Testing for cointegration with threshold adjustment in the presence of structural breaks

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Abstract

In this paper, we develop new threshold cointegration tests with SETAR and MTAR adjustment allowing for the presence of structural breaks in the equilibrium equation. We propose a simple procedure to simultaneously estimate the previously unknown breakpoint and test the null hypothesis of no cointegration. Thereby, we extend the well-known residual-based cointegration test with regime shift introduced by Gregory and Hansen (1996a) to include forms of nonlinear adjustment. We derive the asymptotic distribution of the test statistics and demonstrate the finite-sample performance of the tests in a series of Monte Carlo experiments. We find a substantial decrease of power of the conventional threshold cointegration tests caused by a shift in the slope coefficient of the equilibrium equation. The proposed tests perform superior in these situations. An application to the 'rockets and feathers' hypothesis of price adjustment in the US gasoline market provides empirical support for this methodology.

Keywords: Cointegration, threshold autoregression, structural change, SETAR, MTAR, asymmetric price transmission
MSC Codes: 62H15, 62M10, 62P20
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1 Introduction

The residual-based threshold cointegration models developed by Enders and Siklos (2001) are a useful addition to the toolbox of researchers working with multivariate time series. They are easy to apply, allow for discontinuous adjustment to a long-run equilibrium and nest linear cointegration in the sense of Engle and Granger (1987) as a special case. The dynamics of the adjustment process are described by a two-regime threshold autoregressive (TAR) model which partitions the residual process according to a threshold value and specifies different coefficients of the leading autoregressive lag for each regime. It can therefore be considered a restricted model under the general class of TAR models described by Tong (1983, 1990). A prominent application in the economics literature is the empirical analysis of asymmetric price transmissions in which case non-stationary price series form a cointegrating relationship and may feature asymmetric adjustment to the long-run equilibrium (Meyer and Cramon-Taubadel (2004), Perdiguero (2013) and citations therein). The speed of adjustment for these processes is usually assumed to depend on the sign and magnitude of the deviations from the long-run equilibrium. Empirical studies ultimately aim to test the null hypothesis of symmetric adjustment against the alternative of asymmetric adjustment (Frey and Manera, 2007). While threshold cointegration models are suitable to study these cases, they do not account for possible structural change in the long-run relationship.

It is well-known that conventional residual-based cointegration tests perform poorly when a cointegration relationship has structural breaks (see, for example, Gregory et al. (1996)). Maki (2012) found that the power property of threshold cointegration tests is more robust to structural breaks than, for example, Engle-Granger cointegration tests assuming linear adjustment. Nevertheless, the power of all residual-based cointegration tests is impaired if the tests do not model structural breaks explicitly. Consequently, it is difficult to provide evidence for the existence of a cointegration relationship. Furthermore, the estimated residual series does not approximate the true equilibrium errors if the cointegrating vector does not account for structural change. Hence, subsequently applied error correction models produce biased adjustment coefficients and tests for asymmetry become invalid.

An extensive body of literature exists on the problem of structural instability in time series. Based on the seminal work of Perron (1989), several unit root tests accounting for structural change have been developed (see, inter alia, Zivot and Andrews (1992), Lumsdaine and Papell (1997) and Lee and Strazicich (2003)). Structural breaks in linear cointegration models are addressed in Gregory and Hansen (1996a,b), Carrion-i Silvestre and Sanso (2006), Arai and Kurozumi (2007), Westerlund and Edgerton (2007), Hatemi-J (2008), Davidson and Monticini (2010), Kejriwal and Perron (2010) and Maki (2012). For comprehensive surveys on structural change in time series models, see Perron (2006) and Aue and Horváth (2013). Gregory and Hansen (1996a), henceforth GH, propose a residual-based cointegration test which accounts for one structural break in the long-run equilibrium equation, i.e. a breakpoint at which the cointegrated system attains a new equilibrium. Their test does not require a pre-specified breakpoint which is rarely known in empirical applications. Instead, a single breakpoint with unknown timing is determined from the data based on one of three structural break models. However, the GH test is only suitable for cointegration models with linear adjustment.[1] We contribute to the literature by extending the GH test to include two forms of non-linear adjustment. These new tests are residual-based and use either a self-exciting threshold autoregressive (SETAR) model or a momentum threshold autoregressive (MTAR) to describe the adjustment toward equilibrium. Thereby, we also robustify both cointegration tests proposed by Enders and Siklos (2001) against a structural break in the long-run equilibrium equation.

We derive the limiting distributions of the test statistics considered in this paper and provide a formal proof. The properties of the proposed test are investigated by Monte Carlo experiments for a variety of models ranging from linear adjustment with no structural break to non-linear adjustment with structural break in the intercept and slope coefficients. The results suggest that a break in the intercept does not influence the power of the threshold cointegration tests enough to justify modelling the structural break. However, a break in the slope coefficients reduces the power of the Enders-Siklos tests substantially such that our proposed tests perform clearly better than their benchmarks. In addition, we find that the unknown breakpoints are estimated accurately by the new procedure.

The methodology is applied to empirical data in the context of the 'rockets and feathers' hypothesis. We use monthly US gasoline market data covering the Global Financial Crisis and a substantial transformation of the US refining industry. We illustrate that empirical evidence for the existence of a long-run relationship between neighbouring stages of the gasoline value-chain can only be provided if we control for a structural break in the cointegrating vector. Using a cointegration model with SETAR

[1]The effects on the power properties of linear cointegration tests, if the equilibrium error follows a nonlinear adjustment process, are reported in Pippenger and Goering (2000).

adjustment and the possibility of structural breaks, we find evidence for asymmetric adjustment from spot gasoline to retail gasoline prices. The MTAR model yields similar results.

The paper is organized as follows. Section 2 describes the models and the cointegration testing procedure, Section 3 presents the asymptotic distributions of the test statistics. Section 4 is devoted to the Monte Carlo simulation study. Section 5 reports the results of the empirical application, and Section 6 concludes with a summary and suggestions for future research.

2 Models and cointegration testing

The long-run equilibrium equation of Engle-Granger cointegration models is given by

$$\begin{aligned} y_t &= \mu + \alpha_1 x_{1t} + \alpha_2 x_{2t} + \cdots + \alpha_m x_{mt} + e_t \\ &= \mu + \alpha' x_t + e_t \end{aligned} \tag{1}$$

where $t = 1, 2, \ldots, T$ is the time series index, y_t and $x_t = (x_{it}, x_{2t}, \ldots, x_{mt})'$ are $I(1)$ variables, μ is an intercept, $\alpha' = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ is a vector of slope coefficients and e_t is the equilibrium error. The null hypothesis of no cointegration is rejected if the residuals obtained from least squared estimation of Equation (1) are mean-zero stationary. Since the parameters μ and α are assumed to be time-invariant, a residual-based cointegration test becomes invalid if the long-run equilibrium is subject to structural change.

Following Perron (1989) and Gregory and Hansen (1996a), we consider three forms of structural change.[2] First, in the C model, a break in the intercept μ is considered. This model captures events that cause a parallel shift of the equilibrium equation. Second, the C/T model adds an additional trend term to the equilibrium equation. Third, in the C/S model, a simultaneous break in the constant and slope parameters is specified. This model allows for the possibility of a complete regime shift at one point

[2] We restrict our analysis to these three models. However, our methodology can easily be adapted for other structural break models, as for example given in Gregory and Hansen (1996b) and Hatemi-J (2008).

in time. The three models are given as follows,

$$\begin{array}{rrcl} (C) & y_t & = & \mu_1 + \mu_2\varphi_{t,\tau} + \alpha' x_t + e_{t\tau} \\ (C/T) & y_t & = & \mu_1 + \mu_2\varphi_{t,\tau} + \delta t + \alpha' x_t + e_{t\tau} \\ (C/S) & y_t & = & \mu_1 + \mu_2\varphi_{t,\tau} + \alpha_1' x_t + \alpha_2' x_t \varphi_{t,\tau} + e_{t\tau} \end{array} \tag{2}$$

where μ_1, μ_2 are constants, $\alpha_1 = (\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1m})'$ and $\alpha_2 = (\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2m})'$ are slope coefficients. The dummy variable $\varphi_{t,\tau}$ is defined as

$$\varphi_{t,\tau} = \begin{cases} 1 & \text{if } t \geq [T\tau] \\ 0 & \text{if } t < [T\tau] \end{cases}, \tag{3}$$

where $\tau \in (0, 1)$ denotes the relative timing of the breakpoint (break fraction), and $[\cdot]$ denotes integer part. The timing of the breakpoint is rarely known in empirical applications so that the GH test is constructed without the need of pre-specified breakpoints. More specifically, a grid search over all possible breakpoint is employed, i.e. the structural change model is repeatedly estimated for each possible break fraction $\tau \in \mathcal{T}$. The set $\mathcal{T}$ can be any compact subset of $(0, 1)$ which excludes endpoint results. GH suggest a lateral trimming of 15 percent ($\mathcal{T} = (0.15, 0.85)$) and, for computational reasons, consider only integer steps. Estimating one of the structural break models in (3) by least squares for each breakpoint yields a sequence of residuals. The GH test applies the ADF test to each sequence and evaluates the null hypothesis of no cointegration based on the smallest values of the t ratios across all $\tau \in \mathcal{T}$. If the null hypothesis is rejected, the break fraction $\hat{\tau}$ corresponding to the infimum statistic is considered to be the most likely breakpoint.

In order to account for asymmetric adjustment, the two-regime SETAR model is now used to describe the adjustment toward equilibrium. The SETAR model for the breakpoint-specific equilibrium error process $e_{t\tau}$ is given by

$$\Delta e_{t\tau} = \rho_1 e_{t-1\tau} \mathbb{1}\{e_{t-1\tau} \geq \lambda\} + \rho_2 e_{t-1\tau} \mathbb{1}\{e_{t-1\tau} < \lambda\} + \sum_{j=1}^{K} \gamma_j \Delta e_{t-j\tau} + \epsilon_{t\tau K}, \tag{4}$$

where $\mathbb{1}\{\cdot\}$ denotes the Heaviside indicator function, the parameter λ is a possibly non-zero threshold value and $\epsilon_{t\tau K}$ is a stationary mean zero error term. The coefficient ρ_1 measures the mean-reversion toward the attractor after a shock greater than or equal to λ whereas ρ_2 measures the mean-reversion toward the cointegrating vector after a shock less than λ. The indicator function in this case is set according to the level of $e_{t-1\tau}$.

In an alternative specification, suggested by Enders and Granger (1998) and Caner and Hansen (2001), the indicator function is set depending on $\Delta e_{t-1\tau}$. The two-regime MTAR model is given by

$$\Delta e_{t\tau} \quad = \quad \rho_1 e_{t-1\tau} \mathbb{1}\{\Delta e_{t-1\tau} \geq \lambda^*\} + \rho_2 e_{t-1\tau} \mathbb{1}\{\Delta e_{t-1\tau} < \lambda^*\} + \sum_{j=1}^{K} \gamma_j \Delta e_{t-j\tau} + \epsilon_{t\tau K}.$$

In this specification, ρ_1 measures the mean-reversion toward the attractor if a shock has momentum greater than or equal to λ^* whereas ρ_2 measures the mean-reversion toward the cointegrating vector if a shock has momentum less than λ^*.

Under the null hypothesis of no cointegration, $\rho_1 = \rho_2 = 0$, the data-generating process (DGP) of $e_{t\tau}$ is symmetric and a unit root is present in both regimes. Models (4) and (5) are a special case of the general class of threshold autoregressive models in that they do not allow for regime-specific deterministic terms and regime-specific dynamics beyond the leading autoregressive lag. This restriction is convenient since it circumvents the problem of having an identified threshold under the null hypothesis resulting in an asymptotic distribution of the test statistic that depends on nuisance parameters (see Caner and Hansen (2001) for a more detailed discussion in the context of MTAR processes with a unit root). Furthermore, the Engle-Granger test for symmetric adjustment ($\rho_1 = \rho_2$) is itself a special case of (4) and (5). Petruccelli and Woolford (1984) show that the stationarity of the SETAR process is ensured if $\rho_1 < 0$, $\rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 1$ for any value λ. In the case of MTAR processes, Lee and Shin (2000) prove that stationarity is ensured if $\rho_1 < 0$, $\rho_2 < 0$, $(1 + \rho_1)(1 + \rho_2) < 1$, $(1+\rho_1)(1+\rho_2)^2 < 1$ and $(1+\rho_1)^2(1+\rho_2) < 1$. Assuming stationarity, Tong (1983, 1990) demonstrated that least squares estimators of ρ_1 and ρ_2 are asymptotically normally distributed. Enders and Siklos (2001) recommend a Wald-type F-test to test the null hypothesis of no cointegration in their model without structural breaks. However, since the F-test can lead to rejection of the null hypothesis when only one coefficient is negative, the test should only be applied if both point estimates have the correct sign and suggest a mean-reverting behaviour. In other words, the one-sided alternative $\rho_1 < 0 \wedge \rho_2 \geq 0$ or $\rho_2 < 0 \wedge \rho_1 \geq 0$ should not lead to rejection of the null hypothesis. Please also see Caner and Hansen (2001) for an extensive discussion of this issue.

In the case of a cointegration model with potential structural break, we propose the following cointegration test: First, an appropriate structural break model is selected from (3) and the cointegrating regression is estimated by least squares for each break fraction $\tau \in \mathcal{T}$.[3] Then, the SETAR or MTAR regression is estimated and the F-statistic, F_τ, is computed for each sequence of residuals. Since the indicators are orthogonal, we can write the test statistic as

$$F_\tau = \frac{t_1^2 + t_2^2}{2}, \tag{5}$$

where t_1 and t_2 are the t ratios for $\hat{\rho}_1$ and $\hat{\rho}_2$ from regression (4) or (5). The null hypothesis of no cointegration is naturally rejected for large values of the F-statistic. Consequently, we use the supremum statistic,

$$F^* = \sup_{\tau \in \mathcal{T}} F_\tau, \tag{6}$$

to evaluate the null hypothesis of no cointegration against the alternative of threshold cointegration with possible structural break. The largest value found in this grid search determines the most likely breakpoint if the null hypothesis is rejected. The supremum statistic is evaluated because it puts the most weight on the alternative hypothesis and corresponds to the model specification with the fastest error correction. Note that the alternative contains as a special case the standard model of cointegration under parameter constancy. This means that rejection of the null hypothesis does not provide evidence concerning the question of whether or not a structural break occurred. However, it should help practitioners to find the correct model specification for cointegrated systems with threshold adjustment and possibility of structural change.

3 Asymptotic distribution

In the following, we present the asymptotic distributions of the test statistics as functionals of Brownian motion. The asymptotic theory for SETAR processes with a unit root was developed in Seo (2008) and the asymptotic theory for MTAR processes with a unit root was developed in Caner and Hansen (2001). Gregory and Hansen (1996a) pro-

[3]Critical values for the test statistic depend on the degree of lateral trimming. Imposing a lateral trimming of 15% ignores potential breakpoints located at the beginning and end of the sample. Subsample analysis should be used to determine if potential structural breaks in this region influence the test decision.

vide important results for cointegration test statistics which are functions of the break fraction parameter τ. These results serve as the building block for our residual-based tests.

For notational convenience we use '$\Rightarrow$' to signify weak convergence of the associated probability measures and '$\equiv$' to denote equivalence in distribution. Continuous stochastic processes such as the Brownian motion $B(s)$ on [0,1] are simply written as B if no confusion will be caused. We also write integrals with respect to the Lebesgue measure such as $\int_0^1 B(s)ds$ simply as $\int_0^1 B$.

Let $\{z_t\}_0^\infty$ be an $(m+1)$-vector integrated process whose data generating process is

$$z_t = z_{t-1} + \xi_t, \qquad t = 1, 2, \ldots \tag{7}$$

where it is assumed that $T^{-1/2} z_0 \xrightarrow{p} 0$ so that z_0 can be treated as either fixed or random and the results do not depend on the initial condition. We partition $z_t = (y_t, x_t')'$ into the scalar variate y_t and the m-vector x_t. The $(m+1)$-vector random sequence $\{\xi_t\}_1^\infty$ is defined on the probability space $(X, \mathcal{F}, P)$ and is assumed to be stationary and ergodic with zero mean and finite variance. $\{\xi_t\}_1^\infty$ is assumed to be a linear process that satisfies the following regularity conditions:

Assumption 1. *The process $\{\xi_t\}_1^\infty$ is generated as $\xi_t = \sum_{j=0}^{\infty} C_j \nu_{t-j}$, $\sum_{j=0}^{\infty} j \, \|C_j\| < \infty$ and $\nu_t \sim iid(0, \Sigma)$, where Σ is a positive definite variance matrix. Further, $E|\nu_t|^r < \infty$ for some $r \geq 4$.*

Assumption (1) ensures the validity of the function central limit theorem (FCLT) for partial sum processes constructed from $\{\xi_t\}$ (see, for example, Theorem 3.4 in Phillips and Solo (1992) and its multivariate extension in Phillips (1995)). Hence, it holds for $s \in [0, 1]$ and as $T \to \infty$ that

$$X_T(s) = T^{-1/2} \sum_{t=1}^{[Ts]} \xi_t \Rightarrow B(s), \tag{8}$$

where $B(s)$ is $(m+1)$-vector Brownian motion with covariance matrix

$$\Omega = \lim_{T \to \infty} T^{-1} E\left(\left(\sum_{t=1}^{T} \xi_t\right)\left(\sum_{t=1}^{T} \xi_t'\right)\right). \tag{9}$$

We partition Ω and B conformably with z_t:

$$B = \begin{bmatrix} B_y \\ B_x \end{bmatrix} \qquad \Omega = \begin{bmatrix} \omega_{11} & \omega_{21}' \\ \omega_{21} & \Omega_{22} \end{bmatrix}. \tag{10}$$

We assume $\Omega_{22} > 0$ and decompose Ω as $\Omega = L'L$, where L is given by

$$L = \begin{bmatrix} l_{11} & 0 \\ l_{21} & L_{22} \end{bmatrix}, \tag{11}$$

with $l_{11} = (\omega_{11} - \omega_{21}'\Omega_{22}^{-1}\omega_{21})^{1/2}$, $l_{21} = \Omega_{22}^{-1/2}\omega_{21}$, and $L_{22} = \Omega_{22}^{1/2}$. Further, we define $W(s)$ to be $(m+1)$-vector standard Brownian motion and from Lemma 2.2 of Phillips and Ouliaris (1990) it follows that $B \equiv L'W$.

Residual-based cointegration tests seek to test the null hypothesis of no cointegration using unit root tests applied to the residuals of the cointegrating regression. Hence, we estimate the cointegrating regression according to one of the structural break models (3) using least squares and apply the SETAR model (4) to the residuals $\hat{e}_{t\tau}$ given that the threshold parameter λ is known, i.e. a fixed value. The cointegration residual series $\hat{e}_{t\tau}$ follows a stochastic trend under the null hypothesis and has no stable distribution. Hence, the exact threshold value is negligible asymptotically. In case of finite samples, we still have to specify threshold values that ensure a sufficiently large number of observations in each regime. Otherwise, we cannot guarantee that both regime-dependent coefficients of the SETAR model are accurately estimated. A guideline for practitioners is to verify that at least 15% of observations are contained in each regime for all $\tau \in \mathcal{T}$.

If we use the MTAR specification in (5) instead, we observe that the threshold parameter λ^* has different properties. The threshold variable $\Delta\hat{e}_{t\tau}$ has a stationary distribution under the null hypothesis and the alternative. Hence, the empirical cdf of $\Delta\hat{e}_{t\tau}$ consistently estimates the cdf of its asymptotic counterpart under both hypotheses. It follows, that any given threshold value corresponds to a probability of $\Delta\hat{e}_{t\tau}$ or its asymptotic counterpart being greater than the threshold. Practitioners should either verify that at least 15% of observations are contained in each regime for all $\tau \in \mathcal{T}$ or should alternatively specify the probability $u \in [0.15, 0.85]$ of the asymptotic counterpart to $\Delta\hat{e}_{t\tau}$ being greater than a threshold λ^* directly.

We assume the lag order K in the auxiliary regression to be large enough to capture the correlation structure of the cointegration residuals. Similar to Said and Dickey (1984), we approximate the infinite order process $\epsilon_{t\tau}$ by a TAR model with finite lag order $\epsilon_{t\tau K}$. Since $\epsilon_{t\tau}$ might have a nonzero MA component, it is necessary to increase K with the sample size. In practice, we can use order selection rules such as AIC, BIC or a general-to-specific pretesting procedure to determine the lag truncation parameter. We follow Chang and Park (2002) and state:

Assumption 2. *K increases with T in such a way that $K = o(T^{1/2})$.*

The following theorem presents the asymptotic distributions of the sup F test statistic for model specifications C, C/T and C/S and SETAR adjustment:

Theorem 1. *If $\{z_t\}_0^\infty$ is generated by (7), Assumptions (1) and (2) hold and τ belongs to a compact subset of $(0,1)$, then as $T \to \infty$*

$$F^*_{SETAR} \Rightarrow \frac{1}{2}\sup_{\tau\in\mathcal{T}}\left\{\frac{\left(\int_0^1 \mathbb{1}\{Q_{\kappa\tau} \geq 0\}Q_{\kappa\tau}dQ_{\kappa\tau}\right)^2}{\kappa_\tau' D_\tau \kappa_\tau \int_0^1 \mathbb{1}\{Q_{\kappa\tau} \geq 0\}Q_{\kappa\tau}^2} + \frac{\left(\int_0^1 \mathbb{1}\{Q_{\kappa\tau} < 0\}Q_{\kappa\tau}dQ_{\kappa\tau}\right)^2}{\kappa_\tau' D_\tau \kappa_\tau \int_0^1 \mathbb{1}\{Q_{\kappa\tau} < 0\}Q_{\kappa\tau}^2}\right\}$$

where

$$\begin{aligned} Q_{\kappa\tau} &= W_y - \left(\int_0^1 W_{x\tau}W_{x\tau}'\right)^{-1}\left(\int_0^1 W_y W_{x\tau}'\right)W_{x\tau} \\ \kappa_\tau &= \left(1, -\left(\int_0^1 W_{x\tau}W_{x\tau}'\right)^{-1}\left(\int_0^1 W_y W_{x\tau}'\right)\right) \end{aligned}$$

Under the alternative of cointegration with two-regime SETAR adjustment, $F^*_{SETAR} \to \infty$ as $T \to \infty$. $Q_{\kappa\tau}$ depends on the model:
a) If the residuals are obtained from least squares estimation of model C, then

$$W_{x\tau} = (W_x', 1, \varphi_\tau)'$$

$$D_\tau = \begin{bmatrix} I_{m+1} & 0 \\ 0 & 0 \end{bmatrix}.$$

b) If the residuals are obtained from least squares estimation of model C/T, then

$$W_{x\tau} = (W_x', 1, s, \varphi_\tau)'$$

$$D_\tau = \begin{bmatrix} I_{m+1} & 0 \\ 0 & 0 \end{bmatrix}.$$

c) If the residuals are obtained from least squares estimation of model C/S, then

$$W_{x\tau} = (W_x', 1, W_x'\varphi_\tau, \varphi_\tau)'$$

$$D_\tau = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & I_m & 0 & (1-\tau)I_m & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\tau)I_m & 0 & (1-\tau)I_m & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

A formal proof of Theorem 1 is provided in the Appendix.

Remark 1. The alternative of cointegration encompasses models with linear adjustment as a special case. Further, it can be inferred from the second part of the proof that the null hypothesis of no cointegration can be rejected for cases with and without structural break.

While the threshold variable in the SETAR model is nonstationary under the null hypothesis, the threshold variable in the MTAR model is stationary under both the null hypothesis and alternative. Hence, we have to treat the MTAR case separately and derive the asymptotic distribution of the $\sup F$ test statistic for cointegration models with MTAR adjustment in Theorem 2.

Theorem 2. *If $\{z_t\}_0^\infty$ is generated by (7), Assumptions (1) and (2) hold and τ belongs to a compact subset of $(0,1)$, then as $T \to \infty$*

$$F^*_{MTAR} \Rightarrow \frac{1}{2} \sup_{\tau \in \mathcal{T}} \left\{ \frac{\left(\int_0^1 Q_{\kappa\tau}(s) dW(s,u) \right)^2}{u \int_0^1 Q^2_{\kappa\tau}(s) ds} + \frac{\left(\int_0^1 Q_{\kappa\tau}(s) \left(dW(s,1) - dW(s,u) \right) \right)^2}{(1-u) \int_0^1 Q^2_{\kappa\tau}(s) ds} \right\}$$

where

$$Q_{\kappa\tau} = W_y - \left(\int_0^1 W_{x\tau} W_{x\tau}' \right)^{-1} \left(\int_0^1 W_y W_{x\tau}' \right) W_{x\tau}$$

Under the alternative of cointegration with two-regime MTAR adjustment, $F^*_{MTAR} \to \infty$ as $T \to \infty$. $Q_{\kappa\tau}$ depends on the model:

a) If the residuals are obtained from least squares estimation of model C, then

$$W_{x\tau} = (W_x', 1, \varphi_\tau)'$$

b) If the residuals are obtained from least squares estimation of model C/T, then

$$W_{x\tau} = (W_x', 1, s, \varphi_\tau)'$$

c) If the residuals are obtained from least squares estimation of model C/S, then

$$W_{x\tau} = (W_x', 1, W_x'\varphi_\tau, \varphi_\tau)'$$

A formal proof of Theorem 2 is provided in the Appendix.

Remark 2. Enders and Siklos (2001) do not provide an asymptotic theory for their tests. The theorems given here are easily adapted to provide the asymptotic distributions for models without structural breaks using $W_{x\tau} = (W_x', 1)'$. The asymptotic distribution of their F-statistic using fixed threshold values and a SETAR model is given as a special case of Theorem 1 of this paper and as a special case of Theorem 2 in Maki and Kitasaka (2015). Theorem 2 is new in the multivariate context. It shows that the cointegration test using MTAR adjustment in Enders and Siklos (2001) depends on the nuisance parameter u. However, critical values obtained for different specifications of u are very similar for the standard model without structural breaks.

4 Simulation results

Critical values and finite sample properties of the $\sup F$ tests are examined by Monte Carlo experiments. In the absence of a structural break, we use a DGP according to Engle and Granger (1987) and Banerjee et al. (1986) which is given for one regressor ($m = 1$) in the form of

$$\begin{array}{lllllll} y_t &=& \mu + \alpha x_{1,t} + e_t & \Delta e_t &=& \rho e_{t-1} + \vartheta_t & \vartheta_t \sim N(0,1) \\ y_t &=& x_{1,t} + \eta_t & \eta_t &=& \eta_{t-1} + \omega_t & \omega_t \sim N(0,1), \end{array} \tag{12}$$

where the parameters of the equilibrium equation are $\mu = 1$ and $\alpha = 2$. First, the null hypothesis of no cointegration is simulated with $\rho = 0$. This enables us to obtain quantiles of the $\sup F$ distribution for different sample sizes. The BIC is used to determine the lag truncation parameter K. Critical values are computed for $10,000$ draws for each sample size. The results are reported in Table 1, Table 2 and Table 3.

The power of the sup F test under structural change is evaluated with a DGP designed in line with Gregory and Hansen (1996a). A slight modification was, however, necessary to allow for a linear trend in the long-run equilibrium equation and asymmetric adjustment to the long-run equilibrium. The following DGP is employed for a bivariate cointegrated system with SETAR adjustment,

$$\begin{array}{lllllll} y_t & = & \mu_t + \delta t + \alpha_t x_{1,t} + e_t & \Delta e_t & = & \begin{cases} \rho_1 e_{t-1} + \vartheta_t & \text{if } e_{t-1} \geq 0 \\ \rho_2 e_{t-1} + \vartheta_t & \text{if } e_{t-1} < 0 \end{cases} & \vartheta_t \sim N(0,1) \\ y_t & = & x_{1,t} + \eta_t & \eta_t & = & \eta_{t-1} + \omega_t & \omega_t \sim N(0,1) \end{array}$$

$$\left[\begin{array}{lll} \mu_t = \mu_1, & \alpha_t = \alpha_1, & t \leq [T\tau] \\ \mu_t = \mu_2, & \alpha_t = \alpha_2, & t > [T\tau] \end{array} \right], \tag{13}$$

in which symmetric adjustment is nested as $\rho_1 = \rho_2$. In the case of MTAR adjustment, the speed of adjustment depends on whether the previous period's change was greater than the median of Δe_t. Thus, we investigate the power for $u = 0.5$. A change in the intercept is modelled by means of an increase from $\mu_1 = 1$ to $\mu_2 = 4$ at the breakpoint, whereas a change in the slope is modelled as an increase from $\alpha_1 = 2$ to $\alpha_2 = 4$. The simulation set-up used for cointegrated systems with symmetric adjustment directly follows Gregory and Hansen (1996a) so that the results for the sup F test can be compared with the results for the GH test.

Table 4 reports the rejection rates under cointegration with symmetric adjustment and structural break in either the intercept or slope. The power of the tests is investigated by generating 2,500 draws for every specification. We find that the sup F tests have generally higher rejection rates than either the Engle-Granger test using the ADF test statistic or threshold cointegration tests without breakpoint estimation. The simulation reveals that the sup F test with SETAR adjustment has comparable power properties to the GH test. The MTAR specification of the sup F test has slightly lower power against the alternative than the GH test. The Enders-Siklos test with SETAR adjustment seems to be rather robust to a break in the intercept but suffers from a drastic reduction in power if a break in the slope is considered. The sup F tests appear to have sufficient power at sample sizes above $T = 100$ and moderate adjustment rate $\rho = -0.5$. As expected, model C outperforms models C/T and C/S if a break in the intercept is considered, while model C/S performs best if the slope changes at one point in the sample.

The simulation results under symmetric adjustment can also be used to analyze the estimation accuracy of the pre-specified breakpoint in the DGP. The timing of the break is varied and takes place either at the beginning ($\tau = 0.25$), the middle ($\tau = 0.5$) or near the end of the series ($\tau = 0.75$). The results are summarized in Table 5 and reveal that breakpoint estimates are in large parts very accurate. In general, it seems that breaks at the beginning of the sample are most difficult to detect and the $\sup F$ tests often indicate a later breakpoint. Breaks in the intercept and the slope are estimated with equal accuracy as long as the correct structural break model is applied. The SETAR model seems to produce slightly more accurate breakpoint estimates than the MTAR model.

The upper panels of Table 6 and Table 7 display the rejection rates under structural stability and asymmetric adjustment. For each combination of autoregressive coefficients, we generate series with sample size $T = 100$. If the series are generated under asymmetric adjustment with a stable cointegrating vector, we find that the $\sup F$ tests operate with less power than the threshold cointegration tests by Enders and Siklos (2001). This is not surprising, considering that wrongly specified breaks in form of additional dummy variables in the equilibrium equation leads to noisy coefficient estimates and thus reduces the test's power against the null hypothesis. Accordingly, the most parsimonious model C performs best among the three structural break models.

Finally, the behaviour of the $\sup F$ test is evaluated under parameter instability and asymmetric adjustment. For that matter, we draw from the DGP in (13). We consider SETAR adjustment in Table 6 and MTAR adjustment in Table 7, respectively.[4] In the second panel, we model a break in the intercept. The $\sup F$ tests have poor power properties and are outperformed by the Enders-Siklos test in each parameter combination. The loss in power of the original threshold cointegration test due to a break in the intercept does not justify the additional parameter estimation and grid search of model C. Models C/T and C/S involve an additional parameter and, as expected, have lower rejection rates. In the third panel, we add a linear trend ($\delta = 1$) to the long-run equilibrium equation. Here, only model C/T is correctly specified and has the highest rejection rates for moderate adjustment. The remaining model candidates do not seem to have power against the null hypothesis under this form of

[4] Please note that different forms of misspecification are evaluated in these simulations. In each panel, only one model is correctly specified while the remaining three are either over- or underspecified.

misspecification. In the last panel, we display the results for a simultaneous break in the intercept and the slope and find the picture to be quite different. All structural break models have more power against the null hypothesis than the Enders-Siklos test. As expected, the power of the correctly specified model C/S exceeds all other structural change models for each parameter combination. In general, we find a break in the slope to have a more substantial impact on the power function than a break in the intercept.

Additionally, we evaluate the GH test under asymmetric adjustment and structural instability. The results are reported in Table 8 and Table 9. We find that its power is slightly lower than the sup F tests' power if adjustment is slow and asymmetric. Since we do not know if this situation is present in a given empirical application, we should consider the sup F test instead of the GH test whenever considerable asymmetries are suspected.

Naturally, all results in this section depend strongly on the particular model specification, the break magnitudes and the signal-to-noise ratio. In practice, we do not know the true model and have to decide between available structural change models. Practitioners should ideally argue for a structural change model based on economic reasoning (asking, for example, what type of event could have occurred during the sample period). However, our simulations clearly show that underspecified models do not have power against the null hypothesis, particularly if the variables are trending or in the presence of structural breaks in the slope coefficient.

5 Empirical application

In this section, we apply the sup F test methodology to study the 'rockets and feathers' hypothesis[5] in the US gasoline market. The 'rockets and feathers' hypothesis describes the adjustment behaviour of prices faced with input price shocks. More precisely, the hypothesis states that prices adjust faster to input price increases than to input price decreases. In the terms of Bacon (1991)'s seminal paper, the price goes up like a rocket, but falls down like a feather. While early studies on the matter (Bacon (1991), Manning (1991), Borenstein et al. (1997)) focused on the short-run asymmetry in the pricing process, the focus quickly shifted to the economically meaningful long-run asymmetry estimated by asymmetric error correction models (Bachmeier and Griffin (2002)).

[5] The name originates from the Bacon (1991) paper entitled: 'Rockets and feathers: the asymmetric speed of adjustment of UK retail gasoline prices to cost changes'

For our empirical illustration, we examine the fuel price transmission at two points of the production chain. First, we analyze the speed of adjustment for deviations from the long-run relationship between crude oil prices and gasoline spot prices (*first stage*). Second, we analyze the pass-through from gasoline spot prices to retail prices (*second stage*). Finally, the direct link between crude oil prices and retail prices is analyzed (*single stage*). Naturally, we expect the speed of adjustment at the first and second stage to be faster than at the single stage transmission. Long-run asymmetry in the sense of the 'rockets and feathers' hypothesis is found if negative deviations from the long-run equilibrium are adjusted faster than positive deviations, i.e. $\rho_1 = \rho^- < \rho^+ = \rho_2$. If the threshold value is specified to be $\lambda = 0$ in the SETAR model, the cointegrated system's adjustment depends on whether input prices or output price are too high relative to the long-run equilibrium. Alternatively, we use the MTAR model to investigate whether a shock having momentum greater than or equal to its median is adjusted faster than a shock with less momentum. This specification of the threshold variable guarantees an equal amount of observations in each regime.

Our sample reaches from January 2006 to December 2013 to include the collapse of commodity prices in 2009 and their subsequent recovery. We observe prices at a monthly frequency yielding a total of 96 observations. The West Texas Intermediate prices (p_t^c), regular gasoline spot prices (p_t^s) and regular gasoline retail prices (p_t^g) are all obtained from the U.S. Energy Information Administration (EIA). Figure 1 depicts the trajectory of the prices and shows volatile behaviour of prices for petroleum products during the Global Financial Crisis. Although all times series in Figure 1 seem to be affected by global events, it does not immediately follow that the long-run relationship between them changes. However, from our simulation study, we know that an existing instability of the cointegrating vector can severely decrease the power of threshold cointegration tests.

First, we estimate a threshold cointegration model according to Enders and Siklos (2001). We specify the long-run equilibrium equations

$$\begin{array}{lrcl} (I) & p_t^s & = & \mu + \alpha\, p_t^c + e_t \\ (II) & p_t^g & = & \mu + \alpha\, p_t^s + e_t \\ (S) & p_t^g & = & \mu + \alpha\, p_t^c + e_t \end{array} \tag{14}$$

where the (I), (II), (S) denote *first stage*, *second stage* and *single stage*, respectively. The coefficients of the cointegrating vector are estimated using least squares and a threshold model is applied to the residuals. All adjustment coefficients have the correct sign allowing the test for cointegration and asymmetric adjustment to be conducted. The results for the SETAR model with $\lambda = 0$ are reported in panel (a) of Table 10 and reveal significant asymmetry in the adjustment process only in the *second stage*. The results for the MTAR model with $u = 0.5$ are reported in panel (b) of Table 10.[6] Here, we also find significant asymmetries in the transmission from spot gasoline to retail gasoline prices. Surprisingly, we do not find sufficient evidence for a long-run relationship between crude oil prices and gasoline spot prices. In contrast, retail gasoline prices and crude oil prices seem to maintain a long-run equilibrium which is a less likely result from an economic perspective than the existence of a crude/spot relationship.

Second, we estimate the long-run equilibrium equations again using the C/S specification since this specification of the $\sup F$ tests shows the most robust performance in the simulation study if the variables do not have a linear time trend. It is the only specification that allows for change in the slope coefficient at one point during the sample period and is best-suited for modelling unspecific regime shift events. The results are reported in panel (b) and panel (d) of Table 10. The null hypothesis of no cointegration can now be rejected at all stages along the gasoline value-chain. The breakpoint is located either at the peak crude oil prices during the Global Financial Crisis or after the prices had begun to recover in 2011. Closer inspection of the time series reveals that the spread between crude oil prices and spot gasoline prices widened sharply around 2011. This period coincides with a substantial transformation of the US refining industry (Kilian, 2016). More specifically, the WTI nexus in Cushing had reached capacity due to a surplus of oil coming from North American shale oil fields. This meant that WTI had to be stockpiled and was trading at a discount compared to other crudes like Brent. The bottleneck in the transportation infrastructure seems to have affected the relationship between WTI prices and spot gasoline prices. While the timing of a potential breakpoint was assumed to be unknown, we can investigate the effects of the structural break ex post. Figure 2 displays the estimated regression line through a scatterplot for each cointegration pair.[7] We observe that the regression lines for the *first stage* changes considerably when we incorporate observations after January

[6] Different choices of u in the interval $[0.3, 0.7]$ lead to the same test results. Since the sample size is rather small, choices of u outside of this interval do not guarantee sufficient observations in each regime.

[7] The estimated breakpoints are taken from panel (b) of Table 10 (SETAR specification), but since the estimated breakpoints in panel (d) (MTAR specification) are almost identical, the results hold for both specifications.

2011. Specifically, the post-break observations are clustered in one spot. In contrast, the estimated breakpoint in the *second stage* does not seem to affect the regression line. Here, the post-break observations are evenly distributed across the regression line. The effects on the *single stage* regression line are similar to the ones obtained for the *first stage* although less pronounced. Turning to the tests for asymmetry, we do not find statistical evidence for asymmetric adjustment processes in the *first stage*. The asymmetry results for the *second stage* and *single stage* remain unchanged.

6 Conclusion

This paper proposed an extension to the GH test to include SETAR and MTAR adjustment. Thereby, we constructed threshold cointegration tests which endogenously determine the location of a structural break in the cointegrating vector and test the null of no cointegration. We derived the limiting distribution for the structural break models C, C/T and C/S and tabulated their critical values which were obtained by Monte Carlo simulations. Analysis of the finite sample properties under the alternative of linear and threshold cointegration revealed that the tests exhibit considerable power gains over the conventional Enders-Siklos tests if a break in the slope coefficient is present. We applied the $\sup F$ tests to US gasoline market data and found evidence for a long-run relationship between prices along the value-chain after we accounted for structural breaks. The results for the SETAR and MTAR models provided evidence for asymmetric price transmission from spot gasoline to retail gasoline.

It has to be noted that the performance of the $\sup F$ test naturally depends on the type of structural break and whether the correct model specification has been chosen. Practitioners need to have a strong prior regarding the true DGP and should examine the robustness of their results across different model specifications. Our test, analogously to Gregory and Hansen (1996a) and its extension in Hatemi-J (2008), assumes that there are no level shifts and broken trends in the individual time series under both the null and alternative hypotheses. Hence, the statistical properties of our test might be distorted in these situations. Harris et al. (2016) suggest to pre-test for the presence of a trend break.

Our framework could be extended in various ways. First, our list of structural break models could be expanded to include multiple structural breaks and alternative structural break models. For example, we could consider a break in the linear trend similar to Gregory and Hansen (1996b) or the possibility of encountering two structural breaks during the sampling period as proposed by Hatemi-J (2008). However, the computational costs of higher dimensional grid searches might restrict these considerations. Moreover, the restriction of a fixed threshold value needs to be relaxed to allow for empirical applications where the threshold value is unknown. Since these extensions are beyond the scope of this paper, we leave them for future research.

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Appendix

Proof of Theorem 1. The asymptotic distribution is derived by adapting the results of Gregory and Hansen (1992) to match the F-statistic process involving a threshold indicator function using results in Maki and Kitasaka (2015). However, Maki and Kitasaka (2015) use a different definition of the threshold parameter space in their SETAR model. The threshold parameter in our model is fixed, i.e. belongs to a trivial compact subset of $\mathbb{R}$ whereas the parameter space in Maki and Kitasaka (2015) is data dependent (see the discussion on threshold parameter space in Section 2.2 of their paper). Indicator functions with threshold parameters defined on compact sets are treated in Seo (2008). The proof only refers to model C/S while the results for the remaining models can be deduced from the results obtained for this model. Hence, we consider the cointegrating regression,

$$y_t = \hat{\alpha}_1' x_t + \hat{\mu}_1 + \hat{\alpha}_2' x_t \varphi_{t,\tau} + \hat{\mu}_2 \varphi_{t,\tau} + \hat{e}_{t\tau}, \tag{15}$$

where $\hat{e}_{t\tau}$ is an integrated process under the null hypothesis of no cointegration and $z_t = (y_t, x_t')'$ is generated according to (7).

Define the $(2m+3)$-vector $X_{t\tau} = (y_t, {x_t}', 1, x_t {\varphi_{t,\tau}}', \varphi_{t,\tau})'$ and partition $X_{t\tau} = (X_{1t\tau}, {X_{2t\tau}}')'$ where $X_{1t\tau} = y_t$ and $X_{2t\tau}$ contains all regressors of (15). Define $\delta_T = diag(T^{-1/2} I_{m+1}, 1, T^{-1/2} I_m, 1)$, $\varphi_\tau(s) = \mathbb{1}\{s > \tau\}$ and $X_\tau(s) = (B(s)', 1, B_x(s)\varphi_\tau(s)', \varphi_\tau(s))'$. Partition $\delta_T = (\delta_{1T}, \delta_{2T})$ in conformity to $X_{t\tau}$.

Next, we partition the $(m+1)$-vector standard Brownian Motion W as $W = (W_y, W_x')'$ where

$$\begin{aligned} W_y &= l_{11}^{-1}\left(B_y - \omega_{21}' \Omega_{22}^{-1} B_x\right) \\ W_x &= \Omega_{22}^{-1/2} B_x. \end{aligned} \tag{16}$$

Furthermore, we define

$$W_{x\tau} = ({W_x}', 1, W_x {\varphi_\tau}', \varphi_\tau)' \tag{17}$$

and $W_\tau = (W_y, {W_{x\tau}}')'$.

First, we consider the least squares estimator of the parameters of the cointegrating regression. It is shown in Gregory and Hansen (1992) using the FCLT and the continuous mapping theorem (CMT, see Billingsley (1999), Theorem 2.7) that

$$T^{-1}\delta_T \sum_{t=1}^{T} X_{t\tau} X_{t\tau}{}' \delta_T \Rightarrow \int_0^1 X_\tau X_\tau{}', \tag{18}$$

where the weak convergence is with respect to the uniform metric over $\tau \in \mathcal{T}$. In the remainder of the proof, we refer to weak convergence results involving the break fraction parameter τ as holding uniformly over τ (see also Arai and Kurozumi (2007) for a similar application).

We define the vector $\hat{\theta}_\tau = (\hat{\alpha}_1', \hat{\mu}_1, \hat{\alpha}_2', \hat{\mu}_2)$ as the least squares estimator of (15) for each τ. It follows from (18) and the CMT that

$$\begin{aligned} T^{-1/2}\delta_{2T}^{-1}\hat{\theta}_\tau &= \left(T^{-1}\delta_{2T} \sum_{t=1}^{T} X_{2t\tau} X_{2t\tau}{}' \delta_{2T}\right)^{-1} \left(T^{-1}\delta_{2T} \sum_{t=1}^{T} X_{2t\tau} X_{1t\tau} \delta_{1T}\right) \\ &\Rightarrow \left(\int_0^1 X_{2\tau} X_{2\tau}{}'\right)^{-1} \left(\int_0^1 X_{2\tau} X_{1\tau}\right). \end{aligned} \tag{19}$$

When we set $\hat{\eta}_\tau = T^{-1/2}\delta_T^{-1}(1, -\hat{\theta}_\tau')' = (1, -\delta_{2T}^{-1}\hat{\theta}_\tau')'$, it follows that

$$\hat{\eta}_\tau \Rightarrow \left(1, -\left(\int_0^1 X_{1\tau} X_{2\tau}{}'\right)\left(\int_0^1 X_{2\tau} X_{2\tau}{}'\right)^{-1}\right)' = \eta_\tau. \tag{20}$$

Next, we state some useful convergence results for the residuals of the cointegrating regression. We define the residual series $\hat{e}_{t\tau} = y_t - \hat{\alpha}_1' x_t - \hat{\mu}_1 - \hat{\alpha}_2' x_t \varphi_{t,\tau} - \hat{\mu}_2 \varphi_{t,\tau}$ which is dependent on τ. Note that $\hat{e}_{t\tau}$ can be expressed as

$$\hat{e}_{t\tau} = T^{1/2}\hat{\eta}_\tau' \delta_T X_{t\tau}. \tag{21}$$

Using Lemma 2.2 of Phillips and Ouliaris (1990) yields

$$T^{-1/2}\hat{e}_{t\tau} \Rightarrow \eta_\tau' X_\tau = l_{11}\kappa_\tau' W_\tau = l_{11} Q_{\kappa\tau}, \tag{22}$$

where

$$\begin{aligned}
\kappa_\tau &= \left(1, -\left(\int_0^1 W_y W'_{x\tau}\right)\left(\int_0^1 W_{x\tau} W'_{x\tau}\right)^{-1}\right)' \\
L\eta_\tau &= l_{11}\kappa_\tau \\
Q_{\kappa\tau} &= W_y - \left(\int_0^1 W_y W'_{x\tau}\right)\left(\int_0^1 W_{x\tau} W'_{x\tau}\right)^{-1} W_{x\tau}.
\end{aligned} \tag{23}$$

The first-differenced residuals are expressed as $\Delta\hat{e}_{t\tau} = T^{1/2}\hat{\eta}'_\tau \delta_T \Delta X_{t\tau}$, where

$$\begin{aligned}
\Delta X_{t\tau} &= \Delta(y_t, {x_t}', 1, x_t{\varphi_{t,\tau}}', \varphi_{t,\tau})' \\
&= (\xi_{1t}, {\xi_{2t}}', 0, x_{t-1}{\Delta\varphi_{t,\tau}}' + \Delta x_t{\varphi_{t,\tau}}', \Delta\varphi_{t,\tau})' \\
&= (\xi_{1t}, {\xi_{2t}}', 0, x_{t-1}\Delta\varphi_{t,\tau} + \xi_{2t}{\varphi_{t,\tau}}', \Delta\varphi_{t,\tau})'
\end{aligned} \tag{24}$$

and

$$\Delta\varphi_{t,\tau} = \begin{cases} 1 & \text{if } t = [T\tau] \\ 0 & \text{if } t \neq [T\tau] \end{cases}. \tag{25}$$

The asymptotic counterpart to $\Delta\varphi_{t,\tau}$ is the differential $d\varphi_\tau(s)$, a Dirac function concentrating the unit mass at the point $s = \tau$ so that

$$\int_a^b f d\varphi_\tau = \lim_{z\uparrow\tau} f(z), \qquad a < \tau < b,$$

for all functions with left-limits. Then, we can define the differential dX_τ by

$$dX_\tau(s) = (dB(s)', 0, B_x(s)'d\varphi_\tau(s) + dB_x(s)'\varphi_\tau(s), d\varphi_\tau(s))'. \tag{26}$$

Under Assumption (1), ξ_t is a stationary linear vector process and consequently, the scalar process $T^{1/2}\hat{\eta}'_\tau\delta_T\Delta X_{t\tau} \Rightarrow T^{1/2}\eta'_\tau\delta_T\Delta X_{t\tau}$ is also a stationary linear process with an intervention outlier at $t = [T\tau]$. Moreover, under Assumption (2) the lag truncation parameter $K \to \infty$ for $T \to \infty$. This means that the error of approximating $\epsilon_{t\tau}$ by a finite AR process becomes small as K grows large. Following Phillips and Ouliaris (1990) we write the infinite order AR representation of the SETAR error term process as $\epsilon_{t\tau} = \sum_{j=0}^{\infty} D_j(T^{1/2}\delta_T\Delta X_{t-j\tau})'\eta_\tau = D(L)(T^{1/2}\delta_T\Delta X_{t\tau})'\eta_\tau$. The lag structure is chosen in a way that $\epsilon_{t\tau}$ is an orthogonal $(0, \sigma^2(\eta,\tau))$ sequence with long-run variance $\sigma^2(\eta,\tau) =$

$D(1)^2\eta_\tau'\Omega_\tau\eta_\tau$. From Lemma 2.1 of Phillips and Ouliaris (1990), it follows that

$$T^{-1/2}\sum_{t=1}^{[Ts]}\epsilon_{t\tau K} = D(L)\eta_\tau'\left(T^{-1/2}\sum_{t=1}^{[Ts]}T^{1/2}\delta_T\Delta X_{t\tau}\right) + o_p(1) \Rightarrow D(1)\eta_\tau' X_\tau(s), \tag{27}$$

where $D(1) = \sum_{j=0}^{\infty} D_j$.

Now, we consider the auxiliary regression. We apply the SETAR model to the residuals according to (4) and compute the test statistics F_τ. Note that the estimated adjustment coefficients might be correlated with the estimated coefficients of the additional lagged differences. Therefore, we write the least squares estimator of $\rho = (\rho_1, \rho_2)'$ in the breakpoint specific notation under the null hypothesis $\rho_1 = \rho_2 = 0$ as $\hat{\rho} = (U_\tau' Q_K U_\tau)^{-1} U_\tau' Q_K \epsilon_\tau$, where

$$U_\tau = \begin{bmatrix} \hat{e}_{0\tau}\mathbb{1}\{\hat{e}_{0\tau} \geq \lambda\} & \hat{e}_{0\tau}\mathbb{1}\{\hat{e}_{0\tau} < \lambda\} \\ \hat{e}_{1\tau}\mathbb{1}\{\hat{e}_{1\tau} \geq \lambda\} & \hat{e}_{1\tau}\mathbb{1}\{\hat{e}_{1\tau} < \lambda\} \\ \vdots & \vdots \\ \hat{e}_{T-1\tau}\mathbb{1}\{\hat{e}_{T-1\tau} \geq \lambda\} & \hat{e}_{T-1\tau}\mathbb{1}\{\hat{e}_{T-1\tau} < \lambda\} \end{bmatrix}, \tag{28}$$

$\epsilon_\tau = (\epsilon_{1\tau}, \epsilon_{2\tau}, \ldots, \epsilon_{T\tau})'$ and $Q_K = I - M_K(M_K' M_K)^{-1} M_K'$ is the projection matrix onto the space orthogonal to the regressors $M_K = (\Delta\hat{e}_{t-1\tau}, \ldots, \Delta\hat{e}_{t-K\tau})$.

We partition the matrix U_τ as $U_\tau = (U_{1\tau}, U_{2\tau})$, then the t ratio of $\hat{\rho}_1$ can be expressed as

$$t_1 = \frac{\hat{\rho}_1}{se(\hat{\rho}_1)} = \frac{\hat{\rho}_1}{(\hat{\sigma}^2(U_{1\tau}'Q_K U_{1\tau})^{-1})^{1/2}} = \frac{U_{1\tau}'Q_K\epsilon_\tau}{\hat{\sigma}(U_{1\tau}'Q_K U_{1\tau})^{1/2}} \tag{29}$$

and similarly the t ratio of $\hat{\rho}_2$ can be expressed as

$$t_2 = \frac{U_{2\tau}'Q_K\epsilon_\tau}{\hat{\sigma}(U_{2\tau}'Q_K U_{2\tau})^{1/2}}. \tag{30}$$

In the remainder of the proof, we focus on t_1. Scaling the t ratio appropriately yields the numerator

$$\begin{aligned} T^{-1}U_{1\tau}'Q_K\epsilon_\tau &= T^{-1}U_{1\tau}'\epsilon_\tau - T^{-1/2}\cdot T^{-1}U_{1\tau}'M_K(T^{-1}M_K'M_K)^{-1}T^{-1/2}M_K'\epsilon_\tau \\ &= T^{-1}U_{1\tau}'\epsilon_\tau + o_p(1) = N_T(\lambda,\tau) + o_p(1) \end{aligned} \tag{31}$$

and the term

$$
\begin{aligned}
T^{-2}U_{1\tau}'Q_K U_{1\tau} &= T^{-2}U_{1\tau}'U_{1\tau} - T^{-1}\cdot T^{-1}U_{1\tau}'M_K(T^{-1}M_K'M_K)^{-1}T^{-1}M_K'U_{1\tau} \\
&= T^{-2}U_{1\tau}'U_{1\tau} + o_p(1) = D_T(\lambda,\tau) + o_p(1). \qquad (32)
\end{aligned}
$$

Finally, we need convergence results for $N_T(\lambda,\tau)$, $D_T(\lambda,\tau)$ and $\hat{\sigma}^2$. Since $x \mapsto x\mathbb{1}\{x \geq \lambda\}$ is a regular function, it follows from (22) and Theorem 3.1 of Park and Phillips (2001) that

$$
\begin{aligned}
T^{-1/2}\hat{e}_{t-1\tau}\mathbb{1}\{\hat{e}_{t-1\tau} \geq \lambda\} &= \hat{\eta}_\tau'\delta_T X_{t-1\tau}\mathbb{1}\{T^{1/2}\hat{\eta}_\tau'\delta_T X_{t-1\tau} \geq \lambda\} \\
&= \hat{\eta}_\tau'\delta_T X_{t-1\tau}\mathbb{1}\{\hat{\eta}_\tau'\delta_T X_{t-1\tau} \geq T^{-1/2}\lambda\} \qquad (33) \\
&\Rightarrow \eta_\tau' X_\tau \mathbb{1}\{\eta_\tau' X_\tau \geq 0\} = l_{11}Q_{\kappa\tau}\mathbb{1}\{Q_{\kappa\tau} \geq 0\}.
\end{aligned}
$$

Thus, Theorem 2.2 of Kurtz and Protter (1991) combined with results (27) and (33) yields

$$
\begin{aligned}
N_T(\lambda,\tau) &= T^{-1}\sum_{t=1}^{T}\mathbb{1}\{\hat{e}_{t-1\tau} \geq \lambda\}\hat{e}_{t-1\tau}\epsilon_{t\tau} \\
&= \hat{\eta}_\tau'\delta_T\sum_{t=1}^{T}\mathbb{1}\{\delta_T\hat{\eta}_\tau' X_{t-1\tau} \geq T^{-1/2}\lambda\}X_{t-1\tau}D(L)(\Delta X_{t\tau})'\delta_T\eta_\tau \\
&\Rightarrow D(1)\eta_\tau'\int_0^1 \mathbb{1}\{\eta_\tau' X_\tau \geq 0\}X_\tau dX_\tau'\eta_\tau \qquad (34) \\
&= D(1)l_{11}^2\int_0^1 \mathbb{1}\{Q_{\kappa\tau} \geq 0\}Q_{\kappa\tau}dQ_{\kappa\tau},
\end{aligned}
$$

while (27), (33) and the CMT yield

$$
\begin{aligned}
D_T(\lambda,\tau) &= T^{-2}\sum_{t=1}^{T}\mathbb{1}\{\hat{e}_{t-1\tau} \geq \lambda\}\hat{e}_{t-1\tau}^2 \\
&= \hat{\eta}_\tau'\delta_T T^{-1}\sum_{t=1}^{T}\mathbb{1}\{\delta_T\hat{\eta}_\tau' X_{t-1\tau} \geq T^{-1/2}\lambda\}X_{t-1\tau}X_{t-1\tau}{}'\delta_T\hat{\eta}_\tau \\
&\Rightarrow \eta_\tau'\int_0^1 \mathbb{1}\{\eta_\tau' X_\tau \geq 0\}X_\tau X_\tau'\eta_\tau \qquad (35) \\
&= l_{11}^2\int_0^1 \mathbb{1}\{Q_{\kappa\tau} \geq 0\}Q_{\kappa\tau}^2.
\end{aligned}
$$

For the variance estimate, $\hat{\sigma}^2$, we note that $\hat{\rho}_1 = O_p(T^{-1})$ and $\hat{\rho}_2 = O_p(T^{-1})$, but $(\hat{\gamma}_j - \gamma_j) = O_p(T^{-1/2})$. Using Lemma 2.2 of Phillips and Ouliaris (1990) yields

$$\begin{aligned}
\hat{\sigma}^2 &= T^{-1}\sum_{t=1}^{T}\left(\Delta\hat{e}_{t\tau} - \hat{\rho}_1\hat{e}_{t-1\tau}\mathbb{1}\{\hat{e}_{t-1\tau} \geq \lambda\} - \hat{\rho}_2\hat{e}_{t-1\tau}\mathbb{1}\{\hat{e}_{t-1\tau} < \lambda\} - \sum_{j=1}^{K}\hat{\gamma}_j\Delta\hat{e}_{t-j\tau}\right)^2 \\
&= T^{-1}\sum_{t=1}^{T}\epsilon_{t\tau}^2 + o_p(1) \Rightarrow D(1)^2\eta_\tau'\Omega_\tau\eta_\tau = D(1)^2 l_{11}^2\kappa_\tau' D_\tau\kappa_\tau, \qquad (36)
\end{aligned}$$

where the long-run covariance matrix is given by

$$\Omega_\tau = \begin{bmatrix} \omega_{11} & \omega_{21}' & 0 & (1-\tau)\omega_{21}' & 0 \\ \omega_{21} & \Omega_{22} & 0 & (1-\tau)\Omega_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ (1-\tau)\omega_{21} & (1-\tau)\Omega_{22} & 0 & (1-\tau)\Omega_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (37)$$

and

$$D_\tau = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & I_m & 0 & (1-\tau)I_m & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\tau)I_m & 0 & (1-\tau)I_m & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (38)$$

Similar results can be obtained for t_2 so that the results (34), (35), (36) combine with the CMT to proof the theorem under the null hypothesis.
Under the alternative, the system is cointegrated so that we have $\hat{\eta}_\tau \xrightarrow{p} \eta_\tau$ and

$$\hat{\eta}_\tau = \eta_\tau + O_p(T^{-1}) \qquad (39)$$

from Phillips and Durlauf (1986), Theorem 4.1. Thus, for the residual series it holds that

$$\hat{e}_{t\tau} = \hat{\eta}_\tau' z_t = \eta_\tau' z_t + O_p(T^{-1/2}) = q_{t\eta\tau} + O_p(T^{-1/2}). \qquad (40)$$

By assumption a stationary SETAR representation of $q_{t\eta\tau}$ exists and is given by

$$q_{t\eta\tau} = a_{11}q_{t-1\eta\tau}\mathbb{1}\{q_{t-1\eta\tau} \geq \lambda\} + a_{12}q_{t-1\eta\tau}\mathbb{1}\{q_{t-1\eta\tau} < \lambda\} + \sum_{j=2}^{\infty}a_j q_{t-j\eta\tau} + \epsilon_{t\eta\tau}^*, \qquad (41)$$

where $\epsilon^*_{t\eta\tau}$ is an orthogonal $(0, \sigma_{\epsilon^*_{\eta\tau}})$ sequence. This can alternatively be written as

$$\Delta q_{t\eta\tau} = \psi_{11} q_{t-1\eta\tau} \mathbb{1}\{q_{t-1\eta\tau} \geq \lambda\} + \psi_{12} q_{t-1\eta\tau} \mathbb{1}\{q_{t-1\eta\tau} < \lambda\} + \sum_{j=2}^{\infty} \psi_j \Delta q_{t-j\eta\tau} + \epsilon^*_{t\eta\tau}. \quad (42)$$

If we consider the t ratio of $\hat{\rho}_1$ and use the expression

$$t_1 = \frac{1}{\hat{\sigma}} \left(\hat{\rho}_1 \left(U'_{1\tau} Q_K U_{1\tau} \right)^{1/2} \right), \quad (43)$$

we find that $\hat{\rho}_1 \xrightarrow{p} \psi_{11} \neq 0$ and $\hat{\sigma}^2 \xrightarrow{p} \sigma^2_{\epsilon^*_{\eta\tau}}$. Further, we observe that

$$U'_{1\tau} Q_K U_{1\tau} = U'_{1\tau} U_{1\tau} - U'_{1\tau} M_K (M'_K M_K)^{-1} M'_K U_{1\tau} = O_p(T) \quad (44)$$

which yields $t_1 = O_p(T^{1/2})$ and similarly $t_2 = O_p(T^{1/2})$. Hence, we immediately see that $F^*_{SETAR} \to \infty$ as $T \to \infty$. □

Proof of Theorem 2. The proof is structured similarly to the proof of Theorem 1. Using the results for the cointegrating regression, we write the AR representation of the MTAR error term process as $\epsilon_{t\tau} = \sum_{j=0}^{\infty} a_j (T^{-1/2} \delta_T \Delta X_{t-j\tau})' \eta_\tau = a(L)(T^{-1/2} \delta_T \Delta X_{t-j\tau})' \eta_\tau$ and have $\epsilon_{t\tau}$ as an orthogonal $(0, \sigma^2(\eta, \tau))$ sequence with $\sigma^2(\eta, \tau) = a(1)^2 \eta'_\tau \Omega_\tau \eta_\tau$. From Lemma 2.1 of Phillips and Ouliaris (1990), it follows that

$$T^{-1/2} \sum_{t=1}^{[Ts]} \epsilon_{t\tau K} = a(L) \eta'_\tau \left(T^{-1/2} \sum_{t=1}^{[Ts]} T^{1/2} \delta_T \Delta X_{t\tau} \right) + o_p(1) \Rightarrow a(1) \eta'_\tau X_\tau, \quad (45)$$

where $a(1) = \sum_{j=0}^{\infty} a_j$.

Now, we apply the MTAR model to the residuals according to (5) and compute the test statistics F_τ. The t ratio of $\hat{\rho}_1$ is written as

$$t_1 = \frac{U'_{1\tau} Q_K \epsilon_\tau}{\hat{\sigma} (U'_{1\tau} Q_K U_{1\tau})^{1/2}} \quad (46)$$

and the t ratio of $\hat{\rho}_2$ is written as

$$t_2 = \frac{U'_{2\tau} Q_K \epsilon_\tau}{\hat{\sigma} (U'_{2\tau} Q_K U_{2\tau})^{1/2}}, \quad (47)$$

where

$$U_\tau = (U_{1\tau}, U_{2\tau}) = \begin{bmatrix} \hat{e}_{0\tau}\mathbb{1}\{\Delta\hat{e}_{0\tau} \geq \lambda^*\} & \hat{e}_{0\tau}\mathbb{1}\{\Delta\hat{e}_{0\tau} < \lambda^*\} \\ \hat{e}_{1\tau}\mathbb{1}\{\Delta\hat{e}_{1\tau} \geq \lambda^*\} & \hat{e}_{1\tau}\mathbb{1}\{\Delta\hat{e}_{1\tau} < \lambda^*\} \\ \vdots & \vdots \\ \hat{e}_{T-1\tau}\mathbb{1}\{\Delta\hat{e}_{T-1\tau} \geq \lambda^*\} & \hat{e}_{T-1\tau}\mathbb{1}\{\Delta\hat{e}_{T-1\tau} < \lambda^*\} \end{bmatrix}. \tag{48}$$

Finally, we need convergence results for $N_T(\lambda^*, \tau)$, $D_T(\lambda^*, \tau)$ and $\hat{\sigma}^2$. The main difference between the asymptotic distribution for the SETAR and the MTAR models lies in the fact that the indicator variable $\Delta\hat{e}_{t\tau}$ has a stationary distribution under the null hypothesis and the alternative. Further, the MTAR decomposition of $\hat{e}_{t-1\tau}$ is not regular and Theorem 3.1 of Park and Phillips (2001) cannot be used. However, from Theorem 1 in Caner and Hansen (2001) it follows that

$$\begin{aligned} T^{-1/2}\sum_{t=1}^{[Ts]} \mathbb{1}\{\Delta\hat{e}_{t-1\tau} \geq \lambda^*\}\epsilon_{t\tau} &= T^{-1/2}\sum_{t=1}^{[Ts]} \mathbb{1}\{G\left(\Delta\hat{e}_{t-1\tau}\right) \geq G\left(\lambda^*\right)\}\epsilon_{t\tau} \\ &= T^{-1/2}\sum_{t=1}^{[Ts]} \mathbb{1}\{U_t \geq G\left(\lambda^*\right)\}\epsilon_{t\tau} \\ &\Rightarrow Q_{\kappa\tau}(s,u) = \sigma(\eta,\tau)W(s,u) \qquad (49) \\ &= a(1)l_{11}(\kappa_\tau' D_\tau \kappa_\tau)^{1/2} W(s,u), \end{aligned}$$

where $G(\cdot)$ is the marginal distribution of $\Delta\hat{e}_{t-1\tau}$ so that $G(\Delta\hat{e}_{t-1\tau}) = U_t \sim U[0,1]$ and $G(\lambda^*) = u$. The standard two-parameter Brownian motion $W(s,u)$ is defined on $(s,u) \in [0,1]^2$. Using Theorem 2.2 of Kurtz and Protter (1991) and (49) yields

$$\begin{aligned} N_T(\lambda^*, \tau) &= T^{-1}\sum_{t=1}^{T} \mathbb{1}\{\Delta\hat{e}_{t-1\tau} \geq \lambda^*\}\hat{e}_{t-1\tau}\epsilon_{t\tau} \\ &= \hat{\eta}_\tau' \delta_T \sum_{t=1}^{T} \mathbb{1}\{G\left(\Delta\hat{e}_{t-1\tau}\right) \geq G\left(\lambda^*\right)\} X_{t-1\tau}\epsilon_{t\tau} \\ &\Rightarrow a(1)l_{11}(\kappa_\tau' D_\tau \kappa_\tau)^{1/2}\eta_\tau' \int_0^1 X_\tau(s)dW(s,u) \qquad (50) \\ &= a(1)l_{11}^2(\kappa_\tau' D_\tau \kappa_\tau)^{1/2} \int_0^1 Q_{\kappa\tau}(s)dW(s,u) \end{aligned}$$

and Theorem 3 of Caner and Hansen (2001) yields

$$\begin{aligned} D_T(\lambda^*, \tau) &= T^{-2} \sum_{t=1}^{T} \mathbb{1}\{\Delta \hat{e}_{t-1\tau} \geq \lambda^*\} \hat{e}_{t-1\tau}^2 \\ &= \hat{\eta}_\tau' \delta_T T^{-1} \sum_{t=1}^{T} \mathbb{1}\{G(\Delta \hat{e}_{t-1\tau}) \geq G(\lambda^*)\} X_{t-1\tau} X_{t-1\tau}{}' \delta_T \hat{\eta}_\tau \\ &\Rightarrow u \eta_\tau' \int_0^1 X_\tau(s) X_\tau'(s) ds \eta_\tau \qquad (51) \\ &= u l_{11}^2 \int_0^1 Q_{\kappa\tau}^2(s) ds. \end{aligned}$$

For the variance estimate, $\hat{\sigma}^2$, Lemma 2.2 of Phillips and Ouliaris (1990) yields

$$\begin{aligned} \hat{\sigma}^2 &= T^{-1} \sum_{t=1}^{T} \epsilon_{t\tau}^2 + o_p(1) \\ &\Rightarrow a(1)^2 l_{11}^2 \kappa_\tau' D_\tau \kappa_\tau. \qquad (52) \end{aligned}$$

The results (50), (51), (52) combine with the CMT to proof

$$t_1 \Rightarrow \frac{\int_0^1 Q_{\kappa\tau}(s) dW(s,u)}{\left(u \int_0^1 Q_{\kappa\tau}^2(s) ds\right)^{1/2}}. \qquad (53)$$

Analogously, we can show that

$$t_2 \Rightarrow \frac{\int_0^1 Q_{\kappa\tau}(s) \left(dW(s,1) - dW(s,u)\right)}{\left((1-u) \int_0^1 Q_{\kappa\tau}^2(s) ds\right)^{1/2}} \qquad (54)$$

holds. Finally, we observe that taking the supremum over all $\tau \in \mathcal{T}$ is a continuous transformation so that we can use the CMT to proof the theorem under the null hypothesis. The proof of the theorem under the alternative is a straightforward adaptation of the results given in the proof of Theorem 1. □

Figure 1: WTI crude oil prices, spot gasoline prices and retail gasoline prices from January 2006 to December 2013

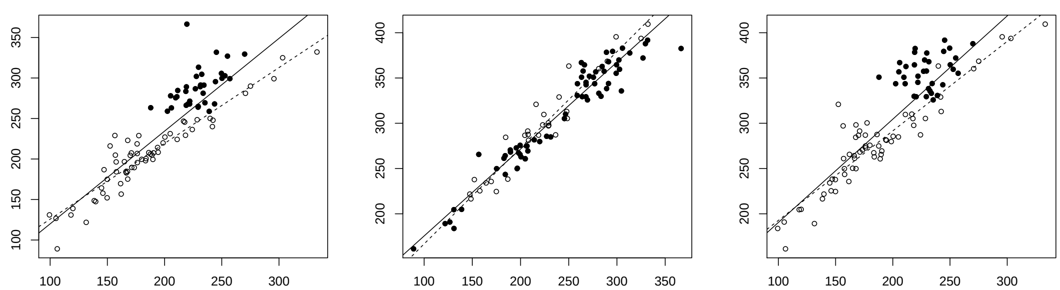

Figure 2: Ex post analysis of structural breaks in the long-run equilibrium equation. The left, middle and right scatterplots display the *first stage*, *second stage* and *single stage* results, respectively. The solid line marks the regression line for the full sample and the dashed line marks the regression line for all observations before the estimated breakpoint. All observations after the breakpoint are marked with black dots.

Table 1: Approximate critical values of F^*_{SETAR}

		C			C/T			C/S		
	T	90%	95%	99%	90%	95%	99%	90%	95%	99%
$m = 1$										
	50	16.01	18.48	24.22	18.80	21.38	27.10	17.52	20.16	25.83
	100	12.73	14.66	19.24	15.58	17.75	22.49	14.44	16.68	21.40
	250	10.80	12.29	15.70	12.99	14.59	18.16	12.52	14.36	17.82
	500	10.13	11.42	14.30	12.11	13.46	16.37	11.76	13.24	16.39
	∞	9.48	10.70	13.45	11.53	12.86	15.74	11.20	12.71	15.69
$m = 2$										
	50	17.63	20.14	26.26	19.84	22.42	28.47	20.49	23.47	29.57
	100	16.19	18.21	23.21	18.69	20.90	25.94	19.24	21.56	26.54
	250	13.33	15.02	18.93	15.50	17.39	21.69	16.47	18.39	23.03
	500	12.22	13.68	17.08	14.06	15.63	19.07	15.22	16.85	20.18
	∞	12.18	13.60	16.88	14.22	15.82	19.33	15.30	16.86	20.45
$m = 3$										
	50	19.80	22.49	28.40	21.71	24.56	30.57	23.94	27.05	34.08
	100	18.20	20.51	25.37	20.40	22.81	28.00	22.87	25.43	30.89
	250	15.37	17.16	21.21	17.31	19.24	23.42	19.81	22.00	26.48
	500	14.15	15.71	19.11	15.88	17.57	21.14	18.44	20.30	24.11
	∞	14.12	15.65	19.03	16.00	17.66	21.23	18.60	20.44	24.09
$m = 4$										
	50	21.19	23.92	29.90	23.22	26.11	32.96	27.33	30.40	37.89
	100	20.13	22.56	27.61	22.42	24.80	29.47	25.98	28.49	34.16
	250	17.36	19.27	23.87	19.21	21.23	26.12	23.26	25.81	30.78
	500	15.77	17.41	20.70	17.41	19.13	22.73	21.46	23.44	27.80
	∞	16.04	17.69	21.28	17.81	19.51	23.12	21.75	23.83	27.95

Note: C, C/T and C/S denote the $\sup F$ tests using the structural break models in (3). m refers to the number of columns of the regressor matrix x_t. The lag truncation parameter is determined using the BIC and maximum lag length $K_{\max} = 8$. Critical values for different order selection rules are not reported but can be obtained from the author upon request.

Table 2: Approximate critical values of F^*_{MTAR}

	u	C 90%	C 95%	C 99%	C/T 90%	C/T 95%	C/T 99%	C/S 90%	C/S 95%	C/S 99%
						$m=1$				
$T=50$										
	0.15	17.49	20.18	26.82	18.72	21.25	26.65	18.12	20.62	26.60
	0.25	16.89	19.41	25.74	18.56	20.86	26.52	17.98	20.43	26.32
	0.50	16.56	19.03	24.57	18.51	20.94	26.52	17.88	20.34	26.00
$T=100$										
	0.15	17.95	21.03	28.71	18.16	20.77	26.43	18.04	21.17	28.37
	0.25	15.56	18.26	24.05	17.03	19.32	24.00	16.55	18.97	24.92
	0.50	14.58	16.86	21.30	16.47	18.92	23.86	15.92	18.28	23.42
$T=250$										
	0.15	19.21	23.19	32.09	18.82	21.85	28.79	19.45	23.15	31.94
	0.25	15.08	17.60	24.11	16.11	18.20	23.65	16.13	18.65	25.27
	0.50	12.85	14.72	18.87	14.75	16.50	20.70	14.33	16.21	20.98
$T=500$										
	0.15	20.83	24.76	35.04	20.90	24.37	32.96	21.56	25.51	35.91
	0.25	15.35	17.67	24.79	16.60	18.98	24.46	16.55	18.95	25.93
	0.50	12.49	14.04	17.37	14.34	15.89	19.51	13.96	15.66	19.37
$T=\infty$										
	0.15	21.59	25.94	36.86	21.52	25.25	34.64	22.52	26.71	37.94
	0.25	15.30	17.48	25.12	16.37	18.67	24.70	16.56	19.01	26.38
	0.50	11.81	13.13	16.39	13.65	14.82	18.08	13.35	14.79	18.27
						$m=2$				
$T=50$										
	0.15	18.18	20.74	26.68	19.72	22.13	28.33	20.23	22.86	29.18
	0.25	17.95	20.36	26.17	19.64	21.99	27.80	20.37	23.05	29.61
	0.50	17.77	20.24	26.40	19.91	22.40	28.33	20.44	23.38	29.22
$T=100$										
	0.15	18.74	21.57	27.97	19.57	22.19	28.05	20.01	22.43	28.48
	0.25	17.21	19.08	25.53	18.84	21.23	26.08	19.62	21.93	27.02
	0.50	16.84	19.15	24.07	18.84	21.09	25.82	19.62	21.13	27.15
$T=250$										
	0.15	19.85	23.35	32.90	19.81	22.51	28.71	20.89	23.90	31.19
	0.25	16.43	18.77	24.80	17.53	19.73	24.53	18.64	21.00	26.47
	0.50	14.70	16.64	20.91	16.48	18.35	22.44	17.46	19.56	24.40
$T=500$										
	0.15	21.24	25.01	35.89	21.49	24.69	31.92	22.52	26.01	35.26
	0.25	16.54	18.96	26.23	17.70	19.77	24.76	18.62	21.19	27.14
	0.50	14.26	15.90	19.66	15.86	17.51	21.06	16.97	18.75	23.10
$T=\infty$										
	0.15	21.88	26.00	38.63	21.79	25.05	32.61	23.17	27.05	37.19
	0.25	16.23	18.99	26.06	17.12	19.07	24.19	18.20	20.87	27.27
	0.50	13.20	14.64	18.18	14.69	16.14	19.47	15.88	17.47	21.79

Note: C, C/T and C/S denote the structural break models in (3). m refers to the number of columns of the regressor matrix x_t. The lag truncation parameter is determined using the BIC and maximum lag length $K_{\max} = 8$. Critical values for different order selection rules are not reported but can be obtained from the author upon request. Critical values for $u = \{0.75, 0.85\}$ are not reported to conserve space. Since the distribution is symmetric in u, the values can easily be inferred from the table.

Table 3: Approximate critical values of F^*_{MTAR}, continued

	u	C 90%	C 95%	C 99%	C/T 90%	C/T 95%	C/T 99%	C/S 90%	C/S 95%	C/S 99%
						$m=3$				
$T=50$										
	0.15	19.72	22.32	27.81	21.40	24.06	30.13	23.59	26.77	33.19
	0.25	19.73	22.47	27.92	21.55	24.10	29.93	23.72	26.87	33.77
	0.50	19.86	22.37	28.41	21.60	24.49	30.78	24.13	27.36	34.55
$T=100$										
	0.15	19.74	22.37	28.26	21.05	23.53	28.71	22.89	25.54	30.77
	0.25	18.90	21.17	26.30	20.50	22.95	27.94	22.77	25.36	30.68
	0.50	18.66	21.03	25.79	20.51	22.89	28.32	22.90	25.71	31.06
$T=250$										
	0.15	20.12	23.50	31.76	20.66	23.36	28.98	22.49	25.25	31.63
	0.25	17.66	20.05	26.40	18.94	21.00	25.55	21.07	23.50	29.01
	0.50	16.52	18.40	22.78	18.17	20.05	24.18	20.52	22.73	27.76
$T=500$										
	0.15	21.79	25.58	34.78	22.22	25.74	33.58	23.45	26.54	34.99
	0.25	17.92	20.37	26.21	19.01	21.17	27.07	20.88	23.27	28.54
	0.50	15.94	17.76	21.83	17.45	19.21	23.49	19.77	21.75	26.03
$T=\infty$										
	0.15	22.14	26.37	36.87	22.20	25.94	34.35	23.42	26.67	36.10
	0.25	17.39	19.98	26.48	18.33	20.30	26.20	20.08	22.46	28.03
	0.50	14.91	16.48	20.49	16.28	17.83	21.55	18.60	20.29	24.59
						$m=4$				
$T=50$										
	0.15	21.12	23.88	29.68	22.90	25.88	31.96	26.96	30.13	36.84
	0.25	21.08	23.84	29.93	23.04	25.89	31.99	27.21	30.20	37.04
	0.50	21.33	24.07	30.30	23.29	26.13	32.80	27.65	30.68	37.73
$T=100$										
	0.15	21.05	23.51	29.54	22.43	24.71	30.01	25.57	28.29	33.88
	0.25	20.52	22.93	27.47	22.31	24.65	29.63	25.77	28.31	34.02
	0.50	20.49	22.93	27.85	22.43	24.91	30.01	26.15	28.71	34.65
$T=250$										
	0.15	21.03	23.88	31.30	21.75	24.39	29.32	24.56	27.12	32.56
	0.25	19.15	21.26	26.57	20.50	22.62	27.36	23.84	26.21	31.76
	0.50	18.24	20.24	24.79	19.83	21.87	26.58	23.61	26.01	31.68
$T=500$										
	0.15	22.38	25.85	35.15	22.59	25.57	33.44	25.12	28.03	35.57
	0.25	18.89	21.28	27.64	19.98	22.11	27.03	23.28	25.52	30.62
	0.50	17.43	19.27	23.32	18.82	20.71	24.71	22.37	24.53	29.27
$T=\infty$										
	0.15	22.50	26.28	36.47	22.37	25.67	33.72	24.82	27.74	35.55
	0.25	18.21	20.51	27.60	19.06	21.15	26.11	22.39	24.59	29.73
	0.50	16.26	17.90	21.89	17.45	19.14	23.09	21.10	23.22	27.89

Table 4: Size-adjusted power of the sup F test under structural change and symmetric adjustment

	$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 2$						$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 4$					
	T = 50			T = 100			T = 50			T = 100		
τ	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
SETAR												
C	0.372	0.377	0.380	0.970	0.970	0.972	0.211	0.217	0.421	0.686	0.627	0.880
C/T	0.224	0.234	0.223	0.875	0.878	0.874	0.107	0.113	0.157	0.485	0.529	0.744
C/S	0.278	0.277	0.290	0.922	0.921	0.934	0.343	0.293	0.334	0.980	0.972	0.966
MTAR												
C	0.297	0.299	0.319	0.893	0.879	0.892	0.167	0.178	0.355	0.516	0.447	0.759
C/T	0.231	0.221	0.218	0.790	0.800	0.797	0.122	0.127	0.172	0.398	0.443	0.653
C/S	0.247	0.238	0.259	0.851	0.819	0.842	0.279	0.265	0.281	0.911	0.892	0.879
EG (c)	0.139	0.096	0.096	0.391	0.274	0.277	0.089	0.060	0.086	0.126	0.100	0.145
EG (c + t)	0.124	0.125	0.116	0.397	0.481	0.434	0.076	0.058	0.096	0.109	0.122	0.187
GH (C)	0.364	0.369	0.374	0.970	0.970	0.973	0.176	0.170	0.375	0.606	0.545	0.870
GH (C/T)	0.240	0.248	0.239	0.879	0.879	0.878	0.101	0.108	0.150	0.411	0.470	0.709
GH (C/S)	0.271	0.271	0.283	0.922	0.921	0.934	0.296	0.257	0.293	0.968	0.963	0.962
Φ_{SETAR}	0.216	0.184	0.186	0.738	0.600	0.727	0.111	0.095	0.148	0.196	0.189	0.303
Φ_{MTAR}	0.194	0.193	0.183	0.699	0.565	0.619	0.092	0.082	0.132	0.182	0.169	0.245

Note: C, C/T and C/S denote the structural break models in (3). EG (c) and EG (c + t) refer to the Engle-Granger test with intercept and intercept plus trend, respectively. GH denotes the Gregory-Hansen test. Φ_{SETAR} and Φ_{MTAR} denote the Enders-Siklos cointegration test with SETAR and MTAR adjustment, respectively. The table is based on 2,500 replications of the DGP described in (13). The autoregressive coefficients are $\rho_1 = \rho_2 = -0.5$, i.e. the adjustment is constant and symmetric.

Table 5: Estimates of the breakpoint under symmetric adjustment

SETAR

	$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 2$						$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 4$					
	T = 50			T = 100			T = 50			T = 100		
τ	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
C	0.32(0.15)	0.53(0.11)	0.70(0.15)	0.28(0.10)	0.51(0.08)	0.74(0.11)	0.34(0.18)	0.55(0.13)	0.72(0.13)	0.28(0.12)	0.54(0.11)	0.75(0.10)
	0.28(0.04)	0.52(0.04)	0.74(0.04)	0.26(0.02)	0.51(0.02)	0.76(0.02)	0.28(0.05)	0.54(0.04)	0.76(0.04)	0.26(0.02)	0.52(0.02)	0.77(0.02)
C/T	0.38(0.19)	0.50(0.16)	0.66(0.22)	0.33(0.16)	0.51(0.11)	0.69(0.16)	0.39(0.19)	0.53(0.15)	0.65(0.20)	0.31(0.15)	0.53(0.11)	0.73(0.13)
	0.28(0.26)	0.50(0.08)	0.74(0.34)	0.27(0.03)	0.51(0.02)	0.75(0.03)	0.28(0.26)	0.52(0.10)	0.74(0.22)	0.27(0.02)	0.52(0.02)	0.75(0.02)
C/S	0.35(0.16)	0.53(0.12)	0.68(0.16)	0.30(0.11)	0.51(0.07)	0.72(0.12)	0.33(0.14)	0.54(0.09)	0.71(0.13)	0.27(0.07)	0.51(0.05)	0.75(0.07)
	0.28(0.18)	0.54(0.04)	0.76(0.12)	0.25(0.02)	0.51(0.02)	0.76(0.03)	0.26(0.14)	0.54(0.04)	0.78(0.08)	0.25(0.02)	0.51(0.02)	0.77(0.01)

MTAR

	$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 2$						$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 4$					
	T = 50			T = 100			T = 50			T = 100		
τ	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
C	0.35(0.18)	0.52(0.14)	0.68(0.18)	0.29(0.11)	0.51(0.08)	0.74(0.10)	0.38(0.22)	0.55(0.21)	0.67(0.21)	0.30(0.17)	0.54(0.17)	0.72(0.17)
	0.28(0.14)	0.52(0.04)	0.74(0.10)	0.26(0.02)	0.51(0.02)	0.75(0.02)	0.28(0.30)	0.54(0.20)	0.74(0.04)	0.27(0.02)	0.52(0.09)	0.77(0.02)
C/T	0.40(0.20)	0.50(0.16)	0.60(0.22)	0.34(0.17)	0.51(0.12)	0.68(0.17)	0.42(0.21)	0.53(0.18)	0.64(0.21)	0.35(0.18)	0.54(0.15)	0.72(0.15)
	0.28(0.34)	0.50(0.08)	0.72(0.38)	0.27(0.04)	0.51(0.02)	0.75(0.05)	0.32(0.32)	0.54(0.14)	0.74(0.26)	0.27(0.13)	0.52(0.07)	0.76(0.02)
C/S	0.38(0.18)	0.53(0.14)	0.67(0.17)	0.31(0.12)	0.51(0.08)	0.72(0.11)	0.36(0.17)	0.52(0.16)	0.66(0.22)	0.27(0.09)	0.50(0.10)	0.72(0.16)
	0.30(0.26)	0.52(0.06)	0.74(0.18)	0.26(0.04)	0.52(0.02)	0.76(0.03)	0.30(0.22)	0.54(0.06)	0.76(0.16)	0.25(0.02)	0.52(0.02)	0.77(0.02)

Note: C, C/T and C/S denote the structural break models in (3). The left panel and right panel report the estimates of the break fraction following a shift in the intercept and a shift in the slope, respectively. Upper rows contain the mean breakpoint estimate and the empirical standard deviation. Lower row contain the median breakpoint and the interquartile range. The autoregressive coefficients are $\rho_1 = \rho_2 = -0.5$, i.e. the adjustment is constant and symmetric.

Table 6: Size-adjusted power of the sup F test (SETAR) under structural change and asymmetric adjustment

	$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 2$												
		C			C/T			C/S			Φ_{SETAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.109	0.060	0.015	0.098	0.052	0.010	0.112	0.055	0.010	**0.123**	**0.070**	**0.019**
	−0.15	0.130	0.072	0.022	0.121	0.065	0.012	0.125	0.066	0.014	**0.160**	**0.086**	**0.023**
	−0.25	0.149	0.084	0.025	0.129	0.069	0.014	0.137	0.075	0.019	**0.185**	**0.113**	**0.031**
−0.05	−0.10	0.133	0.076	0.021	0.128	0.068	0.012	0.127	0.067	0.014	**0.174**	**0.093**	**0.024**
	−0.25	0.178	0.104	0.034	0.163	0.089	0.019	0.168	0.093	0.024	**0.276**	**0.170**	**0.052**
−0.10	−0.15	0.195	0.114	0.035	0.176	0.103	0.022	0.176	0.103	0.025	**0.338**	**0.205**	**0.060**
	−0.25	0.258	0.156	0.051	0.228	0.131	0.029	0.236	0.142	0.036	**0.477**	**0.313**	**0.114**
Size:		0.135	0.073	0.023	0.143	0.085	0.022	0.126	0.080	0.021	0.120	0.060	0.012
	$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 2$												
		C			C/T			C/S			Φ_{SETAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.104	0.052	0.013	0.108	0.051	0.009	0.106	0.054	0.013	**0.119**	**0.062**	**0.014**
	−0.15	0.125	0.062	0.014	0.120	0.061	0.012	0.117	0.061	0.014	**0.143**	**0.067**	**0.018**
	−0.25	0.140	0.072	0.016	0.132	0.068	0.017	0.128	0.068	0.016	**0.153**	**0.084**	**0.022**
−0.05	−0.10	0.129	0.062	0.013	0.122	0.065	0.012	0.118	0.063	0.014	**0.147**	**0.080**	**0.019**
	−0.25	0.174	0.094	0.023	0.159	0.086	0.022	0.155	0.082	0.020	**0.182**	**0.108**	**0.032**
−0.10	−0.15	0.187	0.109	0.023	0.166	0.090	0.018	0.166	0.084	0.023	**0.210**	**0.118**	**0.037**
	−0.25	0.244	0.145	0.041	0.211	0.120	0.029	0.209	0.113	0.032	**0.258**	**0.158**	**0.053**
	$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 4,\ \delta = 1$												
		C			C/T			C/S			Φ_{SETAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.118	0.061	0.012	0.121	0.056	0.010	**0.135**	**0.068**	**0.013**	0.122	0.064	0.012
	−0.15	0.120	0.061	0.011	0.122	0.065	0.012	**0.136**	**0.069**	**0.013**	0.124	0.064	0.012
	−0.25	0.119	0.061	0.012	**0.136**	**0.071**	**0.015**	0.135	0.068	0.012	0.124	0.064	0.013
−0.05	−0.10	0.122	0.061	0.012	0.131	0.066	0.013	**0.135**	**0.068**	**0.014**	0.125	0.064	0.012
	−0.25	0.123	0.061	0.011	**0.167**	**0.090**	**0.018**	0.134	0.070	0.012	0.125	0.066	0.014
−0.10	−0.15	0.123	0.063	0.012	**0.179**	**0.102**	**0.022**	0.137	0.069	0.012	0.126	0.067	0.014
	−0.25	0.123	0.063	0.012	**0.217**	**0.132**	**0.031**	0.137	0.070	0.012	0.126	0.067	0.014
	$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 4$												
		C			C/T			C/S			Φ_{SETAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.247	0.176	0.096	0.206	0.141	0.061	**0.339**	**0.262**	**0.148**	0.111	0.057	0.011
	−0.15	0.275	0.192	0.108	0.225	0.160	0.069	**0.383**	**0.297**	**0.169**	0.112	0.058	0.013
	−0.25	0.283	0.200	0.110	0.235	0.165	0.076	**0.408**	**0.313**	**0.186**	0.118	0.062	0.015
−0.05	−0.10	0.280	0.203	0.117	0.232	0.162	0.072	**0.398**	**0.307**	**0.176**	0.116	0.058	0.011
	−0.25	0.316	0.231	0.131	0.261	0.180	0.082	**0.465**	**0.362**	**0.220**	0.130	0.068	0.017
−0.10	−0.15	0.335	0.249	0.138	0.261	0.189	0.091	**0.505**	**0.398**	**0.240**	0.134	0.069	0.017
	−0.25	0.365	0.282	0.162	0.295	0.208	0.107	**0.565**	**0.450**	**0.282**	0.143	0.081	0.022

Note: C, C/T and C/S denote the structural break models in (3). Φ_{SETAR} denotes the Enders-Siklos cointegration test with SETAR adjustment. The table is based on 2,500 replications of the DGP described in (13) with sample size $T = 100$. The breakpoint occurs mid-sample, i.e. $\tau = 0.5$. The test with the highest rejection rates is highlighted in boldface.

Table 7: Size-adjusted power of the sup F test (MTAR) under structural change and asymmetric adjustment

$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 2$													
		C			C/T			C/S			Φ_{MTAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.090	0.043	0.007	0.096	0.045	0.008	0.094	0.050	0.008	**0.112**	**0.054**	**0.013**
	−0.15	0.098	0.050	0.008	0.117	0.064	0.014	0.107	0.054	0.009	**0.206**	**0.109**	**0.030**
	−0.25	0.135	0.073	0.014	0.152	0.082	0.016	0.142	0.072	0.013	**0.378**	**0.224**	**0.066**
−0.05	−0.10	0.090	0.048	0.009	0.115	0.059	0.014	0.099	0.056	0.012	**0.168**	**0.091**	**0.024**
	−0.25	0.152	0.077	0.015	0.161	0.089	0.017	0.153	0.081	0.015	**0.430**	**0.269**	**0.083**
−0.10	−0.15	0.115	0.064	0.012	0.139	0.074	0.017	0.129	0.067	0.013	**0.323**	**0.192**	**0.055**
	−0.25	0.189	0.100	0.019	0.198	0.106	0.020	0.192	0.110	0.016	**0.557**	**0.365**	**0.119**
Size:		0.085	0.042	0.007	0.074	0.039	0.006	0.093	0.041	0.009	0.109	0.060	0.014
$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 2$													
		C			C/T			C/S			Φ_{MTAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.090	0.044	0.009	**0.110**	**0.057**	0.010	0.099	0.051	0.012	0.108	0.051	**0.013**
	−0.15	0.104	0.053	0.013	0.117	0.064	0.010	0.106	0.050	0.012	**0.155**	**0.083**	**0.014**
	−0.25	0.151	0.070	0.018	0.151	0.086	0.014	0.146	0.074	0.014	**0.233**	**0.128**	**0.033**
−0.05	−0.10	0.151	0.070	0.018	0.151	0.086	0.014	0.146	0.074	0.014	**0.233**	**0.128**	**0.033**
	−0.25	0.166	0.078	0.017	0.159	0.091	0.016	0.160	0.078	0.017	**0.255**	**0.142**	**0.039**
−0.10	−0.15	0.134	0.067	0.015	0.136	0.073	0.015	0.143	0.066	0.013	**0.212**	**0.115**	**0.026**
	−0.25	0.205	0.105	0.023	0.196	0.103	0.025	0.194	0.104	0.021	**0.305**	**0.178**	**0.047**
$\mu_1 = 1,\ \mu_2 = 1,\ \alpha_1 = 2,\ \alpha_2 = 4,\ \delta = 1$													
		C			C/T			C/S			Φ_{MTAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.099	0.050	0.006	0.103	0.049	0.010	0.114	0.053	0.006	**0.120**	**0.062**	**0.013**
	−0.15	0.102	0.049	0.006	**0.129**	0.063	**0.015**	0.114	0.053	0.006	0.120	**0.065**	0.014
	−0.25	0.102	0.047	0.006	**0.162**	**0.088**	**0.019**	0.110	0.055	0.006	0.123	0.063	0.014
−0.05	−0.10	0.101	0.047	0.006	**0.119**	**0.064**	**0.015**	0.113	0.053	0.007	0.114	0.057	0.013
	−0.25	0.101	0.048	0.007	**0.175**	**0.096**	**0.022**	0.112	0.054	0.006	0.123	0.064	0.015
−0.10	−0.15	0.102	0.047	0.007	**0.146**	**0.078**	**0.016**	0.112	0.054	0.007	0.122	0.064	0.015
	−0.25	0.103	0.048	0.007	**0.197**	**0.113**	**0.028**	0.112	0.057	0.006	0.123	0.065	0.014
$\mu_1 = 1,\ \mu_2 = 4,\ \alpha_1 = 2,\ \alpha_2 = 4$													
		C			C/T			C/S			Φ_{MTAR}		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.196	0.132	0.073	0.192	0.135	0.060	**0.303**	**0.232**	**0.113**	0.085	0.040	0.009
	−0.15	0.219	0.156	0.082	0.223	0.154	0.071	**0.367**	**0.281**	**0.140**	0.096	0.050	0.009
	−0.25	0.256	0.193	0.107	0.250	0.173	0.084	**0.435**	**0.330**	**0.181**	0.122	0.063	0.013
−0.05	−0.10	0.213	0.156	0.082	0.219	0.145	0.070	**0.348**	**0.262**	**0.136**	0.093	0.046	0.010
	−0.25	0.273	0.204	0.115	0.257	0.182	0.090	**0.458**	**0.350**	**0.200**	0.126	0.069	0.016
−0.10	−0.15	0.265	0.191	0.106	0.240	0.175	0.084	**0.422**	**0.326**	**0.180**	0.115	0.062	0.012
	−0.25	0.301	0.220	0.132	0.282	0.202	0.100	**0.503**	**0.397**	**0.228**	0.134	0.079	0.018

Note: C, C/T and C/S denote the structural break models in (3). Φ_{MTAR} denotes the threshold cointegration test with MTAR adjustment. The table is based on 2,500 replications of the DGP described in (13) with sample size $T = 100$. The breakpoint occurs mid-sample, i.e. $\tau = 0.5$. The test with the highest rejection rates is highlighted in boldface.

Table 8: Size-adjusted power of the GH test under structural change and SETAR adjustment

	$\mu_1 = 1$, $\mu_2 = 4$, $\alpha_1 = 2$, $\alpha_2 = 2$									
		C			C/T			C/S		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.114	0.062	0.011	0.110	0.052	0.010	0.117	0.058	0.014
	−0.15	0.129	0.069	0.015	0.116	0.060	0.012	0.135	0.061	0.016
	−0.25	0.139	0.083	0.018	0.128	0.068	0.014	0.153	0.069	0.018
−0.05	−0.10	0.136	0.071	0.018	0.124	0.066	0.012	0.135	0.066	0.016
	−0.25	0.174	0.101	0.023	0.159	0.086	0.019	0.175	0.087	0.019
−0.10	−0.15	0.192	0.108	0.030	0.177	0.097	0.022	0.184	0.093	0.021
	−0.25	0.247	0.150	0.037	0.208	0.125	0.032	0.239	0.128	0.029
	$\mu_1 = 1$, $\mu_2 = 1$, $\alpha_1 = 2$, $\alpha_2 = 4$, $\delta = 1$									
		C			C/T			C/S		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.114	0.056	0.008	0.110	0.052	0.010	0.131	0.059	0.009
	−0.15	0.113	0.055	0.008	0.116	0.060	0.012	0.130	0.058	0.010
	−0.25	0.114	0.055	0.008	0.128	0.068	0.014	0.130	0.058	0.010
−0.05	−0.10	0.114	0.058	0.009	0.124	0.066	0.012	0.129	0.058	0.009
	−0.25	0.114	0.058	0.009	0.159	0.086	0.019	0.131	0.061	0.008
−0.10	−0.15	0.114	0.059	0.009	0.177	0.097	0.022	0.133	0.058	0.009
	−0.25	0.114	0.059	0.009	0.208	0.125	0.032	0.134	0.060	0.009
	$\mu_1 = 1$, $\mu_2 = 4$, $\alpha_1 = 2$, $\alpha_2 = 4$									
		C			C/T			C/S		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.202	0.134	0.060	0.182	0.104	0.039	0.293	0.197	0.088
	−0.15	0.221	0.154	0.064	0.192	0.113	0.042	0.330	0.222	0.100
	−0.25	0.236	0.165	0.069	0.204	0.126	0.047	0.348	0.242	0.108
−0.05	−0.10	0.230	0.152	0.072	0.199	0.119	0.047	0.346	0.237	0.112
	−0.25	0.263	0.186	0.087	0.229	0.140	0.056	0.402	0.292	0.137
−0.10	−0.15	0.279	0.198	0.096	0.238	0.147	0.060	0.445	0.315	0.161
	−0.25	0.307	0.226	0.109	0.262	0.171	0.072	0.510	0.379	0.193

Note: C, C/T and C/S denote the structural break models in (3). The table is based on 2,500 replications of the DGP described in (13) with sample size $T = 100$. The breakpoint occurs mid-sample, i.e. $\tau = 0.5$.

Table 9: Size-adjusted power of the GH test under structural change and MTAR adjustment

	$\mu_1 = 1$, $\mu_2 = 4$, $\alpha_1 = 2$, $\alpha_2 = 2$									
		C			C/T			C/S		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.110	0.055	0.011	0.106	0.051	0.010	0.110	0.048	0.013
	−0.15	0.140	0.074	0.017	0.133	0.068	0.015	0.139	0.067	0.016
	−0.25	0.198	0.116	0.026	0.177	0.103	0.025	0.195	0.096	0.022
−0.05	−0.10	0.134	0.069	0.014	0.124	0.062	0.012	0.125	0.062	0.015
	−0.25	0.223	0.132	0.032	0.191	0.112	0.029	0.214	0.109	0.023
−0.10	−0.15	0.184	0.105	0.024	0.165	0.090	0.022	0.177	0.086	0.020
	−0.25	0.281	0.164	0.049	0.229	0.134	0.038	0.248	0.136	0.031
	$\mu_1 = 1$, $\mu_2 = 1$, $\alpha_1 = 2$, $\alpha_2 = 4$, $\delta = 1$									
		C			C/T			C/S		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.115	0.055	0.008	0.106	0.051	0.010	0.129	0.057	0.010
	−0.15	0.117	0.058	0.008	0.133	0.068	0.015	0.132	0.057	0.009
	−0.25	0.120	0.058	0.008	0.177	0.103	0.025	0.132	0.058	0.008
−0.05	−0.10	0.116	0.057	0.009	0.124	0.062	0.012	0.131	0.057	0.009
	−0.25	0.118	0.059	0.008	0.191	0.112	0.029	0.134	0.056	0.008
−0.10	−0.15	0.117	0.058	0.009	0.165	0.090	0.022	0.132	0.056	0.009
	−0.25	0.116	0.058	0.008	0.229	0.134	0.038	0.137	0.057	0.010
	$\mu_1 = 1$, $\mu_2 = 4$, $\alpha_1 = 2$, $\alpha_2 = 4$									
		C			C/T			C/S		
ρ_1	ρ_2	10%	5%	1%	10%	5%	1%	10%	5%	1%
−0.025	−0.05	0.195	0.130	0.062	0.176	0.104	0.040	0.297	0.201	0.088
	−0.15	0.225	0.155	0.075	0.201	0.120	0.050	0.359	0.245	0.116
	−0.25	0.279	0.192	0.094	0.236	0.149	0.061	0.450	0.329	0.157
−0.05	−0.10	0.219	0.153	0.074	0.198	0.113	0.049	0.343	0.236	0.111
	−0.25	0.293	0.205	0.100	0.247	0.159	0.063	0.481	0.360	0.175
−0.10	−0.15	0.269	0.190	0.096	0.233	0.144	0.059	0.435	0.312	0.156
	−0.25	0.322	0.231	0.115	0.270	0.175	0.072	0.549	0.400	0.211

Note: C, C/T and C/S denote the structural break models in (3). The table is based on 2,500 replications of the DGP described in (13) with sample size $T = 100$. The breakpoint occurs mid-sample, i.e. $\tau = 0.5$.

Table 10: Long-run adjustment along the gasoline value-chain

SETAR

Panel (a): No structural break

	μ		α		ρ^+	ρ^-	Φ_{SETAR}	$\rho^+ = \rho^-$	
(I)	5.49		1.145		−0.225	−0.153	3.97	-	
(II)	79.50		0.960		−0.567	−0.887	21.28***	2.029***	
(S)	76.38		1.141		−0.251	−0.326	7.20**	0.245	

Panel (b): Structural break model C/S

	μ_1	μ_2	α_1	α_2	ρ^+	ρ^-	F^*_{SETAR}	$\rho^+ = \rho^-$	break
(I)	32.54	90.52	0.932	−0.216	−0.578	−0.551	14.79**	0.017	01/11
(II)	60.49	22.38	1.062	−0.123	−0.630	−1.018	25.93***	2.817**	10/08
(S)	93.80	195.44	0.989	−0.698	−0.453	−0.588	16.66***	0.549	02/11

MTAR

Panel (c): No structural break

	μ		α		ρ^+	ρ^-	Φ_{MTAR}	$\rho^+ = \rho^-$	
(I)	5.49		1.140		−0.162	−0.243	3.95	-	
(II)	79.50		0.960		−0.437	−0.871	21.51***	3.647**	
(S)	76.38		1.140		−0.226	−0.333	7.21**	0.510	

Panel (d): Structural break model C/S

	μ_1	μ_2	α_1	α_2	ρ^+	ρ^-	F^*_{MTAR}	$\rho^+ = \rho^-$	break
(I)	32.86	81.55	0.930	−0.179	−0.634	−0.406	16.97**	1.474	12/10
(II)	62.92	20.31	1.046	−0.106	−0.453	−0.993	26.04***	5.544***	09/08
(S)	93.80	195.44	0.989	−0.698	−0.448	−0.556	16.14**	0.367	02/11

Note: μ (α) denotes the intercept (slope coefficient) of the long-run equilibrium equation without structural break. μ_1 (α_1) and μ_2 (α_2) denote the intercept (slope coefficient) of the long-run equilibrium equation before the break and after the break, respectively. δ is the linear trend coefficient. Φ_{SETAR} and Φ_{MTAR} denote the F-statistic based on the null hypothesis $H_0 : \rho^+ = \rho^- = 0$, respectively. We conduct bootstrap F-tests with 600 replications to test the null hypothesis $\rho^+ = \rho^-$.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

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