since the exact state of the variables is generally unknown. From that perspective, finding evidence for quantile cointegration but not finding evidence for linear cointegration does not contradict the weak form efficiency of gold and silver markets. In fact, given our results, a statistical arbitrage strategy based on the weakly linked gold and silver prices under the assumption of a single constant coefficient (gold-silver spread trading) would be very risky.

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7 Appendix

Figure 10: Supply of gold (left) and silver (right) in tonnes from 1977 to 2016. Data taken from the GFMS gold and silver surveys (1991 - 2017).

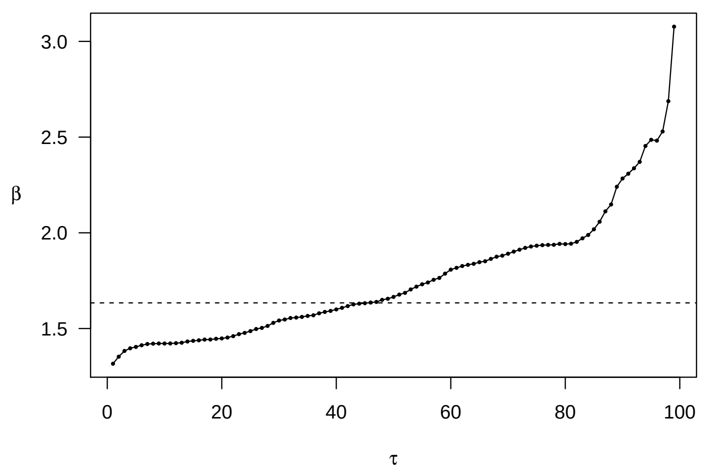

Figure 11: Estimation results for the slope coefficient in the lead-lag augmented quantile cointegration regression (futures contract prices). The DOLS estimate serves as a benchmark (dashed line).

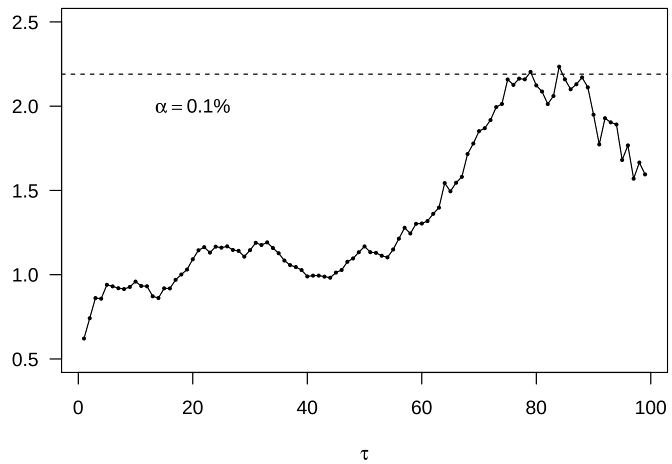

Figure 12: The estimated $Y^*_{n\tau}$ statistics for the 1% to 99% quantile (futures contract prices). The critical value for the 0.1% significance level is 2.19.

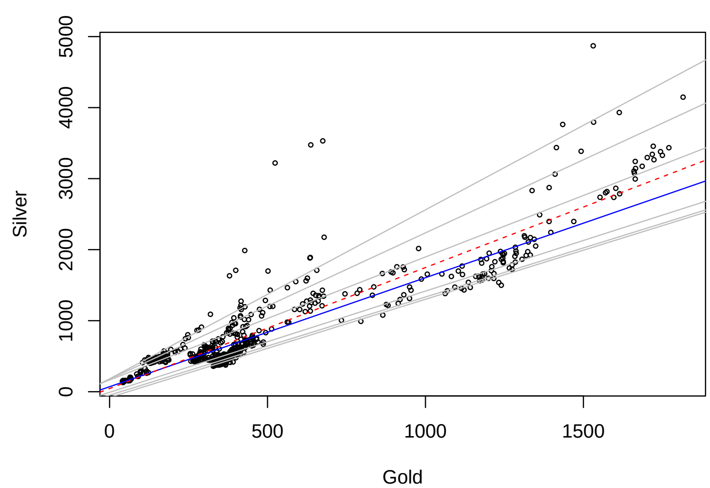

Figure 13: Scatterplots of monthly gold and silver spot prices. The median regression line is drawn in blue, the quantiles regression lines are displayed in gray for $\tau \in \{95, 90, 75, 25, 10, 5\}$ and the DOLS estimate serves as a benchmark (red dashed line).

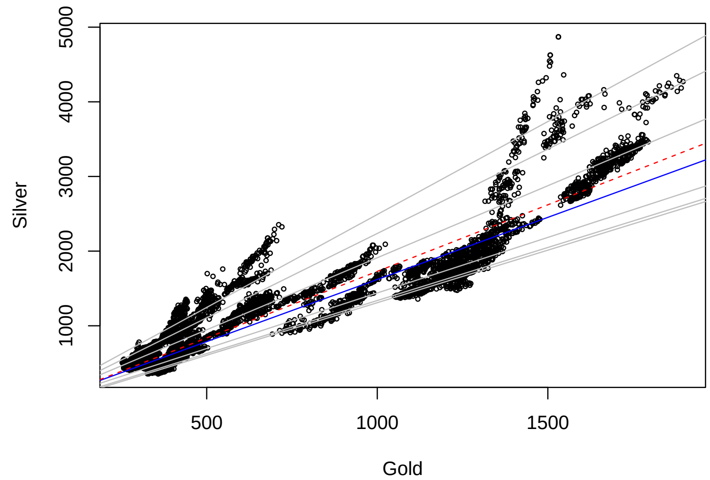

Figure 14: Scatterplots of daily gold and silver spot prices. The median regression line is drawn in blue, the quantiles regression lines are displayed in gray for $\tau \in \{95, 90, 75, 25, 10, 5\}$ and the DOLS estimate serves as a benchmark (red dashed line).

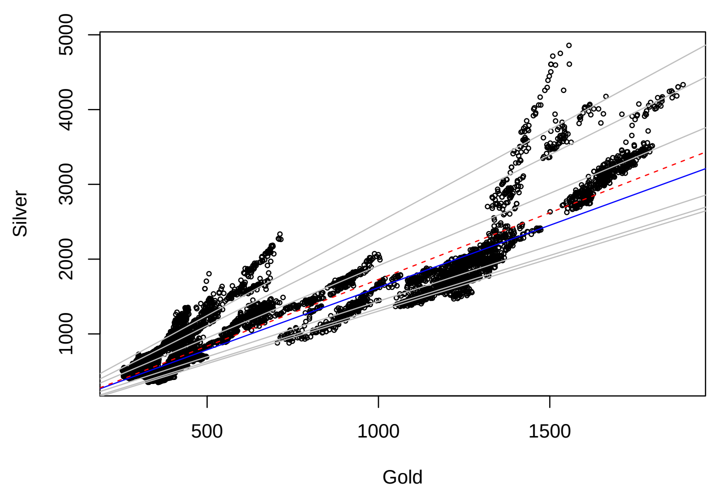

Figure 15: Scatterplots of gold and silver futures prices. The median regression line is drawn in blue, the quantiles regression lines are displayed in gray for $\tau \in \{95, 90, 75, 25, 10, 5\}$ and the DOLS estimate serves as a benchmark (red dashed line).

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